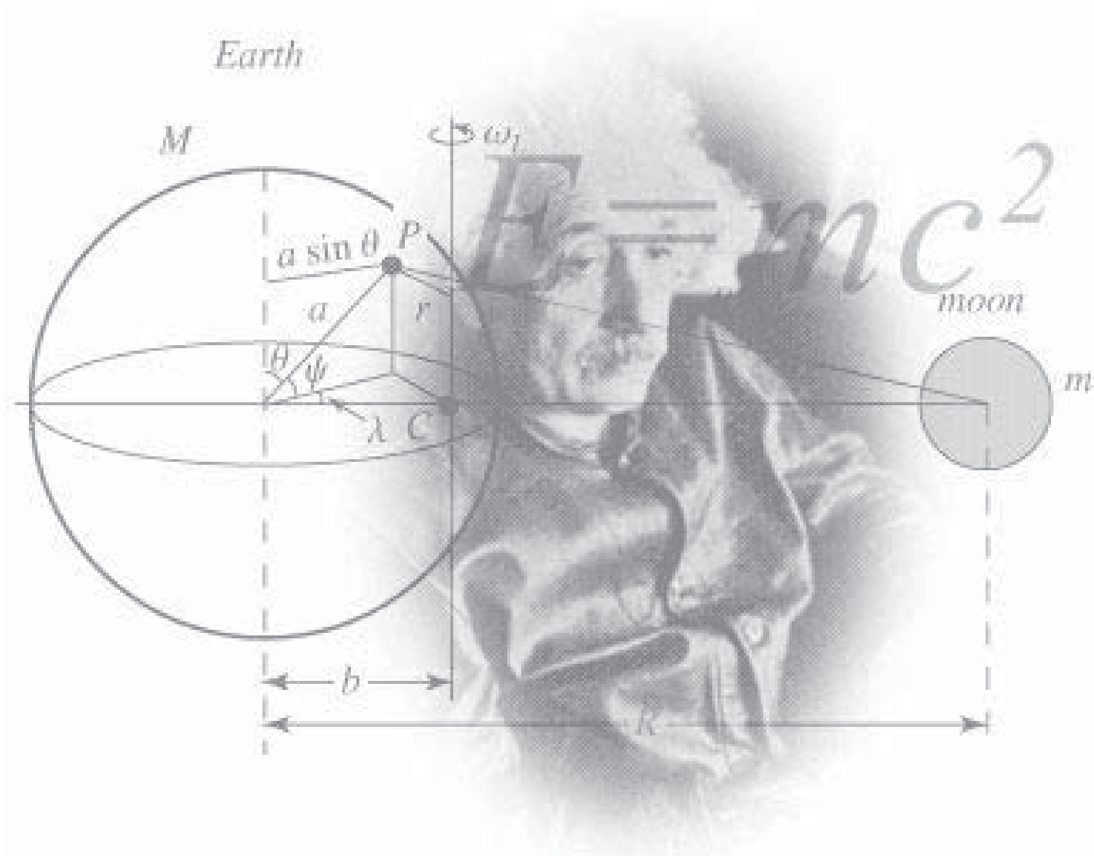
AP Physics 1 – Practice Workbook – Book 1

Mechanics, Waves and Sound, Electrostatics and DC Circuits



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IMPORTANT:

This book is a compilation of all the problems published by College Board in AP Physics B and AP Physics C that **were** appropriate for the AP B level as well as problems from AAPT's Physics Bowl and U.S. Physics Team Qualifying Exams organized by topic.

DISCLAIMER

The Multiple Choice Questions in this workbook have been compiled and modified from previous AP Physics B and C examinations and Physics Bowl exams. They are **not** meant to be representative of the new AP Physics courses.

The Free-Response Questions have not been edited and might not represent the topics covered nor the style of questions in the new exams.

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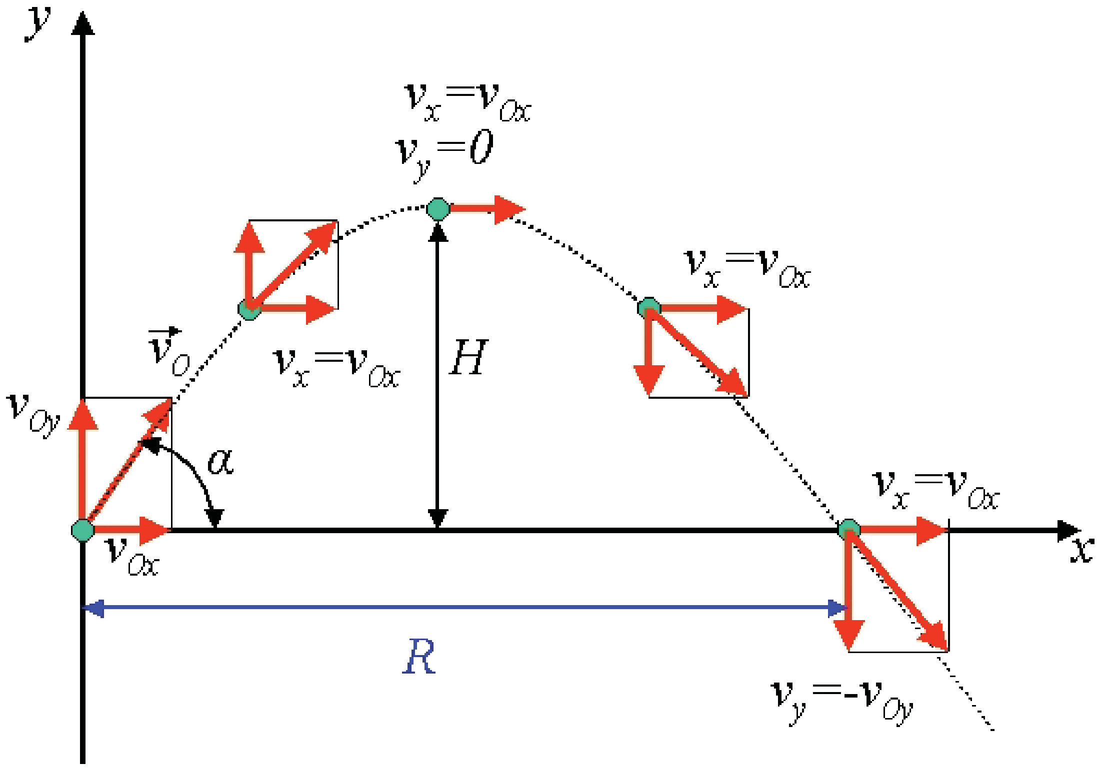
The answers as presented are not the only method to solving many of these problems and physics teachers may present slightly different methods and/or different symbols and variables in each topic, but the underlying physics concepts are the same and we ask you read the solutions with an open mind and use these differences to expand your problem solving skills.

Finally, we *are* fallible and if you find any typographical errors, formatting errors or anything that strikes you as unclear or unreadable, please let us know so we can make the necessary announcements and corrections.

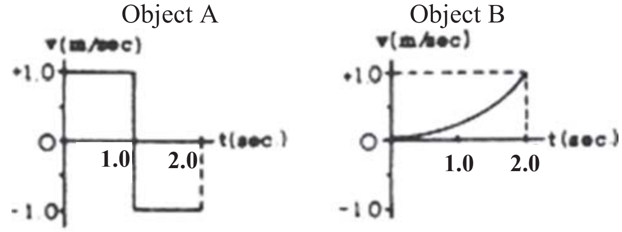
Chapter 1

Kinematics



AP Physics Multiple Choice Practice – Kinematics

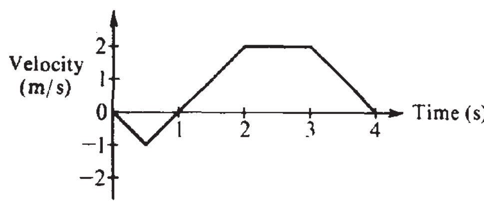
Questions 1 – 3 relate to two objects that start at x = 0 at t = 0 and move in one dimension independently of one another. Graphs, of the velocity of each object versus time are shown below



1. Which object is farthest from the origin at t = 2 seconds.
(A) A (B) B (C) they are in the same location at t = 2 seconds (D) They are the same distance from the origin, but in opposite directions

2. Which object moves with constant non-zero acceleration?
(A) A (B) B (C) both A and B (D) neither A nor B

3. Which object is in its initial position at t = 2 seconds?
(A) A (B) B (C) both A and B (D) neither A nor B

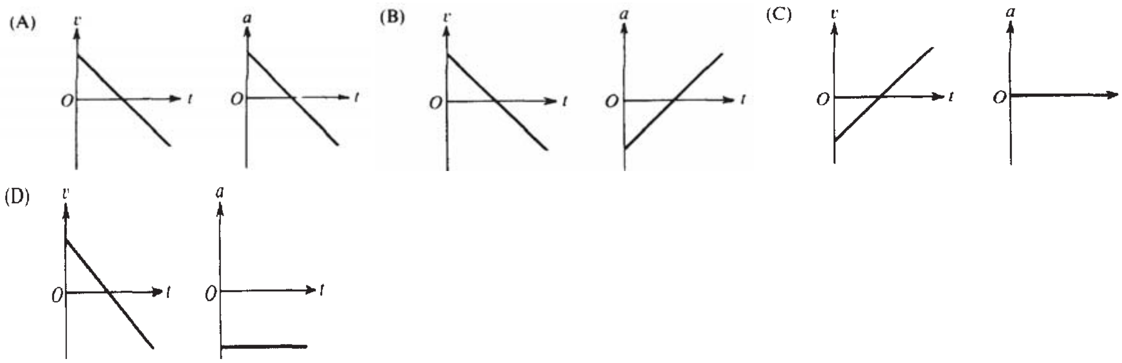


4. The graph above shows the velocity versus time for an object moving in a straight line. At what time after t = 0 does the object again pass through its initial position?
(A) 1 s (B) Between 1 and 2 s (C) 2 s (D) Between 2 and 3 s

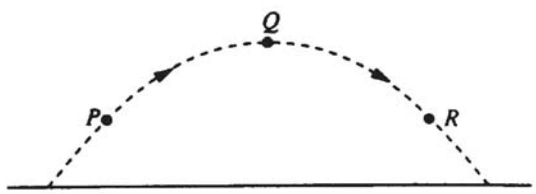
5. A body moving in the positive x direction passes the origin at time t = 0. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At t = 1 second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at t = 11 seconds is
(A) + 99m (B) + 36m (C) – 36 m (E) – 99 m

6. A diver initially moving horizontally with speed v dives off the edge of a vertical cliff and lands in the water a distance *d* from the base of the cliff. How far from the base of the cliff would the diver have landed if the diver initially had been moving horizontally with speed 2v?
(A) *d* (B) $\sqrt{2d}$ (C) *2d* (D) *4d*

7. A projectile is fired with initial velocity v_o at an angle θ_0 with the horizontal and follows the trajectory shown above. Which of the following pairs of graphs best represents the vertical components of the velocity and acceleration, v and a, respectively, of the projectile as functions of time t?



Questions 8-9

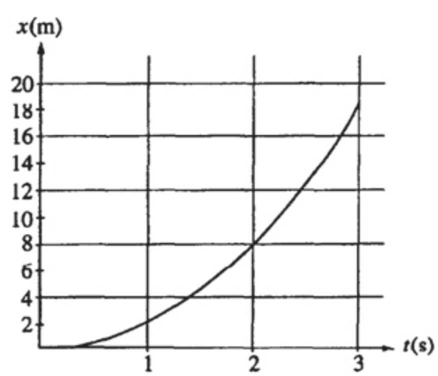


A ball is thrown and follows the parabolic path shown above. Air friction is negligible. Point Q is the highest point on the path. Points P and R are the same height above the ground.

8. How do the speeds of the ball at the three points compare?
(A) $v_P < v_Q < v_R$ (B) $v_R < v_Q < v_P$ (C) $v_Q < v_R < v_P$ (D) $v_Q < v_P = v_R$

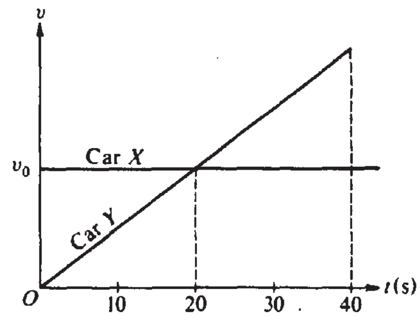
9. Which of the following diagrams best shows the direction of the acceleration of the ball at point P?

(A) ↗ (B) → (C) ↘ (D) ↓



10. The graph above represents position x versus time t for an object being acted on by a constant force. The average speed during the interval between 1 s and 2 s is most nearly
(A) 2 m/s (B) 4 m/s (C) 5 m/s (D) 6 m/s

Questions 11 – 12

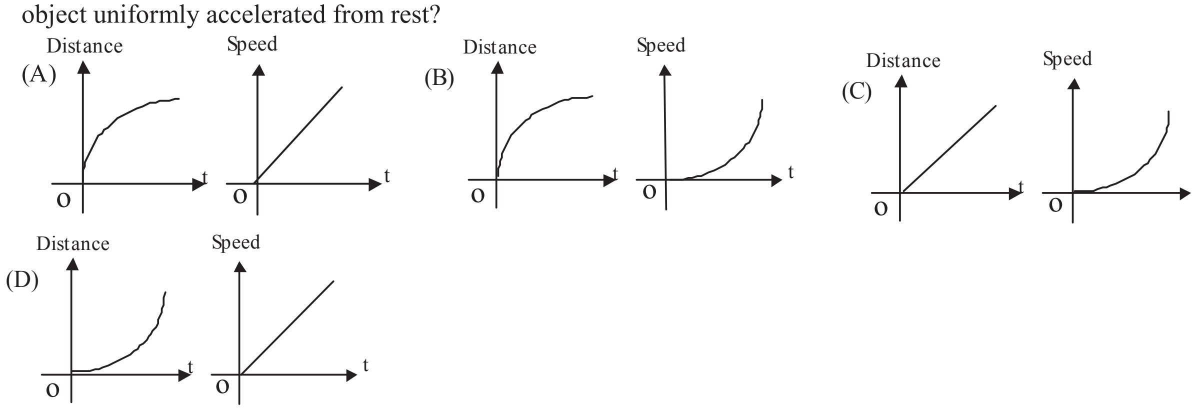


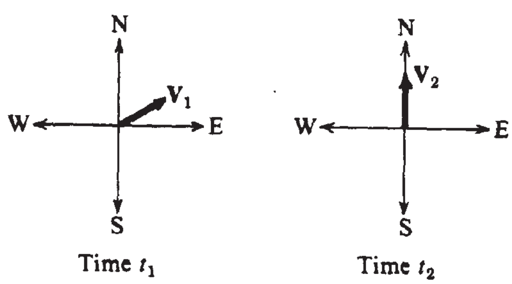
At time t = 0, car X traveling with speed v_0 passes car Y which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above.

11. Which of the following is true at time t = 20 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Car X is accelerating faster than car Y.

12. From time t = 0 to time t = 40 seconds, the areas under both curves are equal. Therefore, which of the following is true at time t = 40 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(d) Car X is accelerating faster than car Y.

13. Which of the following pairs of graphs shows the distance traveled versus time and the speed versus time for an object uniformly accelerated from rest?

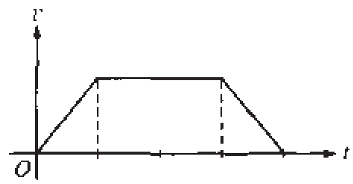




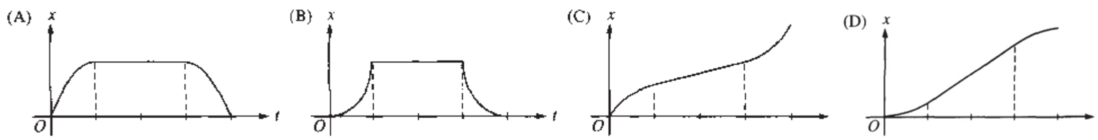
14. Vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ shown above have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. The average acceleration of the object between time t_1 and t_2 was
(A) directed north (B) directed west (C) directed north of east (D) directed north of west

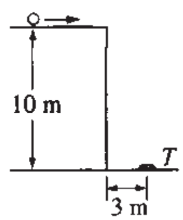
15. The velocity of a projectile at launch has a horizontal component v_h and a vertical component v_v. Air resistance is negligible. When the projectile is at the highest point of its trajectory, which of the following shows the vertical and horizontal components of its velocity and the vertical component of its acceleration?

	Vertical Velocity	Horizontal Velocity	Vertical Acceleration
(A)	v_v	v_h	0
(B)	0	v_h	0
(C)	0	0	g
(D)	0	v_h	g



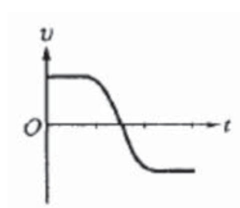
16. The graph above shows the velocity *v* as a function of time *t* for an object moving in a straight line. Which of the following graphs shows the corresponding displacement *x* as a function of time *t* for the same time interval?



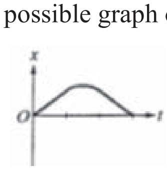
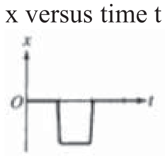
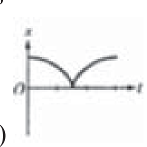
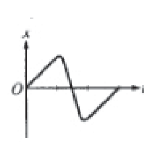


17. A target *T* lies flat on the ground 3 m from the side of a building that is 10 m tall, as shown above. A student rolls a ball off the horizontal roof of the building in the direction of the target. Air resistance is negligible. The horizontal speed with which the ball must leave the roof if it is to strike the target is most nearly

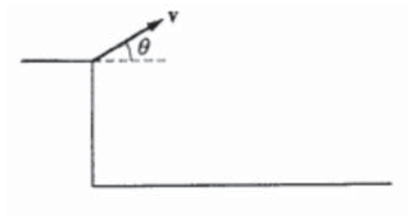
(A) 3/10 m/s (B) $\sqrt{2}$ m/s (C) $\frac{3}{\sqrt{2}}$ m/s (D) 3 m/s



18. The graph above shows velocity v versus time t for an object in linear motion. Which of the following is a possible graph of position x versus time t for this object?

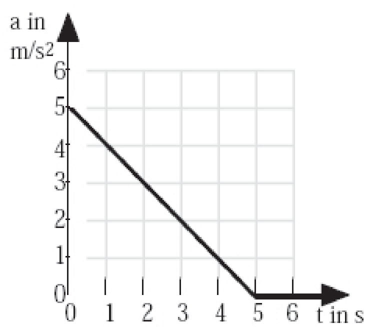
(A)  (B)  (C)  (D) 

19. A student is testing the kinematic equations for uniformly accelerated motion by measuring the time it takes for light-weight plastic balls to fall to the floor from a height of 3 m in the lab. The student predicts the time to fall using g as 9.80 m/s^2 but finds the measured time to be 35% greater. Which of the following is the most likely cause of the large percent error?
(A) The acceleration due to gravity is 70% greater than 9.80 m/s^2 at this location.
(B) The acceleration due to gravity is 70% less than 9.80 m/s^2 at this location.
(C) Air resistance increases the downward acceleration.
(D) The acceleration of the plastic balls is not uniform.



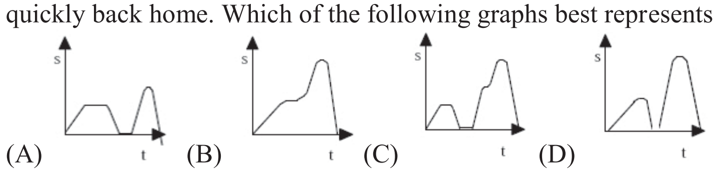
Note: Figure not drawn to scale.

20. An object is thrown with velocity **v** from the edge of a cliff above level ground. Neglect air resistance. In order for the object to travel a maximum horizontal distance from the cliff before hitting the ground, the throw should be at an angle θ with respect to the horizontal of
(A) greater than 60° above the horizontal
(B) greater than 45° but less than 60° above the horizontal
(C) greater than zero but less than 45° above the horizontal
(D) greater than zero but less than 45° below the horizontal



21. Starting from rest at time t = 0, a car moves in a straight line with an acceleration given by the accompanying graph. What is the speed of the car at t = 3 s?
(A) 1.0 m/s (B) 2.0 m/s (C) 6.0 m/s (D) 10.5 m/s

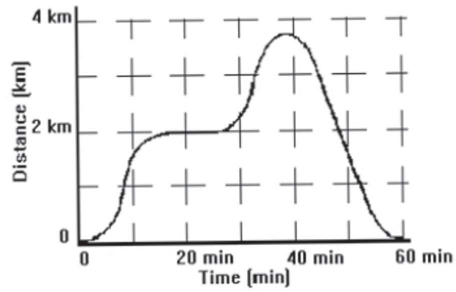
22. A child left her home and started walking at a constant velocity. After a time she stopped for a while and then continued on with a velocity greater than she originally had. All of a sudden she turned around and walked very quickly back home. Which of the following graphs best represents the distance versus time graph for her walk?



(A) (B) (C) (D)

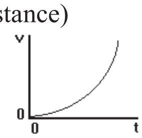
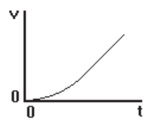
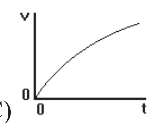
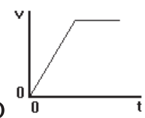
23. A whiffle ball is tossed straight up, reaches a highest point, and falls back down. Air resistance is not negligible. Which of the following statements are true?
I. The ball's speed is zero at the highest point.
II. The ball's acceleration is zero at the highest point.
III. The ball takes a longer time to travel up to the highest point than to fall back down.

(A) I only (B) II only (C) I & II only (D) I & III only



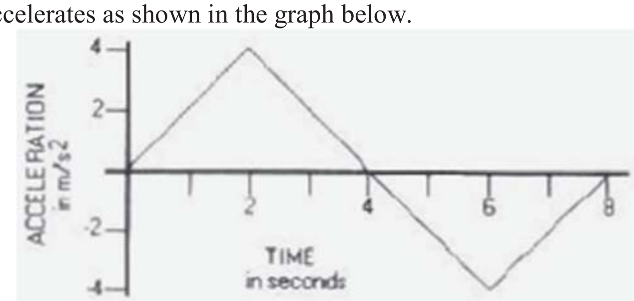
24. Above is a graph of the distance vs. time for car moving along a road. According the graph, at which of the following times would the automobile have been accelerating positively?
(A) 0, 20, 38, & 60 min. (B) 5, 12, 29, & 35 min. (C) 5, 29, & 57 min. (D) 12, 35, & 41 min.

25. A large beach ball is dropped from the ceiling of a school gymnasium to the floor about 10 meters below. Which of the following graphs would best represent its velocity as a function of time? (do not neglect air resistance)

(A)  (B)  (C)  (D 

Questions 26-27

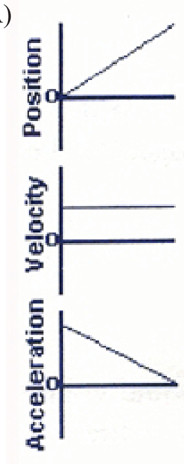
A car starts from rest and accelerates as shown in the graph below.

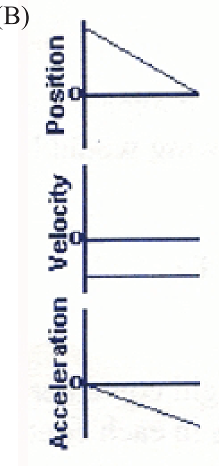


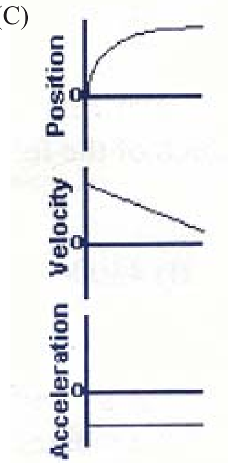
26. At what time would the car be moving with the greatest velocity?
(B) 2 seconds (C) 4 seconds (D) 6 seconds (E) 8 seconds

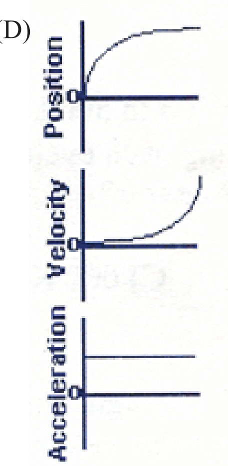
27. At what time would the car be farthest from its original starting position?
(A) 2 seconds (B) 4 seconds (C) 6 seconds (D) 8 seconds

28. Which of the following sets of graphs might be the corresponding graphs of Position, Velocity, and Acceleration vs. Time for a moving particle?

(A) 

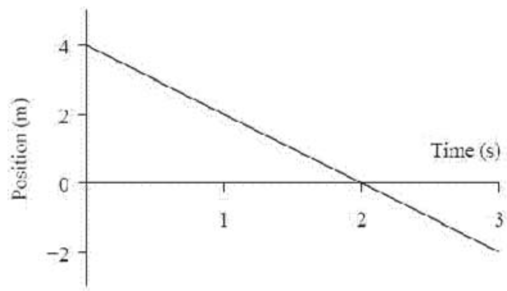
(B) 

(C) 

(D) 

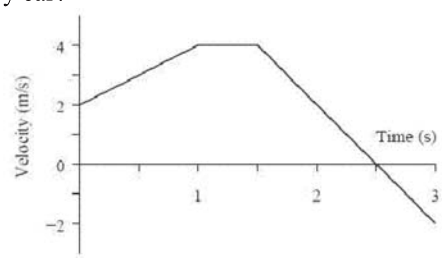
29. An object is thrown with a fixed initial speed v_0 at various angles α relative to the horizon. At some constant height h above the launch point the speed v of the object is measured as a function of the initial angle α. Which of the following best describes the dependence of v on α? (Assume that the height h is achieved, and assume that there is no air resistance.)
(A) v will increase monotonically with α.
(B) v will increase to some critical value v_{max} and then decrease.
(C) v will remain constant, independent of α.
(D) v will decrease to some critical value v_{min} and then increase.

30. The position vs. time graph for an object moving in a straight line is shown below. What is the instantaneous velocity at $t = 2$ s?



(A) – 2 m/s (B) ½ m/s (C) 0 m/s (D) 2 m/s

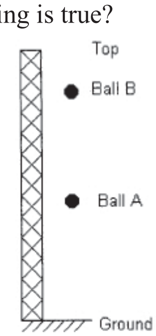
31. Shown below is the velocity vs. time graph for a toy car moving along a straight line. What is the maximum displacement from start for the toy car?



(A) 5 m (B) 6.5 m (C) 7 m (D) 7.5 m

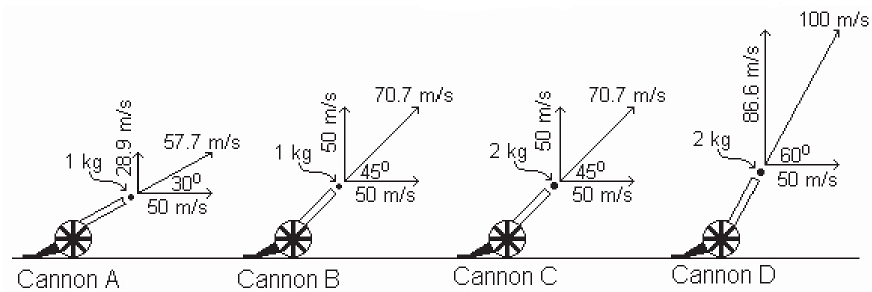
32. An object is released from rest and falls a distance h during the first second of time. How far will it fall during the next second of time?
(A) h (B) $2h$ (C) $3h$ (D) $4h$

33. Two identical bowling balls A and B are each dropped from rest from the top of a tall tower as shown in the diagram below. Ball A is dropped 1.0 s before ball B is dropped but both balls fall for some time before ball A strikes the ground. Air resistance can be considered negligible during the fall. After ball B is dropped but before ball A strikes the ground, which of the following is true?



(A) The distance between the two balls decreases.
(B) The velocity of ball A increases with respect to ball (B)
(C) The velocity of ball A decreases with respect to ball (B)
(D) The distance between the two balls increases.

34. The diagram below shows four cannons firing shells with different masses at different angles of elevation. The horizontal component of the shell's velocity is the same in all four cases. In which case will the shell have the greatest range if air resistance is neglected?



(A) cannon A (B) cannon B only (C) cannon C only (D) cannon D

35. Starting from rest, object 1 falls freely for 4.0 seconds, and object 2 falls freely for 8.0 seconds. Compared to object 1, object 2 falls:
(A) half as far (B) twice as far (C) three times as far (D) four times as far

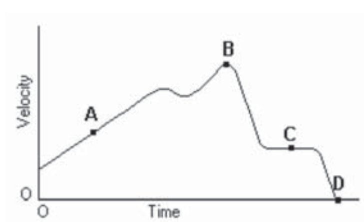
36. A car starts from rest and uniformly accelerates to a final speed of 20.0 m/s in a time of 15.0 s. How far does the car travel during this time?
(A) 150 m (B) 300 m (C) 450 m (D) 600 m

37. An arrow is aimed horizontally, directly at the center of a target 20 m away. The arrow hits 0.050 m below the center of the target. Neglecting air resistance, what was the initial speed of the arrow?
(A) 20 m/s (B) 40 m/s (C) 100 m/s (D) 200 m/s

38. A rocket near the surface of the earth is accelerating vertically upward at 10 m/s^2. The rocket releases an instrument package. Immediately after release the acceleration of the instrument package is:
(A) 20 m/s^2 up (B) 10 m/s^2 up (C) 0 (D) 10 m/s^2 down

39. A ball which is dropped from the top of a building strikes the ground with a speed of 30 m/s. Assume air resistance can be ignored. The height of the building is approximately:
(A) 15 m (B) 30 m (C) 45 m (D) 75 m

40. In the absence of air resistance, if an object were to fall freely near the surface of the Moon,
(A) its acceleration would gradually decrease until the object moves with a terminal velocity.
(B) the acceleration is constant.
(C) it will fall with a constant speed.
(D) the acceleration is zero

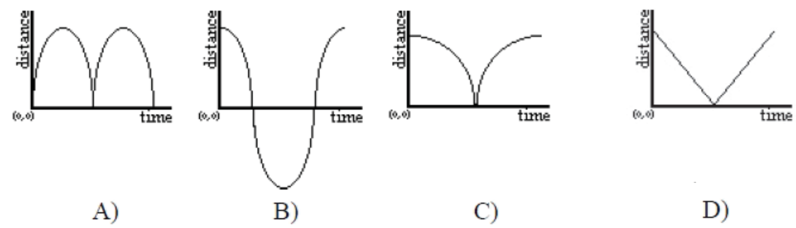


41. Given the graph of the velocity vs. time of a duck flying due south for the winter. At what point did the duck stop its forward motion?
(A) A (B) B (C) C (D) D

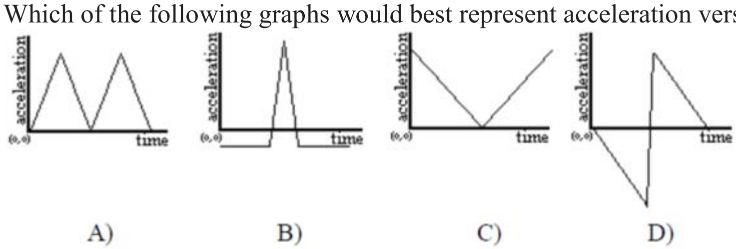
Questions 42-43

The following TWO questions refer to the following information. An ideal elastic rubber ball is dropped from a height of about 2 meters, hits the floor and rebounds to its original height.

42. Which of the following graphs would best represent the distance above the floor versus time for the above bouncing ball?

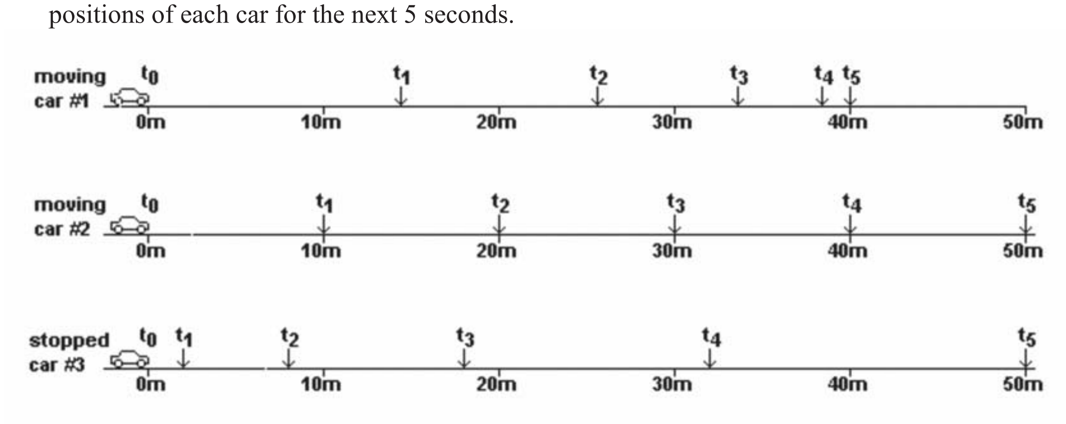


43. Which of the following graphs would best represent acceleration versus time for the bouncing ball?



Questions 44-46

The following TWO questions refer to the following information. At t_0, two cars moving along a highway are side-by-side as they pass a third car stopped on the side of the road. At this moment the driver of the first car steps on the brakes while the driver of the stopped car begins to accelerate. The diagrams below show the positions of each car for the next 5 seconds.

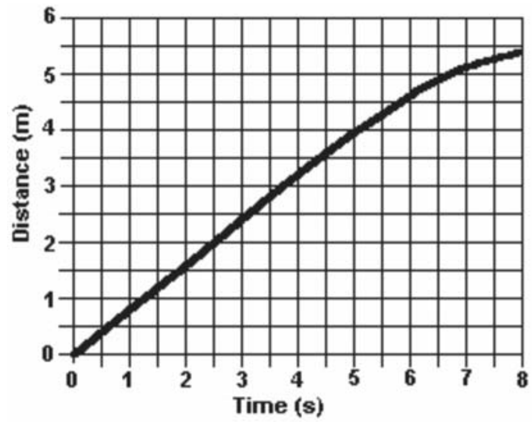


44. During which time interval would cars #2 and #3 be moving at the same average speed?
(A) t_0 to t_1 (B) t_1 to t_2 (C) t_2 to t_3 (D) t_3 to t_4

45. Which of the three cars had the greatest average speed during these 5 seconds?
(A) car #2 and car #3 had the same average speed (B) car #2
(C) all three cars had the same average speed (C) car #3

46. If car #3 continues to constantly accelerate at the same rate what will be its position at the end of 6 seconds?
(A) 24 m (B) 68 m (C) 72 m (D) 78 m

Questions 47-48



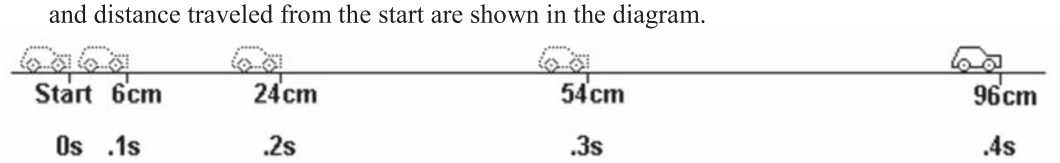
47. The graph represents the relationship between distance and time for an object that is moving along a straight line. What is the instantaneous speed of the object at $t = 5.0$ seconds?
    (A) 0.0 m/s (B) 0.8 m/s (C) 2.5 m/s (D) 4.0 m/s

48. Between what times did the object have a non-zero acceleration?
    (A) 0 s on (B) 0 s to 5 s (C) the object was not accelerating at any time (D) 5 s to 8 s

49. If a ball is thrown directly upwards with twice the initial speed of another, how much higher will it be at its apex?
    (A) 8 times (B) 2 times (C) 4 times (D) 2 times

Questions 50-51

The diagram below represents a toy car starting from rest and uniformly accelerating across the floor. The time and distance traveled from the start are shown in the diagram.

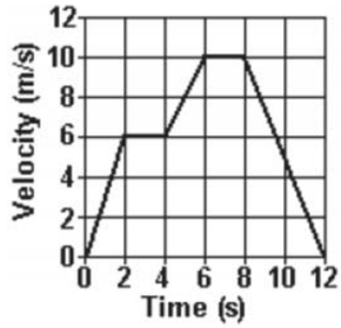


50. What was the acceleration of the cart during the first 0.4 seconds?
    (A) 25 m/s^2 (B) 9.8 m/s^2 (C) 50 m/s^2 (D) 12 m/s^2

51. What was the instantaneous velocity of the cart at 96 centimeters from the start?
    (A) 0.6 m/s (B) 4.8 m/s (C) 1.9 m/s (D) 60 m/s (E) 2.4 m/s

Questions 52-53

The motion of a circus clown on a unicycle moving in a straight line is shown in the graph below



52. What would be the acceleration of the clown at 5 s?
(A) 1.6 m/s^2 (B) 8.0 m/s^2 (C) 2.0 m/s^2 (D) 3.4 m/s^2

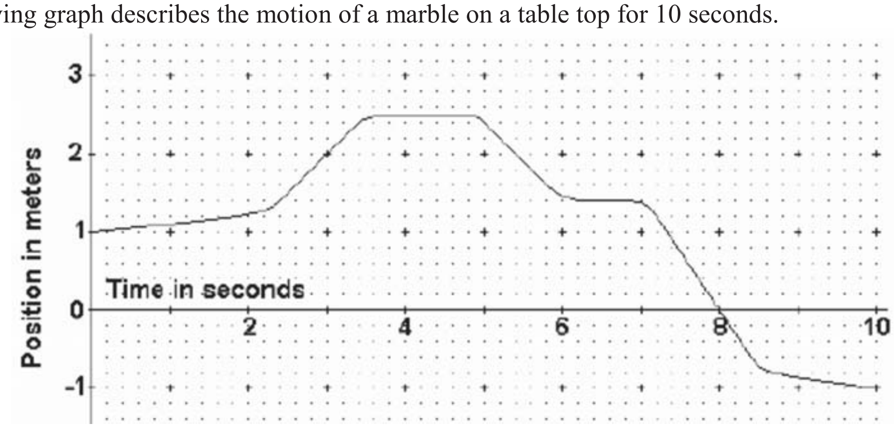
53. After 12 seconds, how far is the clown from her original starting point?
(A) 0 m (B) 10 m (C) 47 m (D) 74 m

54. When an object falls freely in a vacuum near the surface of the earth
(A) the terminal velocity will be greater than when dropped in air
(B) the velocity will increase but the acceleration will be zero
(C) the acceleration will constantly increase
(D) the acceleration will remain constant

55. Two arrows are launched at the same time with the same speed. Arrow A at an angle greater than 45 degrees, and arrow B at an angle less than 45 degrees. Both land at the same spot on the ground. Which arrow arrives first?
(A) arrow A arrives first (B) arrow B arrives first (C) they both arrive together
(D) it depends on the elevation where the arrows land

Questions 56-57

The accompanying graph describes the motion of a marble on a table top for 10 seconds.

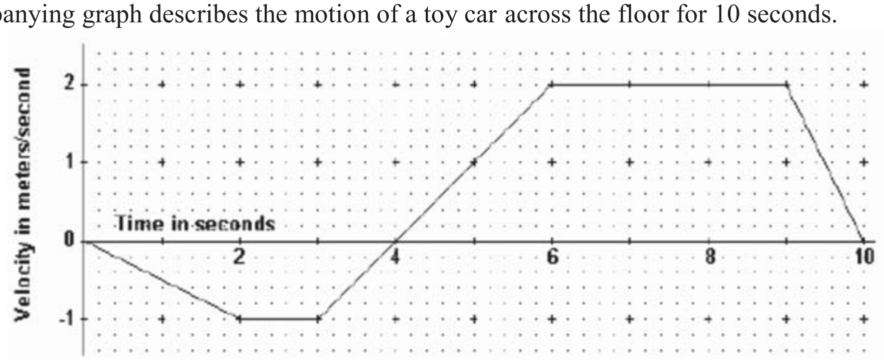


56. For which time interval(s) did the marble have a negative velocity?
(A) from t = 8.0 s to t = 10.0 s only (B) from t = 6.9 s to t = 10.0 s only
(C) from t = 4.8 s to t = 10.0 s only (D) from t = 4.8 s to t = 6.2 s and from t = 6.9 s to t = 10.0 s only

57. For which time interval(s) did the marble have a positive acceleration?
(A) from t = 0.0 s to t = 8.0 s only (B) from t = 0.0 s to t = 3.6 only
(C) from t = 3.8 s to t = 4.8 s and t = 6.2 s to t = 6.8 s only
(D) from t = 2.0 s to t = 2.5 s, from t = 5.8 s to t = 6.2 s, and from t = 8.4 s to t = 8.8 s only

58. What is the marbles average acceleration between t = 3.1 s and t = 3.8 s
(A) -2.0 m/s^2 (B) 0.8 m/s^2 (C) 2.0 m/s^2 (D) 3.0 m/s^2

Questions 59-60
The accompanying graph describes the motion of a toy car across the floor for 10 seconds.



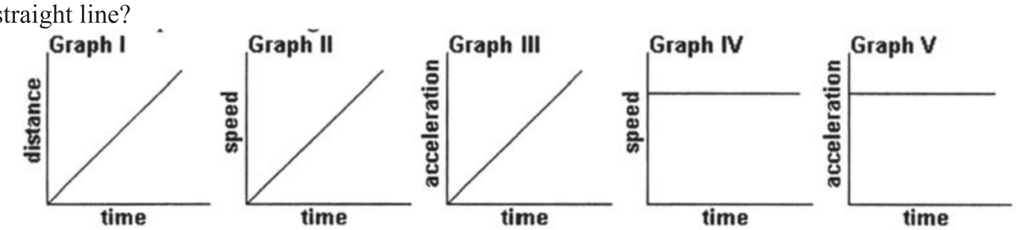
59. What is the acceleration of the toy car at t = 4 s?
(A) –1 m/s^2 (B) 0 m/s^2 (C) 1 m/s^2 (D) 2 m/s^2

60. What was the total displacement of the toy car for the entire 10 second interval shown?
(A) 0 meters (B) 6.5 meters (C) 9 meters (D) 10 meters

61. An object is thrown upwards with a velocity of 30 m/s near the surface of the earth. After two seconds what would be the direction of the displacement, velocity and acceleration?

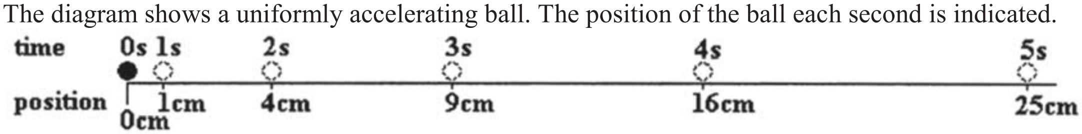
	Displacement	velocity	acceleration
(A)	up	up	up
(B)	up	up	down
(C)	up	down	down
(D)	up	down	up

62. Which of the following graphs could correctly represent the motion of an object moving with a constant speed in a straight line?



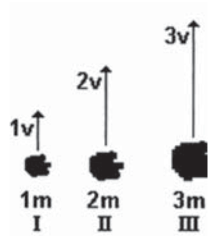
(A) Graph I only (B) Graphs II and V only (C) Graph II only (D) Graphs I and IV only

Questions 63-64
The diagram shows a uniformly accelerating ball. The position of the ball each second is indicated.

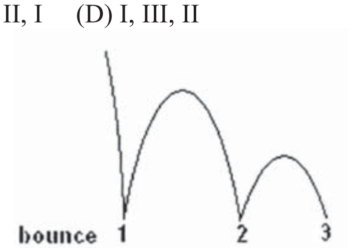


63. What is the average speed of the ball between 3 and 4 seconds?
(A) 3.0 cm/s (B) 7.0 cm/s (C) 3.5 cm/s (D) 12.5 cm/s

64. Which of the following is closest to the acceleration of the ball?
(A) 1 cm/s^2 (B) 4 cm/s^2 (C) 2 cm/s^2 (D) 5 cm/s^2



65. Three stones of different mass (1 *m, 2m* & *3m)* are thrown vertically upward with different velocities (l v, 2v & 3v respectively). The diagram indicates the mass and velocity of each stone. Rank from high to low the maximum height of each stone. Assume air resistance is negligible.
(A) I, II, III (B) II, I, III (C) III, II, I (D) I, III, II



66. A rubber ball bounces on the ground as shown. After each bounce, the ball reaches one-half the height of the bounce before it. If the time the ball was in the air between the first and second bounce was 1 second. What would be the time between the second and third bounce?
(A) 0.50 sec (B) 0.71 sec (C) 1.0 sec (D) 1.4 sec

67. The driver of a car makes an emergency stop by slamming on the car's brakes and skidding to a stop. How far would the car have skidded if it had been traveling twice as fast?
(A) 4 times as far (B) the same distance (C) 2 times as far (D)the mass of the car must be known

68. A snail is moving along a straight line. Its initial position is $x_0 = -5$ meters and it is moving away from the origin and slowing down. In this coordinate system, the signs of the initial position, initial velocity and acceleration, respectively, are

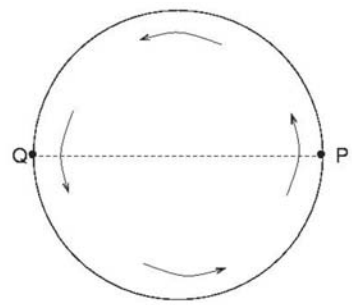
Choice	x_0	v_0	a
(A)	–	+	+
(B)	–	–	+
(C)	–	–	–
(D)	–	+	–

69. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) they strike the ground more than half a second apart.
(D) they strike the ground with the same kinetic energy.

70. A cart is initially moving at 0.5 m/s along a track. The cart comes to rest after traveling 1 m. The experiment is repeated on the same track, but now the cart is initially moving at 1 m/s. How far does the cart travel before coming to rest?
(A) 1 m (B) 2 m (C) 3 m (D) 4 m

71. During an interval of time, a tennis ball is moved so that the angle between the velocity and the acceleration of the ball is kept at a constant 120º. Which statement is true about the tennis ball during this interval of time?
(A) Its speed increases and it is changing its direction of travel.
(B) Its speed decreases and it is changing its direction of travel.
(C) Its speed remains constant, but it is changing its direction of travel.
(D) Its speed remains constant and it is not changing its direction of travel.

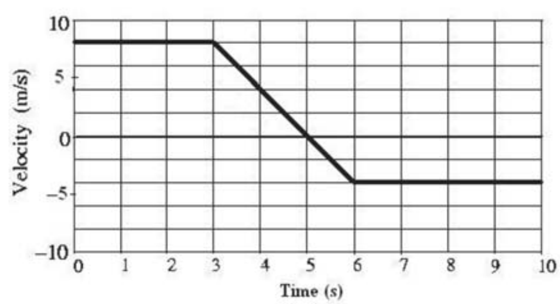
<u>Questions 72-73</u>
A particle continuously moves in a circular path at constant speed in a counterclockwise direction. Consider a time interval during which the particle moves along this circular path from point P to point Q. Point Q is exactly half-way around the circle from Point P.



72. What is the direction of the average velocity during this time interval?
(A) → (B) ← (C) ↑ (D) The average velocity is zero.

73.What is the direction of the average acceleration during this time interval?
(A) → (B) ← (C) ↓ (D) The average acceleration is zero.

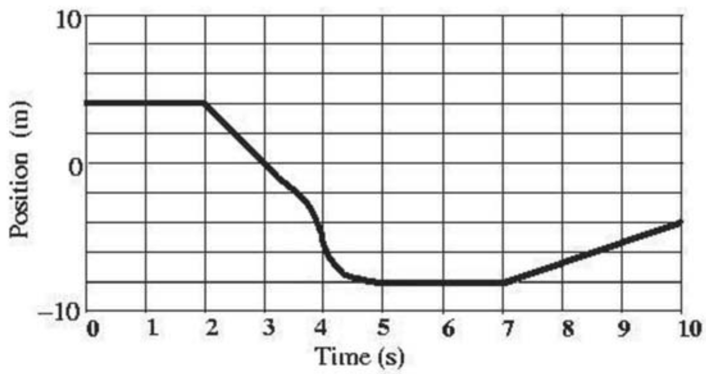
<u>Questions 74-75</u>
The velocity vs. time graph for the motion of a car on a straight track is shown in the diagram. The thick line represents the velocity. Assume that the car starts at the origin $x = 0$.



74. At which time is the car the greatest distance from the origin?
(A) t = 10 s (B) t = 5s (C) t = 3s (D) t = 0 s

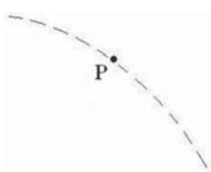
75. What is the average speed of the car for the 10 second interval?
(A) 1.20 m/s (B) 1.40 m/s (C) 3.30 m/s (D) 5.00 m/s

76. Consider the motion of an object given by the position vs. time graph shown. For what time(s) is the speed of the object greatest?



(A) At all times from $t = 0.0\ s \rightarrow t = 2.0\ s$ (B) At time $t = 3.0\ s$ (C) At time $t = 4.0\ s$
(D) At time $t = 8.5\ s$

77. The free fall trajectory of an object thrown horizontally from the top of a building is shown as the dashed line in the figure. Which sets of arrows best correspond to the directions of the velocity and of the acceleration for the object at the point labeled P on the trajectory?



	velocity	acceleration
(A)	↘	↓
(B)	→	↓
(C)	→	↖
(D)	↘	↙

78. A toy car moves 3.0 m to the North in one second. The car then moves at 9.0 m/s due South for two seconds. What is the average speed of the car for this three second trip?
(A) 4.0 m/s (B) 5.0 m/s (C) 6.0 m/s (D) 7 m/s

79. Two automobiles are 150 kilometers apart and traveling toward each other. One automobile is moving at 60 km/h and the other is moving at 40 km/h. In how many hours will they meet?
(A) 1.5 (B) 1.75 (C) 2.0 (D) 2.5

80. Is it possible for an object's velocity to increase while its acceleration decreases?
(A) No, because if acceleration is decreasing the object will be slowing down
(B) No, because velocity and acceleration must always be in the same direction
(C) Yes, an example would be a falling object near the surface of the moon
(D) Yes, an example would be a falling object in the presence of air resistance

<u>Questions 81-82</u>

During a recent winter storm, bales of hay had to be dropped from an airplane to a herd of cattle below. Assume the airplane flew horizontally at an altitude of 180 m with a constant velocity of 50 m/s and dropped one bale of hay every two seconds. It is reasonable to assume that air resistance will be negligible for this situation.

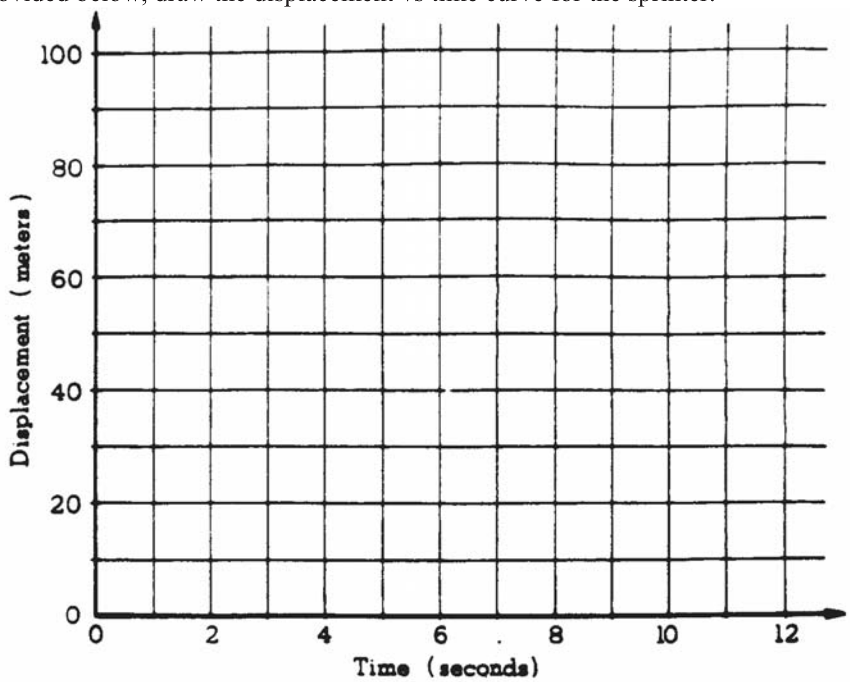
81. As the bales are falling through the air, what will happen to their distance of separation?
(A) the distance of separation will increase
(B) the distance of separation will decrease
(C) the distance of separation will remain constant
(D) the distance of separation will depend on the mass of the bales

82. About how far apart from each other will the bales land on the ground?
(A) 300 m (B) 180 m (C) 100 m (D) 50 m

AP Physics Free Response Practice – Kinematics

1982B1. The first meters of a 100-meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.

a. Determine the sprinter's constant acceleration during the first 2 seconds.
b. Determine the sprinters velocity after 2 seconds have elapsed.
c. Determine the total time needed to run the full 100 meters.
d. On the axes provided below, draw the displacement vs time curve for the sprinter.



2006B2. A world-class runner can complete a 100 m dash in about 10 s. Past studies have shown that runners in such a race accelerate uniformly for a time t and then run at constant speed for the remainder of the race. A world-class runner is visiting your physics class. You are to develop a procedure that will allow you to determine the uniform acceleration a and an approximate value of t for the runner in a 100 m dash. By necessity your experiment will be done on a straight track and include your whole class of eleven students.

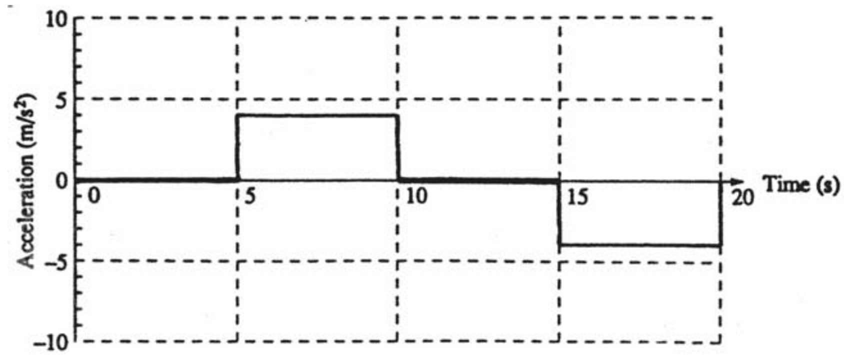
(a) By checking the line next to each appropriate item in the list below, select the equipment, other than the runner and the track, that your class will need to do the experiment.

____Stopwatches ____Tape measures ____ Rulers ____ Masking tape

____Metersticks ____ Starter's pistol ____ String ____ Chalk

(b) Outline the procedure that you would use to determine a and t, including a labeled diagram of the experimental setup. Use symbols to identify carefully what measurements you would make and include in your procedure how you would use each piece of the equipment you checked in part (a).

(c) Outline the process of data analysis, including how you will identify the portion of the race that has uniform acceleration, and how you would calculate the uniform acceleration.

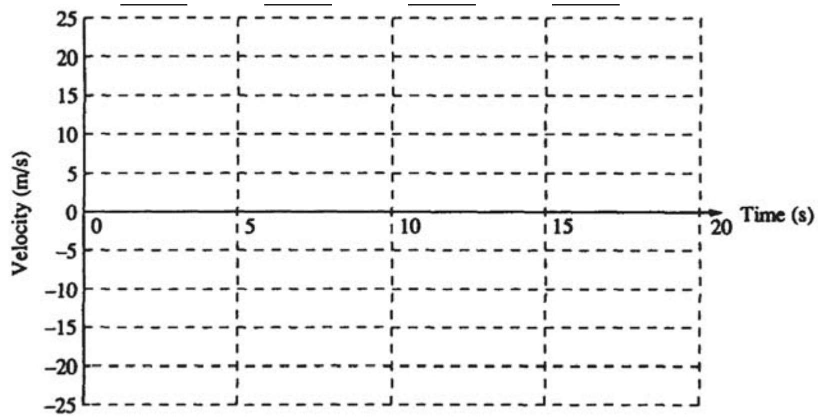


1993B1 (modified) A student stands in an elevator and records his acceleration as a function of time. The data are shown in the graph above. At time t = 0, the elevator is at displacement x = 0 with velocity v = 0. Assume that the positive directions for displacement, velocity, and acceleration are upward.

a. Determine the velocity v of the elevator at the end of each 5-second interval.

i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
v (m/s)	_____	_____	_____	_____



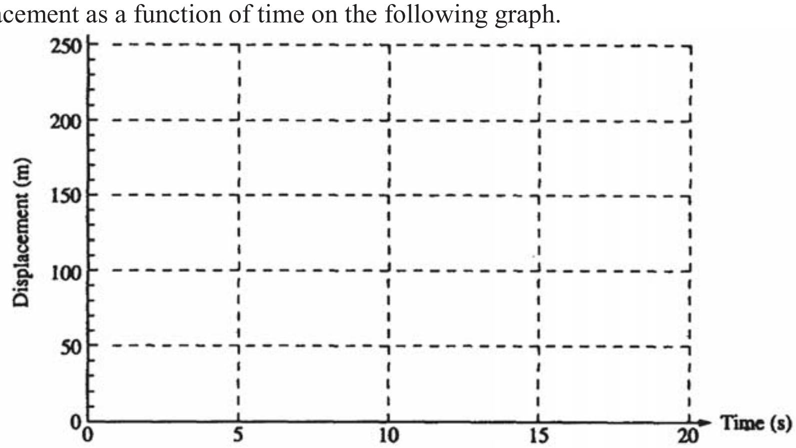
ii. Plot the velocity as a function of time on the following graph.

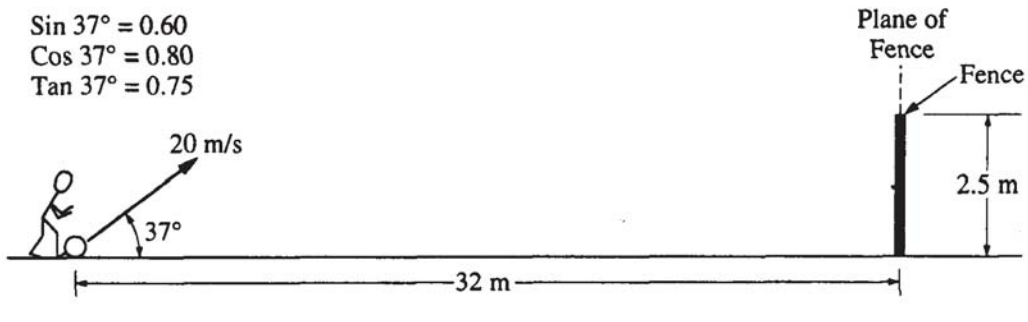
b. Determine the displacement x of the elevator above the starting point at the end of each 5-second interval.

i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
x (m)	_____	_____	_____	_____

ii. Plot the displacement as a function of time on the following graph.





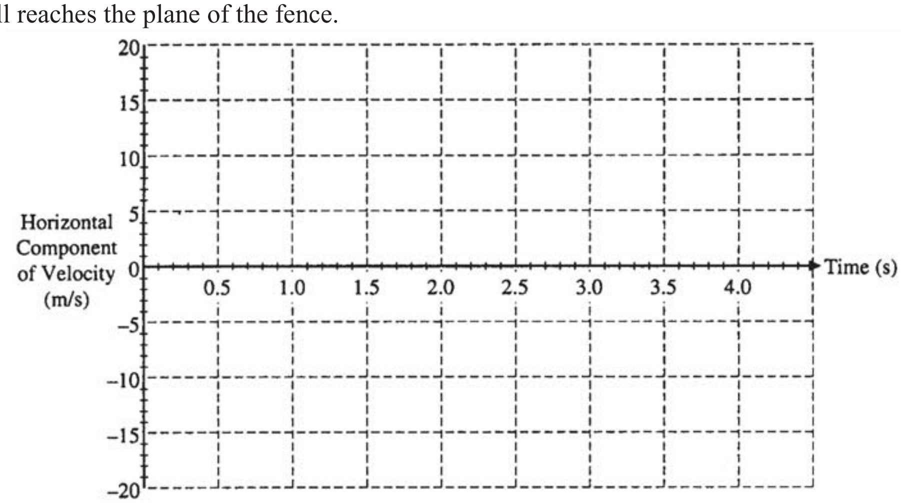
Note: Diagram not drawn to scale.

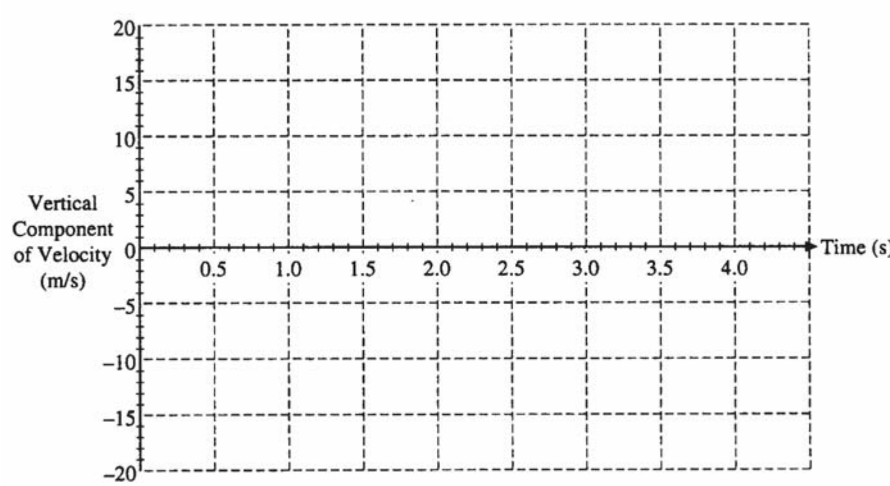
1994B1 (modified) A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in flight and air resistance is negligible.

a. Determine the time it takes for the ball to reach the plane of the fence.

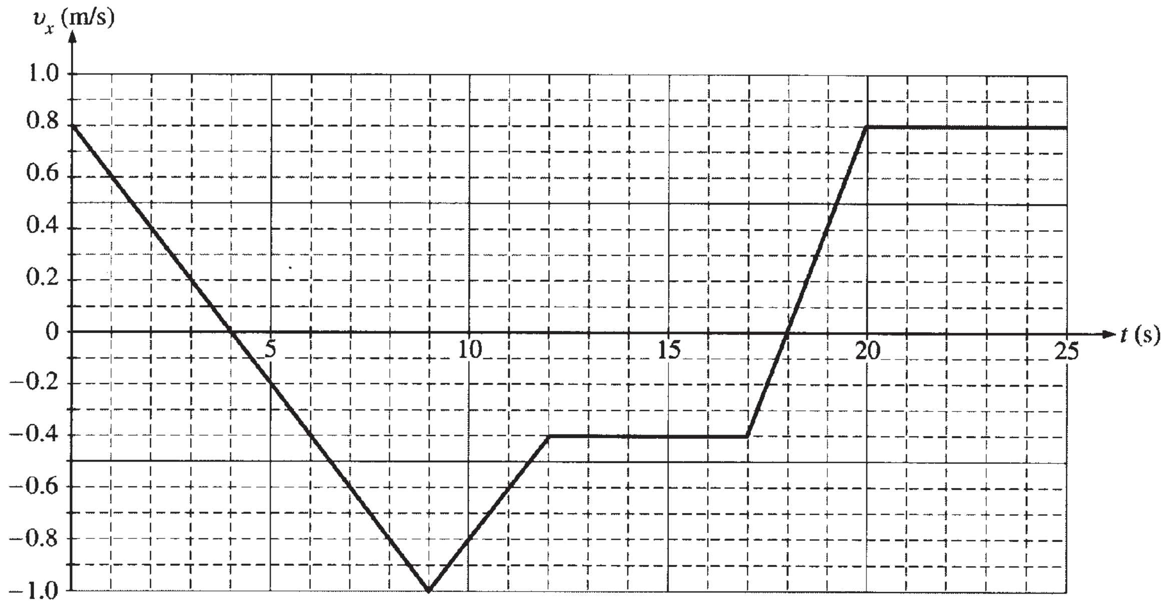
b. Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

c. On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.





2000B1 (modified) A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.

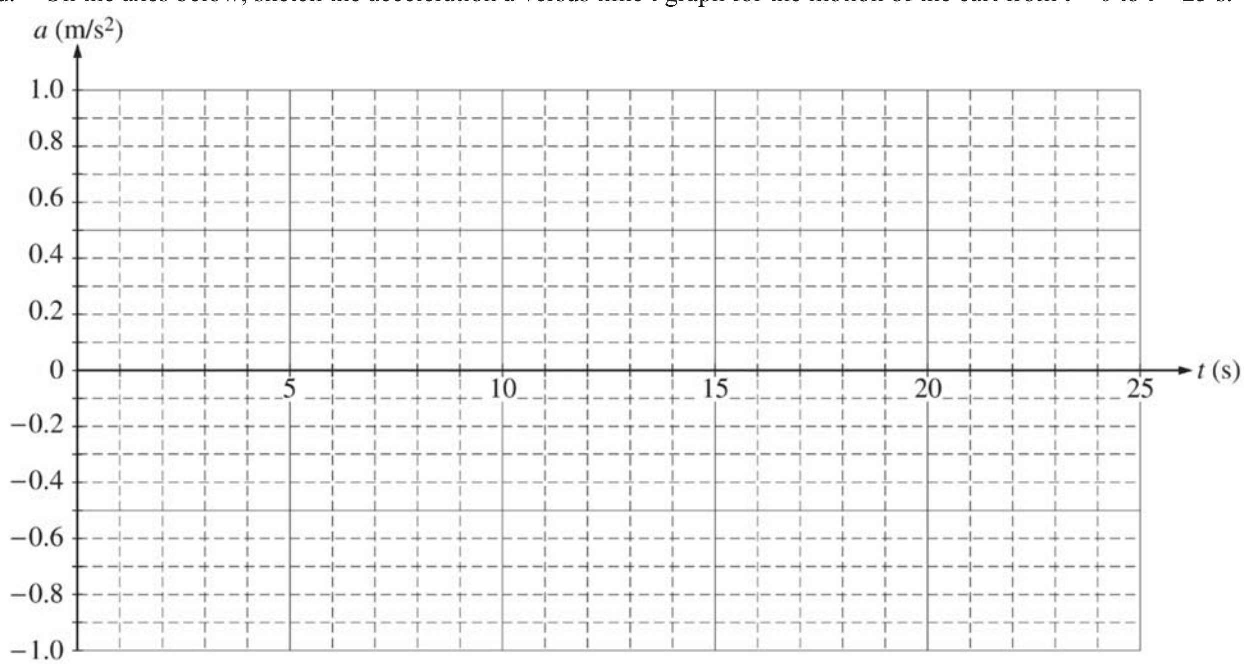


a. Indicate every time t for which the cart is at rest.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

c. Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.

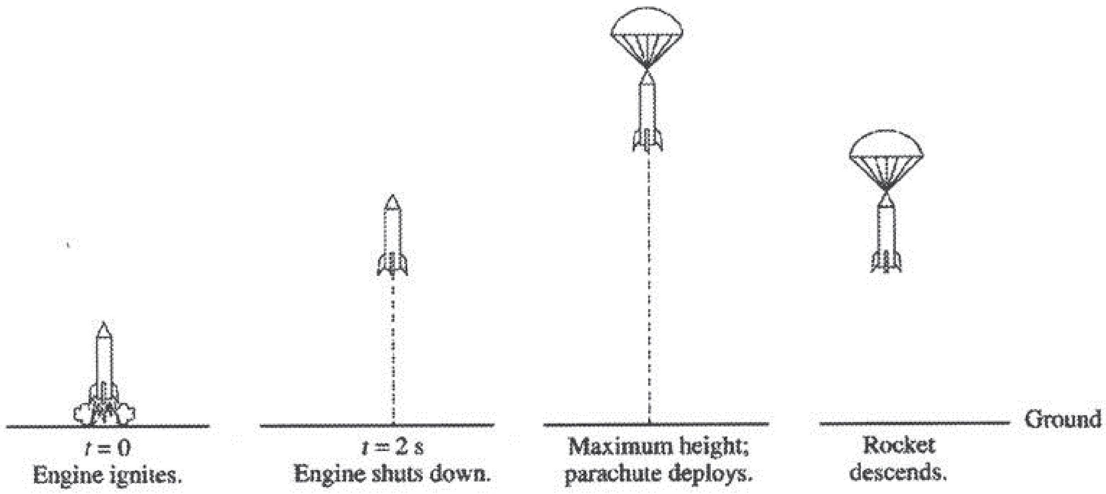
d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.



e. From t = 25 s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following:

i. The time from when the cart leaves the track until it first hits the floor

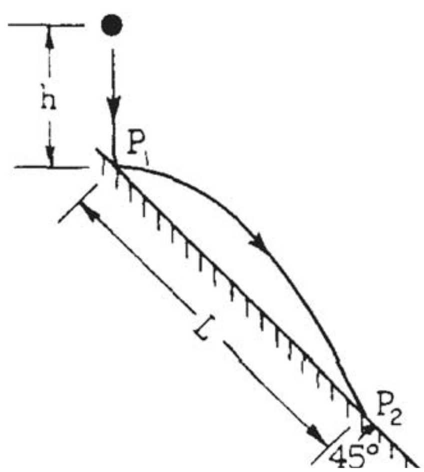
ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor



Note: Figures not drawn to scale.

2002B1 (modified) A model rocket is launched vertically with an engine that is ignited at time t = 0, as shown above. The engine provides an upward acceleration of 30 m/s^2 for 2.0 s. Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

a. Determine the speed of the rocket after the 2 s firing of the engine.
b. What maximum height will the rocket reach?
c. At what time after t = 0 will the maximum height be reached?

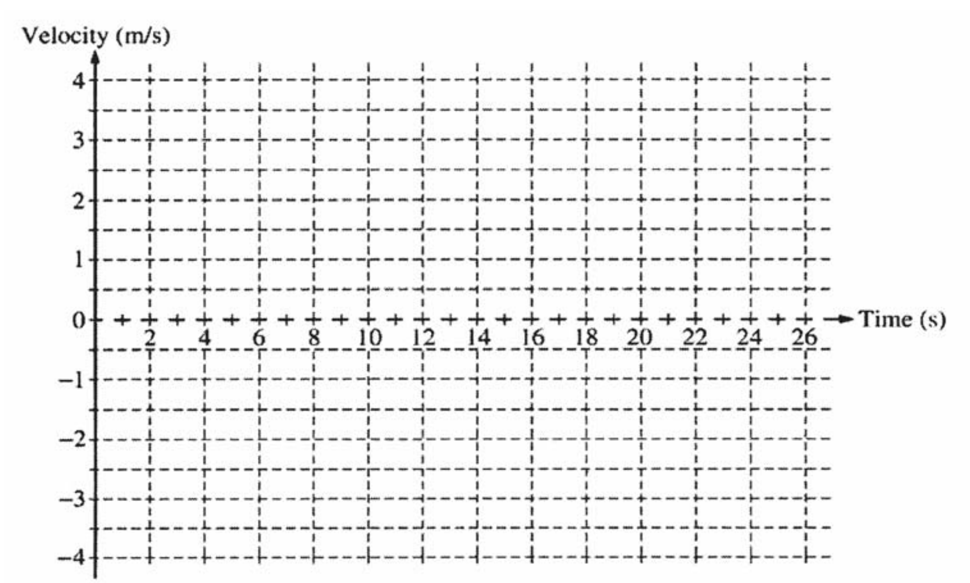


*1979M1 (modified) A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces horizontally off the plane at point P_1 with the same speed with which it struck the plane and strikes the plane again at point P_2. In terms of g and h determine each of the following quantities:

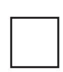
a. The speed of the ball just after it first bounces off the plane at P_1.
b. The time the ball is in flight between points P_1 and P_2.
c. The distance L along the plane from P_1 to P_2.
d. The speed of the ball just before it strikes the plane at P_2.

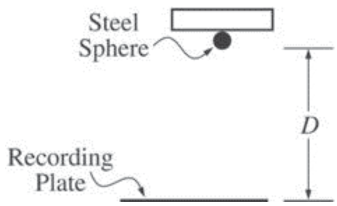
2005B1 (modified) The vertical position of an elevator as a function of time is shown above.

a. On the grid below, graph the velocity of the elevator as a function of time.



b. i. Calculate the average acceleration for the time period t = 8 s to t = 10 s.
ii. On the box below that represents the elevator, draw a vector to represent the direction of this average acceleration.

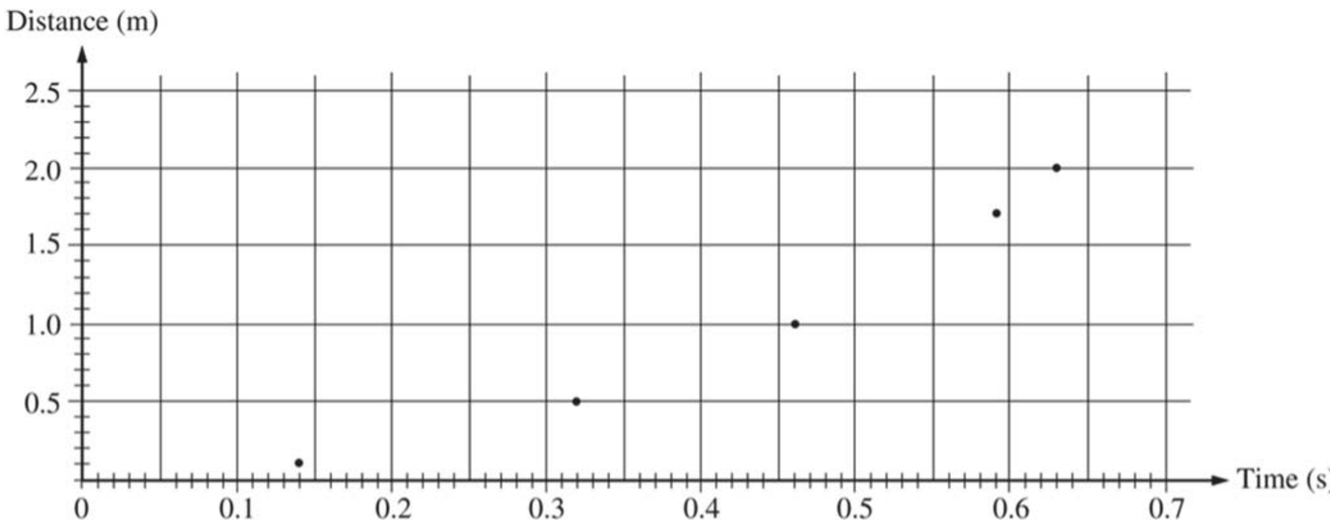




2006Bb1. A student wishing to determine experimentally the acceleration g due to gravity has an apparatus that holds a small steel sphere above a recording plate, as shown above. When the sphere is released, a timer automatically begins recording the time of fall. The timer automatically stops when the sphere strikes the recording plate.

The student measures the time of fall for different values of the distance D shown above and records the data in the table below. These data points are also plotted on the graph.

Distance of Fall (m)	0.10	0.50	1.00	1.70	2.00
Time of Fall (s)	0.14	0.32	0.46	0.59	0.63

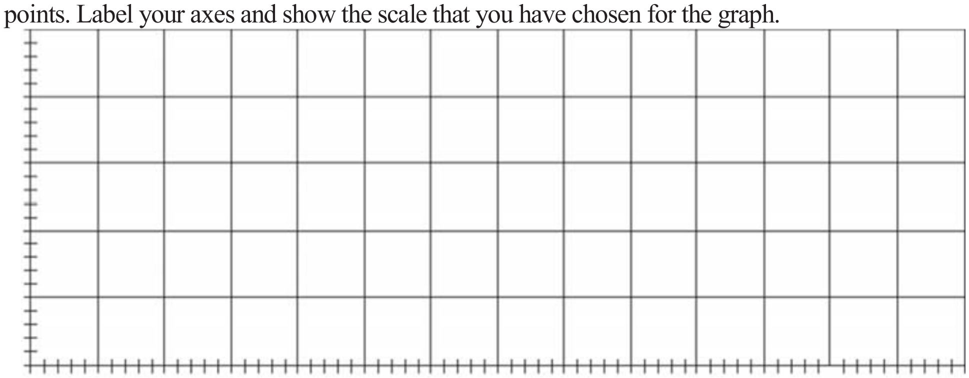


(a) On the grid above, sketch the smooth curve that best represents the student's data

The student can use these data for distance D and time t to produce a second graph from which the acceleration g due to gravity can be determined.

(b) If only the variables D and t are used, what quantities should the student graph in order to produce a linear relationship between the two quantities?

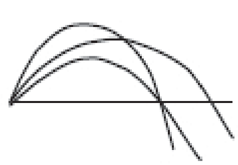
(c) On the grid below, plot the data points for the quantities you have identified in part (b), and sketch the best straight-line fit to the points. Label your axes and show the scale that you have chosen for the graph.



(d) Using the slope of your graph in part (c), calculate the acceleration g due to gravity in this experiment.

(e) State one way in which the student could improve the accuracy of the results if the experiment were to be performed again. Explain why this would improve the accuracy.

ANSWERS - AP Physics Multiple Choice Practice – Kinematics

	Solution	Answer
1.	Area bounded by the curve is the displacement By inspection of particle A the positive area between 0 and 1s will be countered by an equal negative area between 1 and 2s.	B
2.	Constant non-zero acceleration would be a straight line with a non-zero slope	D
3.	Area bounded by the curve is the displacement By inspection of particle A the positive area between 0 and 1s will be countered by an equal negative area between 1 and 2s.	A
4.	Area bounded by the curve is the displacement By inspection the negative area between 0 and 1s will be countered by an equal negative area sometime between 1 and 2s.	B
5.	Between 0 and 1 s; $d_1 = vt$; from 1 to 11 seconds; $d_2 = v_0t + ½ at^2$; $d = d_1 + d_2$	C
6.	The time in the air for a horizontal projectile is dependent on the height and independent of the initial speed. Since the time in the air is the same at speed v and at speed 2v, the distance (d = vt) will be twice as much at a speed of 2v	C
7.	The acceleration is constant and negative which means the slope of the velocity time graph must have a constant negative slope. (Only one choice has the correct acceleration anyway)	D
8.	At the top of its path, the vertical component of the velocity is zero, which makes the speed at the top a minimum. With symmetry, the projectile has the same speed when at the same height, whether moving up or down.	D
9.	g points down in projectile motion. Always.	D
10.	Average speed = total distance/total time = (8 m – 2 m)/(1 second)	D
11.	The area under the curve is the displacement. There is more area under the curve for Car X.	A
12.	Area under the curve is the displacement. Car Y is moving faster as they reach the same point.	B
13.	Uniformly accelerated means the speed-time graph should be a stright line with non-zero slope. The corresponding distance-time graph should have an increasing slope (curve upward)	D
14.	Acceleration is proportional to Δv. $\Delta v = v_2 - v_1 = v_2 + (-v_1)$	D
15.	horizontal velocity v_x remains the same thorughout the flight. g remains the same as well.	D
16.	A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows an increasing slope, then a constant slope, then a decreasing slope (to zero)	D
17.	For a horizontal projectile, the initial speed does not affect the time in the air. Use $v_{0y} = 0$ with 10 m = $½ gt^2$ to get $t = \sqrt{2}$ For the horzontal part of the motion; v = d/t	C
18.	A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows a constant slope, then a decreasing slope to zero, becoming negative and increasing, then a constant slope. Note this is an analysis of the *values* of v, not the slope of the graph itself	A
19.	By process of elimination (A and B are unrealistic; C is wrong, air resistance should decrease the acceleration)	D
20.	 The 45° angle gives the maximum horizontal travel to the original elevation, but the smaller angle causes the projectile to have a greater horizontal component of velocity, so given the additional time of travel allows such a trajectory to advance a greater horizontal distance. In other words given enough time the smaller angle of launch gives a parabola which will eventual cross the parabola of the 45° launch.	C

21. The area under the curve of an acceleration-time graph is the change in speed. D

22. The slope of the line represents her velocity. Beginning positive and constant, going to zero, then positive and larger than the initial, then negative while the line returns to the time axis B

23. Positive acceleration is an increasing slope (including negative slope increasing toward zero) or upward curvature C

24. Positive acceleration is an increasing slope (including negative slope increasing toward zero) or upward curvature C

25. With air resistance, the acceleration (the slope of the curve) will decrease toward zero as the ball reached terminal velocity. Note: without air resistance, choice (A) would be correct C

26. Since for the first 4 seconds, the car is accelerating positively the entire time, the car will be moving fastest just beofre slowing down after t = 4 seconds. C

27. The area under the curve represents the change in velocity. The car begins from rest with an increasing positive velocity, after 4 seconds the car begins to slow and the area under the curve from 4 to 8 seconds couters the increase in velocity form 0 to 4 seconds, bringing the car to rest. However, the car never changed direction and was moving away from its original starting position the entire time. D

28. The velocity-time graph should represent the slope of the position-time graph and the acceleration-time graph should represent the slope of the velocity-time graph C

29. It's a surprising result, but while both the horizontal and vertical components change at a given height with varying launch angle, the *speed* $(v_x^2 + v_y^2)^{1/2}$ will be independent of α (try it!) C

30. Instantaneous velocity is the slope of the line at that point A

31. Displacement is the area under the curve. Maximum displacement is just before the car turns around at 2.5 seconds. C

32. From the equation $d = \frac{1}{2} at^2$, displacement is proportional to time squared. Traveling from rest for twice the time gives 4 times the displacement (or 4 h). Since the object already travelled h in the first second, during the time interval from 1 s to 2 s the object travelled the remaining 3h C

33. Looking at choices A and D eliminates the possibility of choices B and C (each ball increases its speed by 9.8 m/s each second, negating those choices anyway). Since ball A is moving faster than ball B at all times, it will continue to pull away from ball B (the relative speed between the balls separates them). D

34. Since they all have the same horizontal component of the shell's velocity, the shell that spends the longest time in the air will travel the farthest. That is the shell launched at the largest angle (mass is irrelevant). D

35. Since (from rest) $d = \frac{1}{2} gt^2$, distance is proportional to time squared. An object falling for twice the time will fall four times the distance. D

36. $\bar{v} = \frac{v_i + v_f}{2} = \frac{d}{t}$ A

37. For a horizontal projectile ($v_{iy} = 0$ m/s) to fall 0.05 m takes (using $0.05 \text{ m} = \frac{1}{2} gt^2$) 0.1 seconds. To travel 20 m in this time requires a speed of d/t = (20 m)/(0.1 s) D

38. Once released, the package is in free-fall (subject to gravity only) D

39. To reach a speed of 30 m/s when dropped takes (using v = at) about 3 seconds. The distance fallen after three seconds is found using $d = \frac{1}{2} at^2$ C

40. Falling on the Moon is no different conceptually than falling on the Earth B

41. Since the line is above the t axis for the entire flight, the duck is always moving in the positive (forward) direction, until it stops at point D — D

42. One could analyze the graphs based on slope, but more simply, the graph of position versus time should represent the actual path followed by the ball as seen on a platform moving past you at constant speed. — C

43. Other than the falling portions ($a = -9.8\ m/s^2$) the ball should have a "spike" in the acceleration when it bounces due to the rapid change of velocity from downward to upward. — B

44. The same average speed would be indicated by the same distance traveled in the time interval — C

45. Average speed = (total distance)/(total time). Cars #2 and #3 travelled the same distance. — A

46. If you look at the distance covered in each time interval you should nitce a pattern: 2 m, 6 m, 10 m, 14 m, 18 m; making the distance in the next second 22 m. — C

47. Instantaneous speed is the slope of the line at that point. — B

48. A non-zero accleeration is inidcated by a curve in the line — D

49. Maximum height of a projectile is found from $v_y = 0$ m/s at max height and $(0\ m/s)^2 = v^2 + 2gh$ and gives $h = v^2/2g$. At twice the initial speed, the height will be 4 times as much — C

50. $d = ½ at^2$ (use any point) — D

51. $v = v_i + at$ — B

52. Acceleration is the slope of the line segment — C

53. Displacement is the area under the line — D

54. In a vacuum, there is no air resistance and hence no terminal velocity. It will continue to accelerate. — D

55. A projectile launched at a smaller angle does not go as high and will fall to the ground first. — B

56. Velocity is the slope of the line. — D

57. Positive acceleration is an upward curvature — D

58. Average acceleration = $\Delta v/\Delta t$ — A

59. Acceleration is the slope of the line segment — C

60. Displacement is the area between the line and the t-axis. Area is negative when the line is below the t-axis. — B

61. After two seconds, the object would be above it's original position, still moving upward, but the acceleration due to gravity is always pointing down — B

62. Constant speed is a constant slope on a position-time graph, a horizontal line on a velocity time graph or a zero value on an acceleration-time graph — D

63. Average speed = total distance divided by total time = (7 cm)/(1 s) — B

64. $d = ½ at^2$ (use any point) — C

65. Maximum height of a projectile is found from $v_y = 0$ m/s at max height and $(0\ m/s)^2 = v^2 + 2gh$ and gives $h = v^2/2g$. Mass is irrelevant. Largest initial speed = highest. — C

66. Using $d = ½ at^2$ shows the height is proportional to the time squared. ½ the maximum height is $\frac{1}{\sqrt{2}}$ times the time. — B

67. Stopping distance is found using $v_f = 0 = v_i^2 + 2ad$ which gives $d = v_i^2/2a$ where stopping distance is proportional to initial speed squared. A

68. Moving away from the origin will maintain a negative position and velocity. Slowing down indicates the acceleration is opposite in direction to the velocity. B

69. Since the first rock is always traveling faster, the relative distance between them is always increasing. A

70. Stopping distance is found using $v_f = 0 = v_i^2 + 2ad$ which gives $d = v_i^2/2a$ where stopping distance is proportional to initial speed squared. B

71. At an angle of 120º, there is a component of the acceleration perpendicular to the velocity causing the direction to change and a component in the opposite direction of the velocity, causing it to slow down. B

72. The displacement is directly to the left. The average velocity is proportional to the displacement B

73. The velocity is initially pointing up, the final velocity points down. The acceleration is in the same direction as $\Delta v = v_f + (-v_i)$ C

74. The car is the greatest distance just before it reverses direction at 5 seconds. B

75. Average speed = (total *distance*)/(total time), the total distance is the magnitude of the area under the line (the area below the t-axis is considered positive) D

76. Speed is the slope of the line. C

77. Velocity is pointing tangent to the path, acceleration (gravity) is downward. A

78. Average speed = (total *distance*)/(total time) D

79. The relative speed between the two cars is $v_1 - v_2 = (60 \text{ km/h}) - (-40 \text{ km/h}) = 100 \text{ km/h}$. They will meet in $t = d/v_{relative} = 150 \text{ km}/100 \text{ km/h}$ A

80. Acceleration is independent of velocity (you can accelerate in any direction while traveling in a ny direction). If the accleration is in the same direction as the velocity, the object is speeding up. D

81. As the first bales dropped will always be traveling faster than the later bales, their relative velocity will cause their separation to always iincrease. A

82. Horizontally, the bales will all travel at the speed of the plane, as gravity will not affect their horizontal motion. D = vt = (50 m/s)(2 seconds apart) C

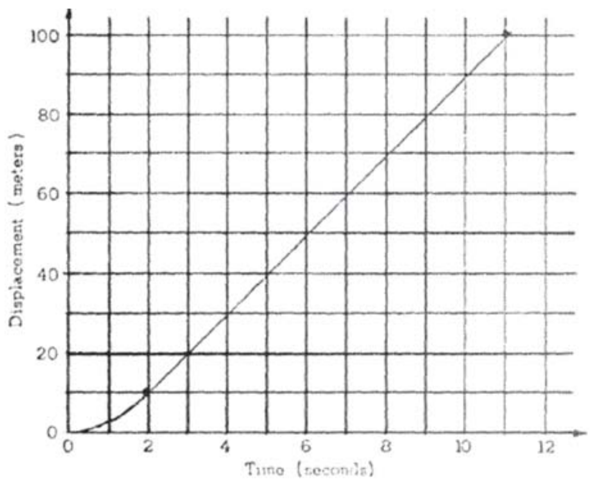
1982B1

a. For the first 2 seconds, while acceleration is constant, $d = \frac{1}{2} at^2$
Substituting the given values d = 10 meters, t = 2 seconds gives $a = 5\ m/s^2$

b. The velocity after accelerating from rest for 2 seconds is given by v = at, so v = 10 m/s

c. The displacement, time, and constant velocity for the last 90 meters are related by d = vt.
To cover this distance takes t = d/v = 9 s. The total time is therefore 9 + 2 = 11 seconds

d.



2006B2

Two general approaches were used by most of the students.
Approach A: Spread the students out every 10 meters or so. The students each start their stopwatches as the runner starts and measure the time for the runner to reach their positions.

Analysis variant 1: Make a position vs. time graph. Fit the parabolic and linear parts of the graph and establish the position and time at which the parabola makes the transition to the straight line.
Analysis variant 2: Use the position and time measurements to determine a series of average velocities ($v_{avg} = \Delta x/\Delta t$) for the intervals. Graph these velocities vs. time to obtain a horizontal line and a line with positive slope. Establish the position and time at which the sloped and horizontal lines intersect.
Analysis variant 3: Use the position and time measurements to determine a series of average accelerations ($\Delta x = v_0 t + \frac{1}{2} at^2$. Graph these accelerations vs. time to obtain two horizontal lines, one with a nonzero value and one at zero acceleration. Establish the position and time at which the acceleration drops to zero.

Approach B: Concentrate the students at intervals at the end of the run, in order to get a very precise value of the constant speed v_f, or at the beginning in order to get a precise value for a. The total distance D is given by $a = \frac{1}{2} a t_u^2 + v_f(T - t_u)$, where T is the total measured run time. In addition $v_f = a t_u$ These equations can be solved for a and t_u (if v_f is measured directly) or v_f and t_u (if a is measured directly). Students may have also defined and used distances, speeds, and times for the accelerated and constant-speed portions of the run in deriving these relationships.

1993B1

a. i. Use the kinematic equation applicable for constant acceleration: $v = v_0 + at$. For each time interval, substitute the initial velocity for that interval, the appropriate acceleration from the graph and a time of 5 seconds.

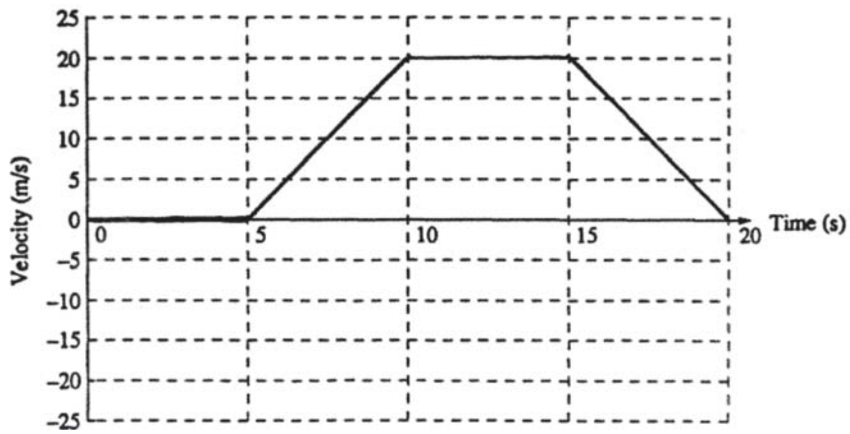
5 seconds: $v = 0 + (0)(5\ s) = 0$

10 seconds: $v = 0 + (4\ m/s^2)(5\ s) = 20\ m/s$

15 seconds: $v = 20\ m/s + (0)(5\ s) = 20\ m/s$

20 seconds: $v = 20\ m/s + (-4\ m/s^2)(5\ s) = 0$

ii.



b. i. Use the kinematic equation applicable for constant acceleration, $x = x_0 + v_0t + ½\ at^2$. For each time interval, substitute the initial position for that interval, the initial velocity for that interval from part (a), the appropriate acceleration, and a time of 5 seconds.

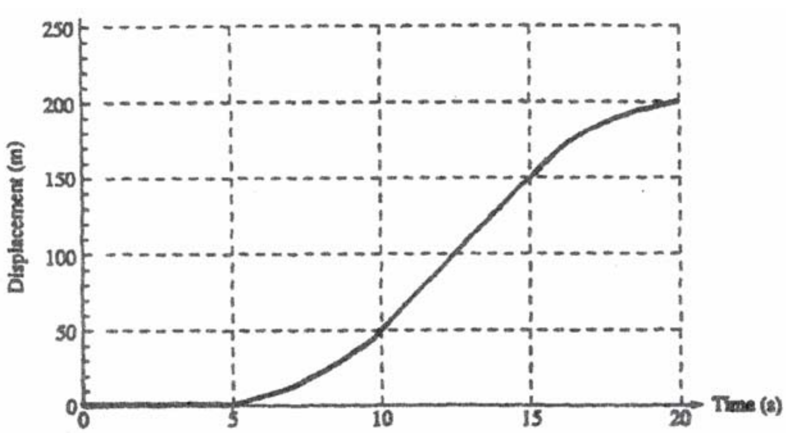
5 seconds: $x = 0 + (0)(5\ s) + ½\ (0)(5\ s)^2 = 0$

10 seconds: $x = 0 + (0)(5\ s) + ½\ (4\ m/s^2)(5\ s)^2 = 50\ m$

15 seconds: $x = 50\ m + (20\ m/s)(5\ s) + ½\ (0)(5\ s)^2 = 150\ m$

20 seconds: $x = 150\ m + (20\ m/s)(5\ s) + ½\ (-4\ m/s^2)(5\ s)^2 = 200\ m$

ii.

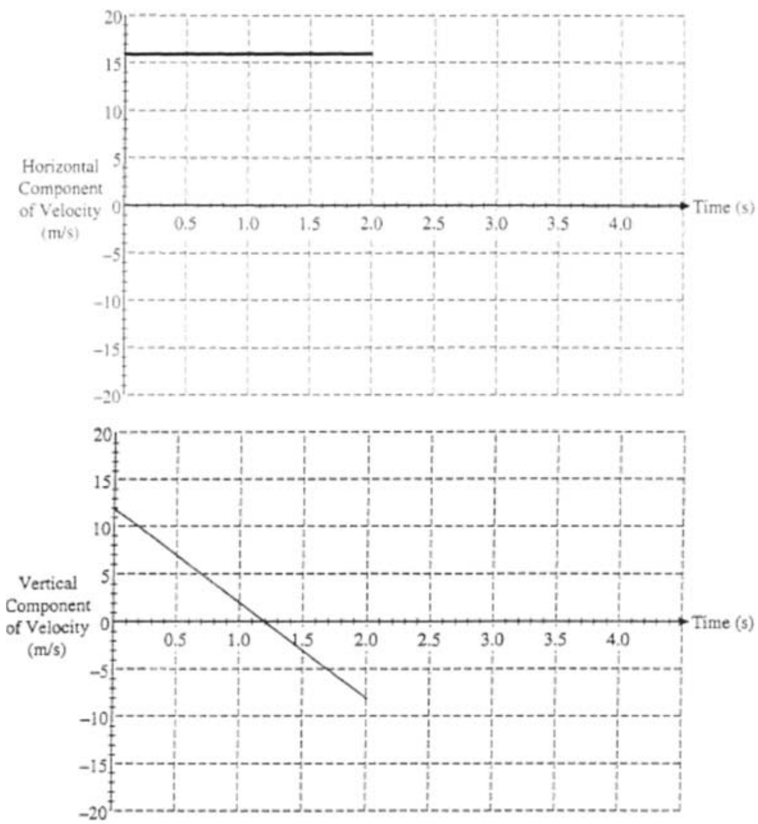


1994B1

a. The horizontal component of the velocity is constant so $v_x t = d$ where $v_x = v_0 \cos\theta = 16$ m/s
$t = d/v = 2$ s

b. The height of the ball during its flight is given by $y = v_{0y}t + \frac{1}{2} gt^2$ where $v_{0y} = v_0 \sin\theta = 12$ m/s and $g = -9.8$ m/s^2 which gives at $t = 2$ s, $y = 4.4$ m. The fence is 2.5 m high so the ball passes above the fence by 4.4 m – 2.5 m = 1.9 m

c.

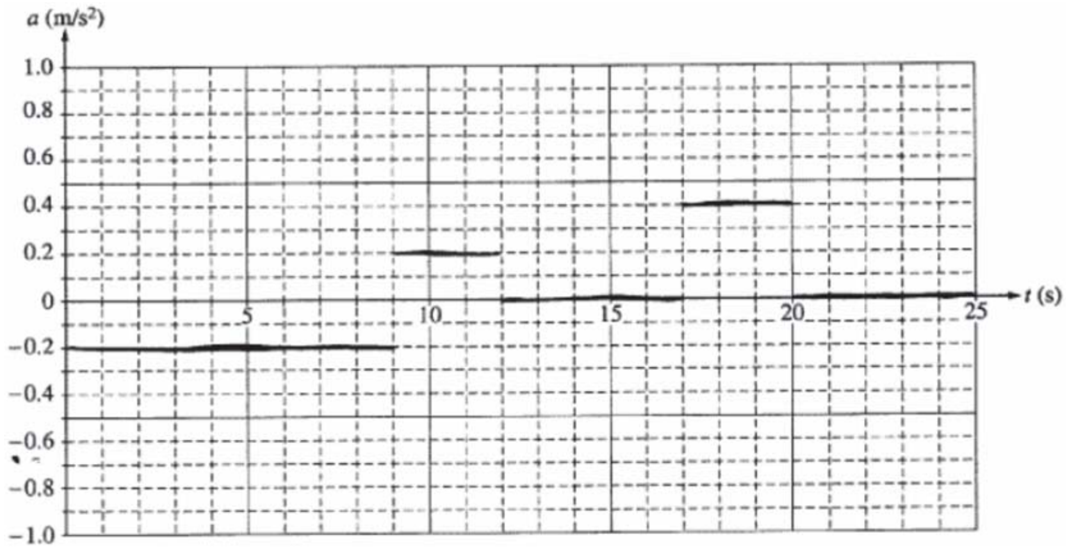


2000B1

a. The car is at rest where the line crosses the t axis. At t = 4 s and 18 s.

b. The speed of the cart increases when the line moves away from the t axis (larger values of v, positive or negative). This occurs during the intervals t = 4 to 9 seconds and t = 18 to 20 seconds.

c. The change in position is equal to the area under the graph. From 0 to 4 seconds the area is positive and from 4 to 9 seconds the area is negative. The total area is –0.9 m. Adding this to the initial position gives $x = x_0 + \Delta x$ = 2.0 m + (–0.9 m) = 1.1 m d.



e. i. $y = \frac{1}{2} gt^2$ ($v_{0y} = 0$ m/s) gives t = 0.28 seconds.
ii. $x = v_x t = 0.22$ m

2002B1

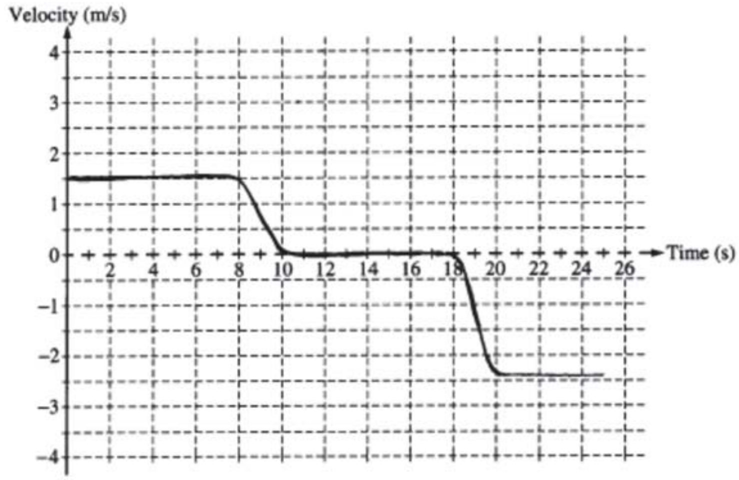
a. $v_1 = v_0 + at = 60$ m/s
b. The height of the rocket when the engine stops firing $y_1 = \frac{1}{2} at^2 = 60$ m
To determine the extra height after the firing stops, use $v_f^2 = 0$ m/s $= v_1^2 + 2(-g)y_2$ giving $y_2 = 180$ m
total height = $y_1 + y_2 = 240$ m
c. To determine the time of travel from when the engine stops firing use $v_f = 0$ m/s $= v_1 + (-g)t_2$ giving $t_2 = 6$ s.
The total time is then 2 s + 6 s = 8 seconds

1979M1

a. The speed after falling a height h is found from $v_f^2 = v_i^2 + 2gh$, where $v_i = 0$ m/s giving $v_f = \sqrt{2gh}$
b/c. During the flight from P_1 to P_2 the ball maintains a horizontal speed of $\sqrt{2gh}$ and travels a horizontal distance of $\frac{L}{\sqrt{2}}$ thus (using d = vt) we have $\frac{L}{\sqrt{2}} = \sqrt{2gh}\, t$. During the same time t the ball travels the same distance vertically given by $\frac{L}{\sqrt{2}} = \frac{1}{2}gt^2$. Setting these expressions equal gives us $\sqrt{2gh}\, t = \frac{1}{2} gt^2$. Solving for t and substituting into the expression of L gives $t = \sqrt{8h/g}$ $and\ L = 4\sqrt{2}h$
d. During the flight from P_1 to P_2 the ball maintains a horizontal speed of $\sqrt{2gh}$ and the vertical speed at P_2 can be found from $v_y = v_i + at$ where $v_i = 0$, a = g and t is the time found above. Once v_x and v_y are known the speed is $\sqrt{v_x^2 + v_y^2}$ giving $v = \sqrt{10gh}$

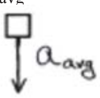
2005B1

a.



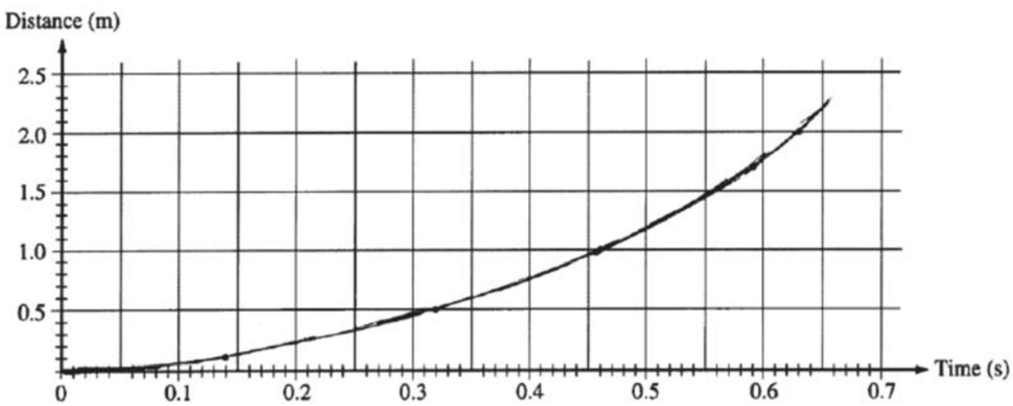
b. i. $a_{avg} = \Delta v/\Delta t = (0 - 1.5\text{ m/s})/(2\text{ s}) = -0.75\text{ m/s}^2$

ii.



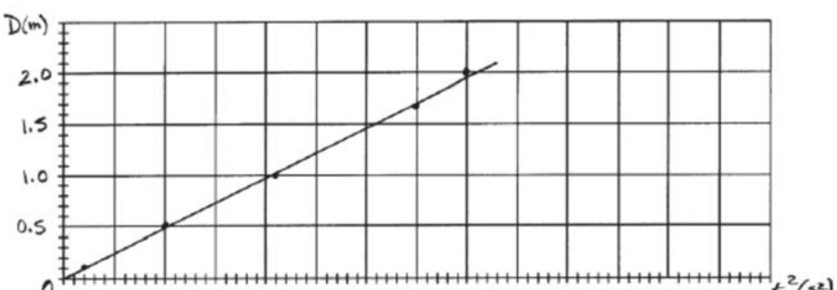
2006Bb1

a.



b. Distance and time are related by the equation $D = \frac{1}{2} gt^2$. To yield a straight line, the quantities that should be graphed are D and t^2 or $\sqrt{D}$ and t.

c.

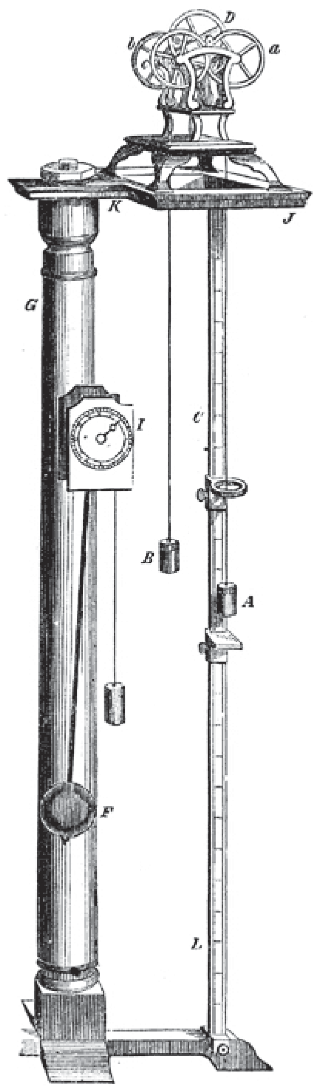


d. The slope of the graph of D vs. t^2 is ½ g. The slope of the line shown is 4.9 m/s^2 giving g = 9.8 m/s^2

e. (example) Do several trials for each value of *D* and take averages. This reduces personal and random error.

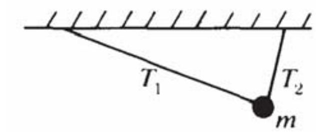
Chapter 2

Dynamics



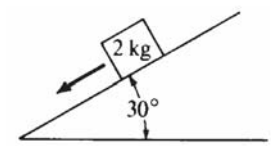
AP Physics Multiple Choice Practice – Dynamics

SECTION A – Linear Dynamics



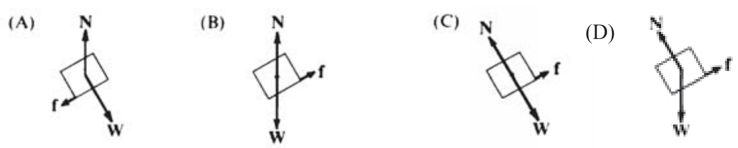
1. A ball of mass m is suspended from two strings of unequal length as shown above. The magnitudes of the tensions T_1 and T_2 in the strings must satisfy which of the following relations?
(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $T_1 < T_2$ (D) $T_1 + T_2 = mg$

Questions 2 – 3

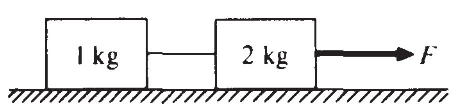


A 2-kg block slides down a 30° incline as shown above with an acceleration of 2 m/s^2.

2. Which of the following diagrams best represents the gravitational force W. the frictional force f, and the normal force N that act on the block?

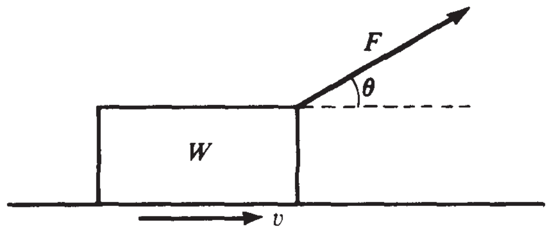


3. Which of the following correctly indicates the magnitudes of the forces acting up and down the incline?
   (A) 20 N down the plane, 16 N up the plane
   (B) 4 N down the plane, 4 N up the plane
   (C) 0 N down the plane, 4 N up the plane
   (D) 10 N down the plane, 6 N up the plane

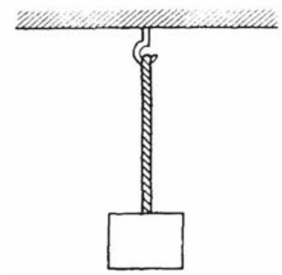


4. When the frictionless system shown above is accelerated by an applied force of magnitude the tension in the string between the blocks is (A) F (B) 2/3 F (C) ½ F (D) 1/3 F

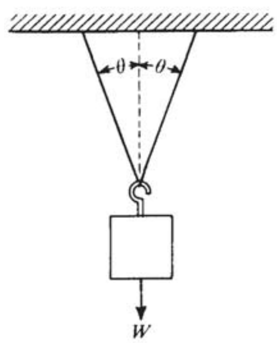
5. A ball falls straight down through the air under the influence of gravity. There is a retarding force F on the ball with magnitude given by F = *bv*, where *v* is the speed of the ball and *b* is a positive constant. The ball reaches a terminal velocity after a time *t*. The magnitude of the acceleration at time *t*/2 is
   (A) Increasing
   (B) Decreasing
   (C) 10 m/s/s
   (D) Zero



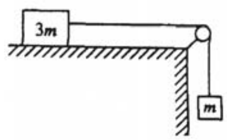
6. A block of weight W is pulled along a horizontal surface at constant speed v by a force F, which acts at an angle of θ with the horizontal, as shown above. The normal force exerted on the block by the surface has magnitude
   (A) greater than W
   (B) greater than zero but less than W
   (C) equal to W
   (D) zero



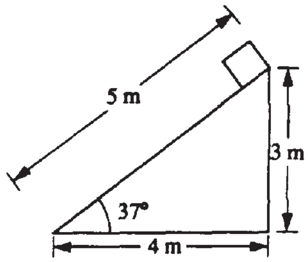
7. A uniform rope of weight 50 N hangs from a hook as shown above. A box of weight 100 N hangs from the rope. What is the tension in the rope?
   (A) 75 N throughout the rope (B) 100 N throughout the rope
   (C) 150 N throughout the rope (D) It varies from 100 N at the bottom of the rope to 150 N at the top.



8. When an object of weight W is suspended from the center of a massless string as shown above, the tension at any point in the string is
   (A) 2Wcosθ (B) ½Wcosθ (D) W/(2cosθ) (E) W/(cosθ)



9. A block of mass 3m can move without friction on a horizontal table. This block is attached to another block of mass m by a cord that passes over a frictionless pulley, as shown above. If the masses of the cord and the pulley are negligible, what is the magnitude of the acceleration of the descending block?
   (A) g/4 (B) g/3 (C) 2g/3 (D) g

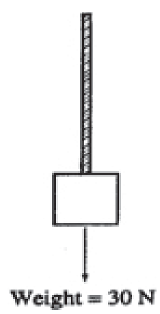


A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 N is placed at the top of the plane and allowed to slide down.

10. The magnitude of the normal force exerted on the block by the plane is
    (A) greater than 20 N
    (B) greater than zero but less than 20 N
    (C) equal to 20 N
    (D) zero

11. **Multiple correct:** Three forces act on an object. If the object is moving to the right in translational equilibrium, which of the following must be true? Select two answers.
    (A) The vector sum of the three forces must equal zero.
    (B) All three forces must be parallel.
    (C) The magnitudes of the three forces must be equal.
    (D) The object must be moving at a constant speed.

12. For which of the following motions of an object must the acceleration always be zero?
    (A) Any motion in a straight line
    (B) Simple harmonic motion
    (C) Any motion at constant speed
    (D) Any single object in motion with constant momentum



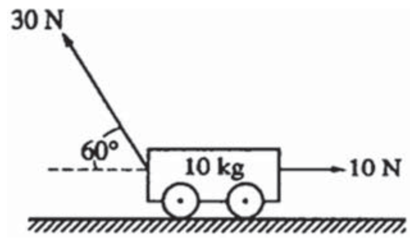
13. A rope of negligible mass supports a block that weighs 30 N, as shown above. The breaking strength of the rope is 50 N. The largest acceleration that can be given to the block by pulling up on it with the rope without breaking the rope is most nearly
    (A) 6.7 m/s^2 (B) 10 m/s^2 (C) 16.7 m/s^2 (D) 26.7 m/s^2

<u>Questions 14-15</u>

A horizontal, uniform board of weight 125 N and length 4 m is supported by vertical chains at each end. A person weighing 500 N is hanging from the board. The tension in the right chain is 250 N.

14. What is the tension in the left chain?
    (A) 125 N (B) 250 N (C) 375 N (D) 625 N

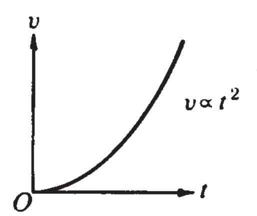
15. Which of the following describes where the person is hanging?
(A) between the chains, but closer to the left-hand chain
(B) between the chains, but closer to the right-hand chain
(C) Right in the middle of the board
(D) directly below one of the chains



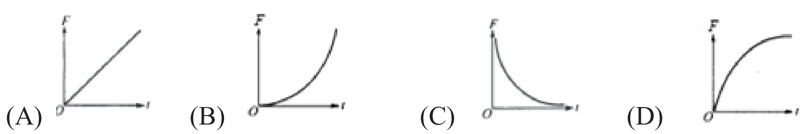
16. **Multiple correct:** The cart of mass 10 kg shown above moves without frictional loss on a level table. A 10 N force pulls on the cart horizontally to the right. At the same time, a 30 N force at an angle of 60° above the horizontal pulls on the cart to the left. Which of the following describes a manner in which this cart could be moving? Select two answers.

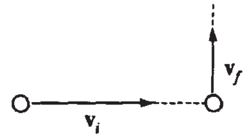
(A) moving left and speeding up
(B) moving left and slowing down
(C) moving right and speeding up
(D) moving right and slowing down

17. Two people are pulling on the ends of a rope. Each person pulls with a force of 100 N. The tension in the rope is:
(A) 0 N (B) 50 N (C) 100 N (D) 200 N

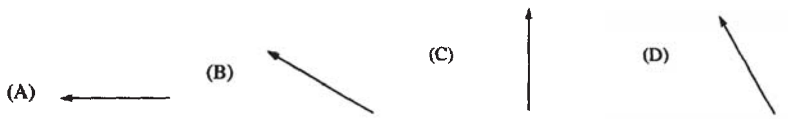


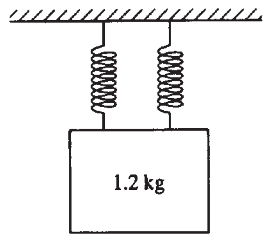
18. The parabola above is a graph of speed v as a function of time t for an object. Which of the following graphs best represents the magnitude F of the net force exerted on the object as a function of time t?



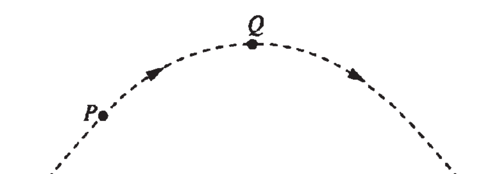


19. A ball initially moves horizontally with velocity v_i, as shown above. It is then struck by a stick. After leaving the stick, the ball moves vertically with a velocity v_f, which is smaller in magnitude than v_i. Which of the following vectors best represents the direction of the average force that the stick exerts on the ball?

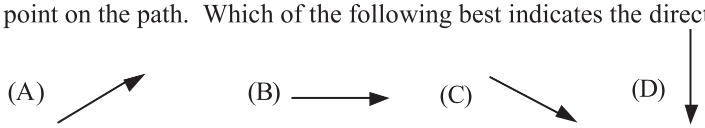


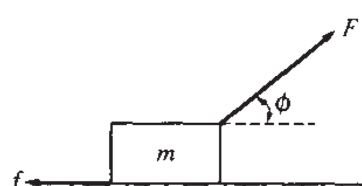


20. Two massless springs, of spring constants k_1 and k_2, are hung from a horizontal support. A block of weight 12 N is suspended from the pair of springs, as shown above. When the block is in equilibrium, each spring is stretched an additional 24 cm. Thus, the equivalent spring constant of the two-spring system is 12 N / 24 cm = 0.5 N/cm. Which of the following statements is correct about k_1 and k_2?
(A) $k_1 = k_2 = 0.25$ N/cm
(B) $1/k_1 + 1/k_2 = 1/(0.5 \text{ N/cm})$
(C) $k_1 - k_2 = 0.5$ N/cm
(D) $k_1 + k_2 = 0.5$ N/cm

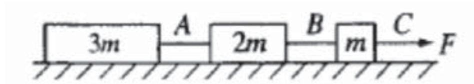


21. A ball is thrown and follows a parabolic path, as shown above. Air friction is negligible. Point Q is the highest point on the path. Which of the following best indicates the direction of the net force on the ball at point P ?

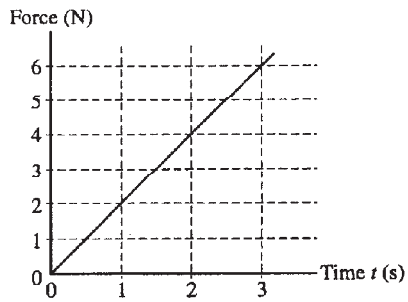




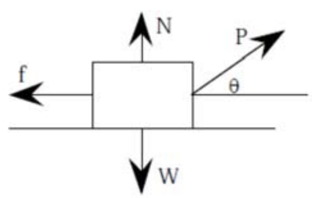
22. A block of mass m is accelerated across a rough surface by a force of magnitude F that is exerted at an angle ϕ with the horizontal, as shown above. The frictional force on the block exerted by the surface has magnitude f. What is the acceleration of the block?
(A) F/m (B) $(F\cos\phi)/m$ (C) $(F-f)/m$ (D) $(F\cos\phi-f)/m$



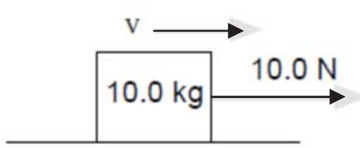
23. Three blocks of masses 3*m*, 2*m*, and*s* are connected to strings *A, B*, and *C* as shown above. The blocks are pulled along a rough surface by a force of magnitude *F* exerted by string *C*. The coefficient of friction between each block and the surface is the same. Which string must be the strongest in order not to break?
(A) *A* (B) *B* (C) *C* (D) They must all be the same strength.



24. A block of mass 3 kg, initially at rest, is pulled along a frictionless, horizontal surface with a force shown as a function of time t by the graph above. The acceleration of the block at t = 2 s is
(A) 4/3 m/s^2 (B) 2 m/s^2 (C) 8 m/s^2 (D) 12 m/s^2

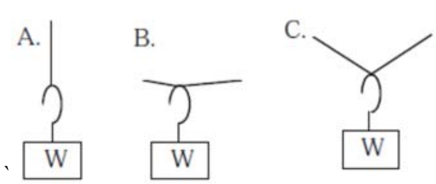


25. A student pulls a wooden box along a rough horizontal floor at constant speed by means of a force *P* as shown to the right. Which of the following must be true?
(A) P > f and N < W.
(B) P > f and N = W.
(C) P = f and N > W.
(D) P = f and N = W.

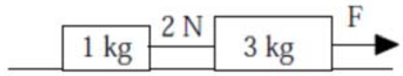


26. The 10.0 kg box shown in the figure to the right is sliding to the right along the floor. A horizontal force of 10.0 N is being applied to the right. The coefficient of kinetic friction between the box and the floor is 0.20. The box is moving with:
(A) acceleration to the left. (B) acceleration to the right.
(C) constant speed and constant velocity. (D) constant speed but not constant velocity.

27. Assume the objects in the following diagrams have equal mass and the strings holding them in place are identical. In which case would the string be most likely to break?



D. All would be equally likely to break

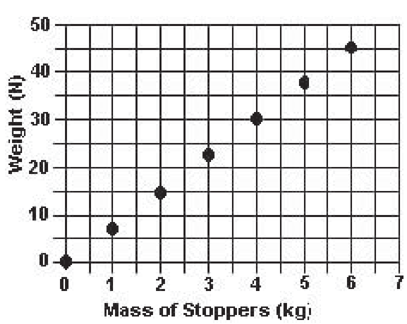


28. Two blocks of mass 1.0 kg and 3.0 kg are connected by a string which has a tension of 2.0 N. A force *F* acts in the direction shown to the right. Assuming friction is negligible, what is the value of *F*?
(A) 2.0 N (B) 4.0 N (C) 6.0 N (D) 8.0 N

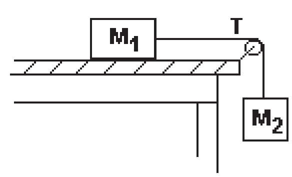
29. A 50-kg student stands on a scale in an elevator. At the instant the elevator has a downward acceleration of 1.0 m/s^2 and an upward velocity of 3.0 m/s, the scale reads approximately
(A) 350 N (B) 450 N (C) 500 N (D) 550 N

30. A tractor-trailer truck is traveling down the road. The mass of the trailer is 4 times the mass of the tractor. If the tractor accelerates forward, the force that the trailer applies on the tractor is
(A) 4 times greater than the force of the tractor on the trailer.
(B) 2 times greater than the force of the tractor on the trailer.
(C) equal to the force of the tractor on the trailer.
(D) ¼ the force of the tractor on the trailer.

31. A wooden box is first pulled across a horizontal steel plate as shown in the diagram A. The box is then pulled across the same steel plate while the plate is inclined as shown in diagram B. How does the force required to overcome friction in the inclined case (B) compare to the horizontal case (A)?
(A) the frictional force is the same in both cases
(B) the inclined case has a greater frictional force
(C) the inclined case has less frictional force
(D) the frictional force increases with angle until the angle is 90º, then drops to zero



32. The graph at left shows the relationship between the mass of a number of rubber stoppers and their resulting weight on some far-off planet. The slope of the graph is a representation of the:
(A) mass of a stopper
(B) density of a stopper
(C) acceleration due to gravity
(D) number of stoppers for each unit of weight



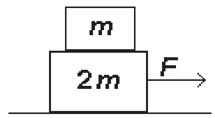
33. Two masses, m_1 and m_2, are connected by a cord and arranged as shown in the diagram with m_1 sliding along on a frictionless surface and m_2 hanging from a light frictionless pulley. What would be the mass of the falling mass, m_2, if both the sliding mass, m_1, and the tension, T, in the cord were known?

(A) $\frac{m_1 g - T}{g}$ (B) $\frac{1}{2}Tg$ (C) $\frac{m_1(T-g)}{(gm_1 - T)}$ (D) $\frac{Tm_1}{(gm_1 - T)}$

34. A mass is suspended from the roof of a lift (elevator) by means of a spring balance. The lift (elevator) is moving upwards and the readings of the spring balance are noted as follows:

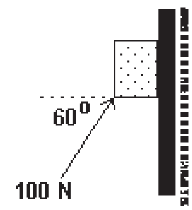
Speeding up: R_U Constant speed: R_C Slowing down: R_D

Which of the following is a correct relationship between the readings?
(A) $R_U > R_C$ (B) $R_U = R_D$ (C) $R_C < R_D$ (D) $R_C < R_D$



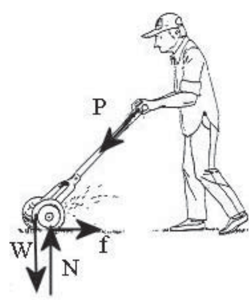
35. A small box of mass *m* is placed on top of a larger box of mass *2m* as shown in the diagram at right. When a force *F* is applied to the large box, both boxes accelerate to the right with the same acceleration. If the coefficient of friction between all surfaces is μ, what would be the force accelerating the smaller mass?
(A) $\frac{F}{3} - mg\mu$ (B) $F - 3mg\mu$ (C) $F - mg\mu$ (D) $\frac{F - mg\mu}{3}$

36. A 6.0 kg block initially at rest is pushed against a wall by a 100 N force as shown. The coefficient of kinetic friction is 0.30 while the coefficient of static friction is 0.50. What is true of the friction acting on the block after a time of 1 second?
(A) Static friction acts upward on the block.
(B) Kinetic friction acts upward on the block
(C) Kinetic friction acts downward on the block.
(D) Static friction acts downward on the block.



37. A homeowner pushes a lawn mower across a horizontal patch of grass with a constant speed by applying a force P. The arrows in the diagram correctly indicate the directions but not necessarily the magnitudes of the various forces on the lawn mower. Which of the following relations among the various force magnitudes, *W, f, N, P* is correct?

(A) $P > f$ and $N > W$
(B) $P < f$ and $N = W$
(C) $P > f$ and $N < W$
(D) $P = f$ and $N > W$

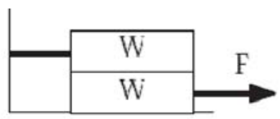


38. A mass, M, is at rest on a frictionless surface, connected to an ideal horizontal spring that is unstretched. A person extends the spring 30 cm from equilibrium and holds it at this location by applying a 10 N force. The spring is brought back to equilibrium and the mass connected to it is now doubled to 2M. If the spring is extended back 30 cm from equilibrium, what is the necessary force applied by the person to hold the mass stationary there?
(A) 20.0 N (B) 14.1 N (C) 10.0 N (D) 7.07 N

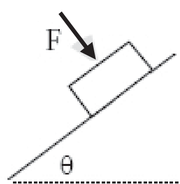
39. A crate of toys remains at rest on a sleigh as the sleigh is pulled up a hill with an increasing speed. The crate is not fastened down to the sleigh. What force is responsible for the crate's increase in speed up the hill?
(A) the contact force (normal force) of the ground on the sleigh
(B) the force of static friction of the sleigh on the crate
(C) the gravitational force acting on the sleigh
(D) no force is needed

40. A box slides to the right across a horizontal floor. A person called Ted exerts a force *T* to the right on the box. A person called Mario exerts a force *M* to the left, which is half as large as the force *T*. Given that there is friction *f* and the box accelerates to the right, rank the sizes of these three forces exerted on the box.
(A) $f < M < T$ (B) $M < f < T$ (C) $M < T < f$ (D) $f = M < T$

41. A spaceman of mass 80 kg is sitting in a spacecraft near the surface of the Earth. The spacecraft is accelerating upward at five times the acceleration due to gravity. What is the force of the spaceman on the spacecraft?
(A) 4800 N (B) 4000 N (C) 3200 N (D) 800 N

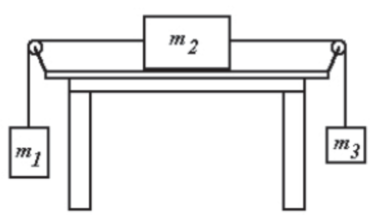


42. Two identical blocks of weight *W* are placed one on top of the other as shown in the diagram above. The upper block is tied to the wall. The lower block is pulled to the right with a force *F*. The coefficient of static friction between all surfaces in contact is μ. What is the largest force *F* that can be exerted before the lower block starts to slip?
(A) μW (B) $2\mu W$ (C) $3\mu W$ (D) $3\mu W/2$



43. A force *F* is used to hold a block of mass *m* on an incline as shown in the diagram (see above). The plane makes an angle of θ with the horizontal and *F* is perpendicular to the plane. The coefficient of friction between the plane and the block is μ. What is the minimum force, *F*, necessary to keep the block at rest?
(A) $mg\cos\theta$ (B) $mg\sin\theta$ (C) $mg\sin\theta/\mu$ (D) $mg(\sin\theta - \mu\cos\theta)/\mu$

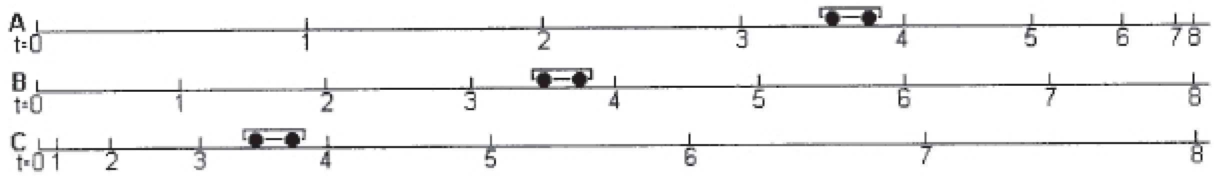
44. When the speed of a rear-drive car is increasing on a horizontal road, what is the direction of the frictional force on the tires?
(A) backward on the front tires and forward on the rear tires
(B) forward on the front tires and backward on the rear tires
(C) forward on all tires
(D) backward on all tires



45. Given the three masses as shown in the diagram above, if the coefficient of kinetic friction between the large mass (m_2) and the table is μ, what would be the upward acceleration of the small mass (m_3)? The mass and friction of the cords and pulleys are small enough to produce a negligible effect on the system.
(A) $g(m_1 + m_2\mu)/(m_1 + m_2 + m_3)$ (B) $g\mu(m_1 + m_2 + m_3)/(m_1 - m_2 - m_3)$
(C) $g\mu(m_1 - m_2 - m_3)/(m_1 + m_2 + m_3)$ (D) $g(m_1 - \mu m_2 - m_3)/(m_1 + m_2 + m_3)$

Questions 47-48

Three identical laboratory carts A, B, and C are each subject to a constant force F_A, F_B, and F_C, respectively. One or more of these forces may be zero. The diagram below shows the position of each cart at each second of an 8.0 second interval.

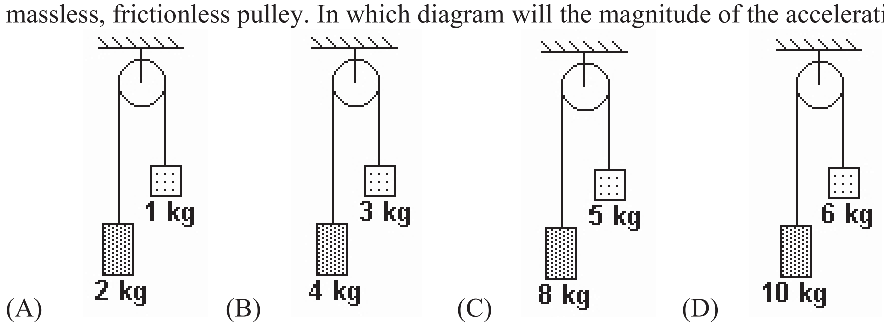


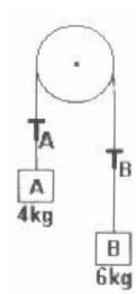
47. Which car has the greatest average velocity during the interval?
(A) A (B) B (C) C (D) all three average velocities are equal

48. How does the magnitude of the force acting on each car compare?
(A) $F_A > F_B > F_C$ (B) $F_A = F_C > F_B$ (C) $F_A > F_C = F_B$ (D) $F_A = F_B > F_C$

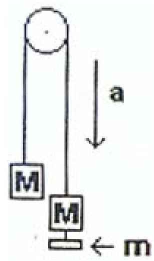
49. A skydiver is falling at terminal velocity before opening her parachute. After opening her parachute, she falls at a much smaller terminal velocity. How does the total upward force before she opens her parachute compare to the total upward force after she opens her parachute?
(A) The ratio of the forces is equal to the ratio of the velocities.
(B) The upward force with the parachute will depend on the size of the parachute.
(C) The upward force before the parachute will be greater because of the greater velocity.
(D) The upward force in both cases must be the same.

50. Each of the diagrams below represents two weights connected by a massless string which passes over a massless, frictionless pulley. In which diagram will the magnitude of the acceleration be the largest?

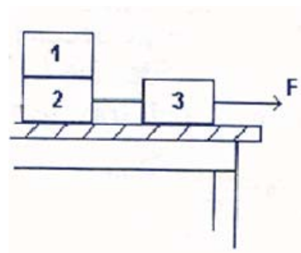




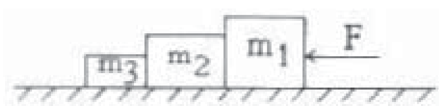
51. A simple Atwood's machine is shown in the diagram above. It is composed of a frictionless lightweight pulley with two cubes connected by a light string. If cube A has a mass of 4.0 kg and cube B has a mass of 6.0 kg, the system will move such that cube B accelerates downwards. What would be the tension in the two parts of the string between the pulley and the cubes?
(A) $T_A = 47$ N ; $T_B = 71$ N (B) $T_A = 47$ N ; $T_B = 47$ N (C) $T_A = 47$ N ; $T_B = 42$ N
(D) $T_A = 39$ N ; $T_B = 39$ N



52. A simple Atwood's machine remains motionless when equal masses M are placed on each end of the chord. When a small mass m is added to one side, the masses have an acceleration a. What is M? You may neglect friction and the mass of the cord and pulley.
(A) $\frac{m(g-a)}{2a}$ (B) $\frac{2m(g-a)}{a}$ (C) $\frac{2m(g+a)}{a}$ (D) $\frac{m(g+a)}{2a}$



53. Block 1 is stacked on top of block 2. Block 2 is connected by a light cord to block 3, which is pulled along a frictionless surface with a force F as shown in the diagram. Block 1 is accelerated at the same rate as block 2 because of the frictional forces between the two blocks. If all three blocks have the same mass m, what is the minimum coefficient of static friction between block 1 and block 2?
(A) 2F/mg (B) F/mg (C) 3F/2mg (D) F/3mg

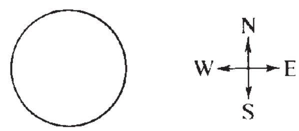


54. Three blocks (m_1, m_2, and m_3) are sliding at a constant velocity across a rough surface as shown in the diagram above. The coefficient of kinetic friction between each block and the surface is μ. What would be the force of m_1 on m_2?
(A) $(m_2 + m_3)g\mu$ (B) $F - (m_2 - m_3)g\mu$ (C) F (D) $m_1g\mu - (m_2 + m_3)g\mu$

SECTION B – Circular Motion

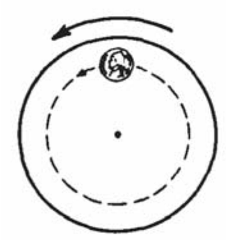
1. **Multiple Correct:** A person stands on a merry-go-round which is rotating at constant angular speed. Which of the following are true about the frictional force exerted on the person by the merry-go-round? Select two answers.
(A) The force is greater in magnitude than the frictional force exerted on the person by the merry-go-round.
(B) The force is opposite in direction to the frictional force exerted on the merry-go-round by the person.
(C) The force is directed away from the center of the merry-go-round.
(D) The force is dependent on the person's mass.

2. A ball attached to a string is whirled around in a horizontal circle having a radius R. If the radius of the circle is changed to 4R and the same centripetal force is applied by the string, the new speed of the ball is which of the following?
(A) One-quarter the original speed
(B) One-half the original speed
(C) The same as the original speed
(D) Twice the original speed

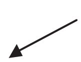


View of Track from Above

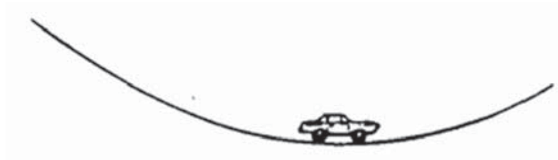
3. A racing car is moving around the circular track of radius 300 meters shown above. At the instant when the car's velocity is directed due east, its acceleration is directed due south and has a magnitude of 3 meters per second squared. When viewed from above, the car is moving
(A) clockwise at 30 m/s
(B) clockwise at 10 m/s
(C) counterclockwise at 30 m/s
(D) counterclockwise at 10 m/s


View from Above

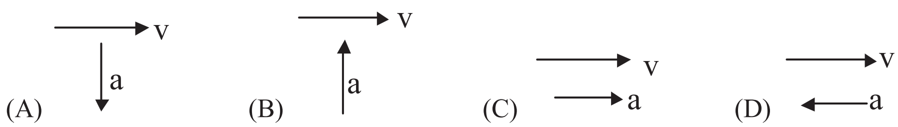
4. The horizontal turntable shown above rotates at a constant rate. As viewed from above, a coin on the turntable moves counterclockwise in a circle as shown. Which of the following vectors best represents the direction of the frictional force exerted on the coin by the turntable when the coin is in the position shown?

(A) (B)  (C)  (D) 

5. In which of the following situations would an object be accelerated? Select two answers.
(A) It moves in a straight line at constant speed.
(B) It moves with uniform circular motion.
(C) It travels as a projectile in a gravitational field with negligible air resistance.
(D) It is at rest.



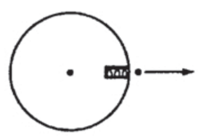
6. An automobile moves at constant speed down one hill and up another hill along the smoothly curved surface shown above. Which of the following diagrams best represents the directions of the velocity and the acceleration of the automobile at the instant that it is at the lowest position. as shown?



7. A car initially travels north and then turns to the left along a circular curve. This causes a package on the seat of the car to slide toward the right side of the car. Which of the following is true of the net force on the package while it is sliding?
(A) The force is directed away from the center of the circle.
(B) There is not enough force directed north to keep the package from sliding.
(C) There is not enough force tangential to the car's path to keep the package from sliding.
(D) There is not enough force directed toward the center of the circle to keep the package from sliding.

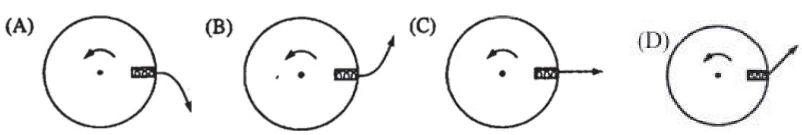
8. A child has a toy tied to the end of a string and whirls the toy at constant speed in a horizontal circular path of radius R. The toy completes each revolution of its motion in a time period T. What is the magnitude of the acceleration of the toy?

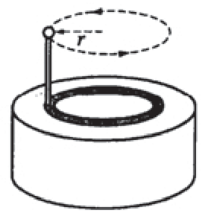
(A) Zero (B) $\dfrac{4\pi^2 R}{T^2}$ (C) $\dfrac{\pi R}{T^2}$ (D) g



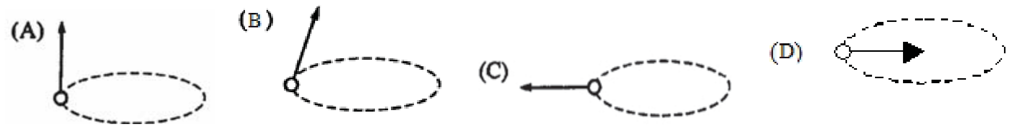
Top View

9. A compressed spring mounted on a disk can project a small ball. When the disk is not rotating, as shown in the top view above, the ball moves radially outward. The disk then rotates in a counterclockwise direction as seen from above, and the ball is projected outward at the instant the disk is in the position shown above. Which of the following best shows the subsequent path of the ball relative to the ground?

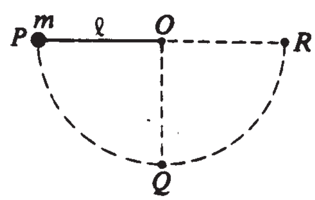




10. A steel ball supported by a stick rotates in a circle of radius r, as shown above. The direction of the net force acting on the ball when it is in the position shown is indicated by which of the following?



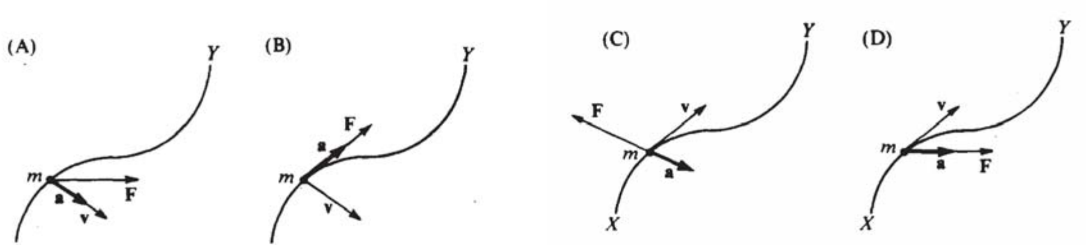
11. Inside a washing machine, the radius of the cylinder where the clothes sit is 0.50 m. In one of its settings the machine spins the cylinder at 2.0 revolutions per second. What is the acceleration of an item of clothing?
(A) 0.080 m/s^2 (B) 1.6 m/s^2 (C) 8.0 m/s^2 (D) 79 m/s^2

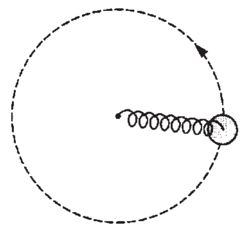


12. A ball of mass m is attached to the end of a string of length Q as shown above. The ball is released from rest from position P. where the string is horizontal. It swings through position Q. where the string is vertical, and then to position R. where the string is again horizontal. What are the directions of the acceleration vectors of the ball at positions Q and R?

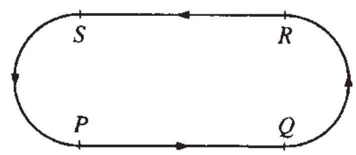
	Position Q	Position R
(A)	Downward	Downward
(B)	Downward	To the right
(C)	Upward	Downward
(D)	Upward	To the left

13. A mass m moves on a curved path from point X to point Y. Which of the following diagrams indicates a possible combination of the net force F on the mass, and the velocity v and acceleration a of the mass at the location shown?

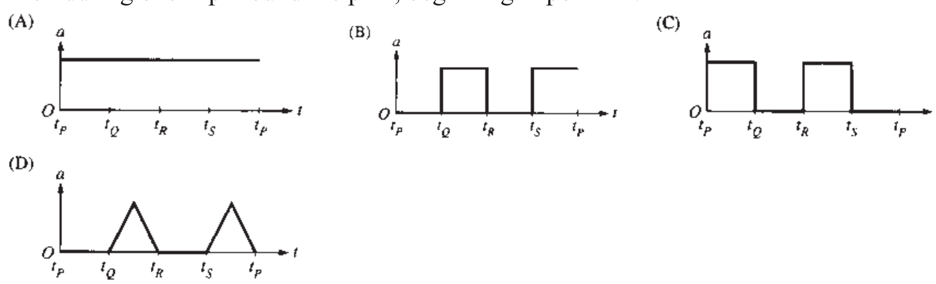




14. A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view above. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible. What is the centripetal force on the disc?
(A) 0.3 N (B) 3N (C) 10 N (D) 300 N



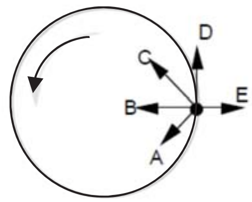
15. A figure of a dancer on a music box moves counterclockwise at constant speed around the path shown above. The path is such that the lengths of its segments, *PQ, QR, RS*, and *SP*, are equal. Arcs *QR* and *SP* are semicircles. Which of the following best represents the magnitude of the dancer's acceleration as a function of time t during one trip around the path, beginning at point *P* ?



16. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
(A) vertically upward
(B) horizontally forward
(C) horizontally backward
(D) zero

17. A car is traveling on a road in hilly terrain, see figure to the right. Assume the car has speed v and the tops and bottoms of the hills have radius of curvature R. The driver of the car is most likely to feel weightless:

(A) at the top of a hill when $v > \sqrt{gR}$
(B) at the bottom of a hill when $v > \sqrt{gR}$
(C) going down a hill when $v = \sqrt{gR}$
(D) at the top of a hill when $v = gR$

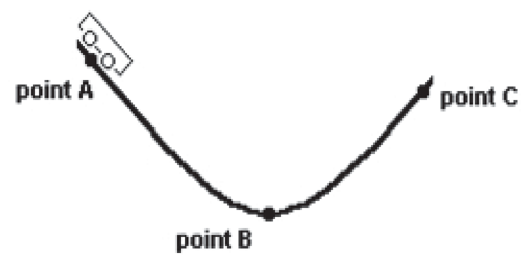


18. An object shown in the accompanying figure moves in uniform circular motion. Which arrow best depicts the net force acting on the object at the instant shown?
    (A) A (B) B (C) C (D) D

19. **Multiple Correct:** A child whirls a ball at the end of a rope, in a uniform circular motion. Which of the following statements is true? Select two answers.
    (A) The speed of the ball is constant
    (B) The velocity is of the ball is constant
    (C) The magnitude of the ball's acceleration is constant
    (D) The net force on the ball is directed radially outwards

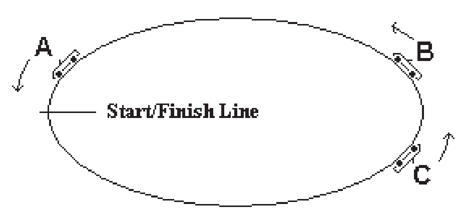
20. An astronaut in an orbiting space craft attaches a mass m to a string and whirls it around in uniform circular motion. The radius of the circle is R, the speed of the mass is v, and the tension in the string is F. If the mass, radius, and speed were all to double the tension required to maintain uniform circular motion would be
    (A) F (B) $2F$ (C) $4F$ (D) $8F$

21. Assume the roller coaster cart rolls along the curved track from point A to point C under the influence of gravity. Assume the friction between the track and the cart is negligible. What would be the direction of the cart's acceleration at point B?
    (A) upward
    (B) downward
    (C) forward
    (D) backward



<u>Questions 22 – 23</u>

The diagram below is a snapshot of three cars all moving counterclockwise during a one lap race on an elliptical track.

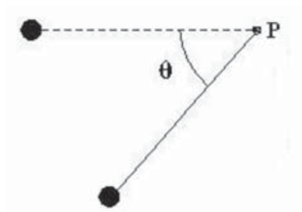


22. Which car has had the lowest average speed during the race so far?
    (A) car A
    (B) car B
    (C) car C
    (D) all three cars have had the same average speed

23. Which car at the moment of the snapshot MUST have a net force acting on it?
    (A) car A
    (B) car B
    (C) car C
    (D) all three cars have net forces acting on them

24. A centripetal force of 5.0 newtons is applied to a rubber stopper moving at a constant speed in a horizontal circle. If the same force is applied, but the radius is made smaller, what happens to the speed, *v*, and the frequency, *f*, of the stopper?

(A) *v* increases and *f* increases
(B) *v* decreases and *f* decreases
(C) *v* increases and *f* decreases
(D) *v* decreases and *f* increases



25. Astronauts on the Moon perform an experiment with a simple pendulum that is released from the horizontal position at rest. At the moment shown in the diagram with $0° < \theta < 90°$, the total acceleration of the mass may be directed in which of the following ways?
(A) straight to the right
(B) straight upward
(C) straight downward
(D) straight along the connecting string toward point P (the pivot)

26. A 4.0 kg mass is attached to one end of a rope 2 m long. If the mass is swung in a vertical circle from the free end of the rope, what is the tension in the rope when the mass is at its highest point if it is moving with a speed of 5 m/s?
(A) 5.4 N (B) 10.8 N (C) 50 N (D) 65.4 N

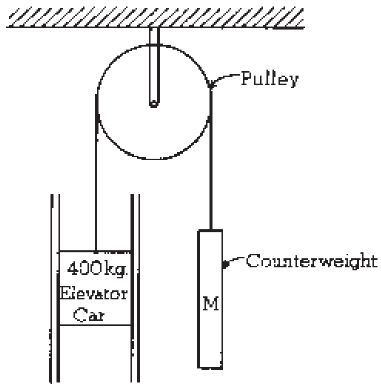
27. A ball of mass *m* is fastened to a string. The ball swings at constant speed in a vertical circle of radius *R* with the other end of the string held fixed. Neglecting air resistance, what is the difference between the string's tension at the bottom of the circle and at the top of the circle?
(A) mg (B) $2mg$ (C) $4mg$ (D) $8mg$

28. An object weighing 4 newtons swings on the end of a string as a simple pendulum. At the bottom of the swing, the tension in the string is 6 newtons. What is the magnitude of the centripetal acceleration of the object at the bottom of the swing?
(A) 0.5 g (B) g (C) 1.5 g (D) 2.5 g

29. Riders in a carnival ride stand with their backs against the wall of a circular room of diameter 8.0 m. The room is spinning horizontally about an axis through its center at a rate of 45 rev/min when the floor drops so that it no longer provides any support for the riders. What is the minimum coefficient of static friction between the wall and the rider required so that the rider does not slide down the wall?
(A) 0.056 (B) 0.11 (C) 0.53 (D) 8.9

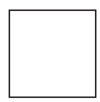
AP Physics Free Response Practice – Dynamics

SECTION A – Linear Dynamics



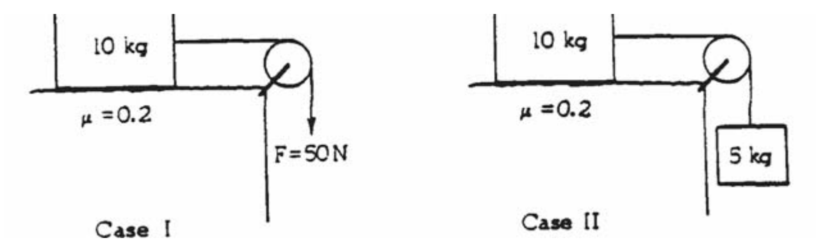
1976B1. The two guide rails for the elevator shown above each exert a constant friction force of 100 newtons on the elevator car when the elevator car is moving upward with an acceleration of 2 meters per second squared. The pulley has negligible friction and mass. Assume g = 10 m/sec^2.

a. On the diagram below, draw and label all forces acting on the elevator car. Identify the source of each force.



b. Calculate the tension in the cable lifting the 400-kilogram elevator car during an upward acceleration of 2 m/sec^2. (Assume g 10 m/sec^2.)

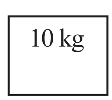
c. Calculate the mass M the counterweight must have to raise the elevator car with an acceleration of 2 m/sec^2.

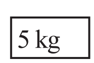


1979B2. A 10-kilogram block rests initially on a table as shown in cases I and II above. The coefficient of sliding friction between the block and the table is 0.2. The block is connected to a cord of negligible mass, which hangs over a massless frictionless pulley. In case I a force of 50 newtons is applied to the cord. In case II an object of mass 5 kilograms is hung on the bottom of the cord. Use g = 10 meters per second squared.

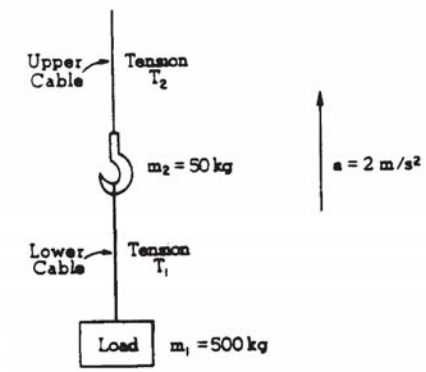
a. Calculate the acceleration of the 10-kilogram block in case I.

b. On the diagrams below, draw and label all the forces acting on each block in case II





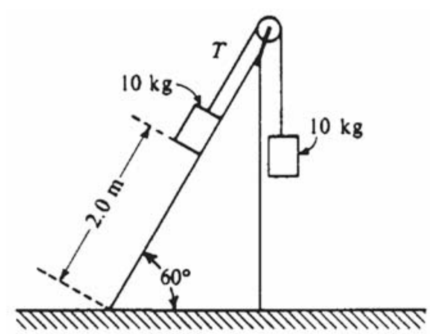
c. Calculate the acceleration of the 10-kilogram block in case II.



1982B2. A crane is used to hoist a load of mass m_1 = 500 kilograms. The load is suspended by a cable from a hook of mass m_2 = 50 kilograms, as shown in the diagram above. The load is lifted upward at a constant acceleration of 2 m/s^2.

a. On the diagrams below draw and label the forces acting on the hook and the forces acting on the load as they accelerate upward

b. Determine the tension T_1 in the lower cable and the tension T_2 in the upper cable as the hook and load are accelerated upward at 2 m/s^2. Use g = 10 m/s^2.

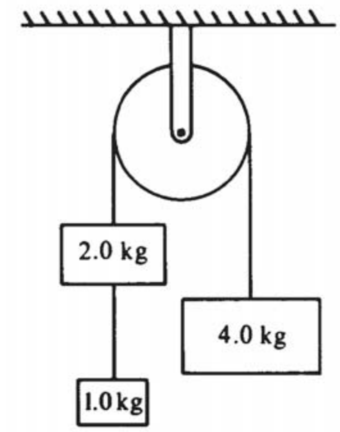


1985B2 (modified) Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use g = 10 m/s^2, sin 60° = 0.87, and cos 60° = 0.50.

a. What is the tension T in the string?

b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

c. Determine the magnitude of the frictional force acting on the box on the plane.



1986B1. Three blocks of masses 1.0, 2.0, and 4.0 kilograms are connected by massless strings, one of which passes over a frictionless pulley of negligible mass, as shown above. Calculate each of the following.

a. The acceleration of the 4-kilogram block
b. The tension in the string supporting the 4-kilogram block
c. The tension in the string connected to the l-kilogram block

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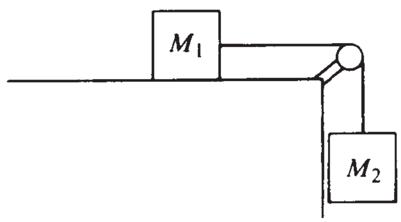


1987B1. In the system shown above, the block of mass M_1 is on a rough horizontal table. The string that attaches it to the block of mass M_2 passes over a frictionless pulley of negligible mass. The coefficient of kinetic friction μ_k between M_1 and the table is less than the coefficient of static friction μ_s

a. On the diagram below, draw and identify all the forces acting on the block of mass M_1.

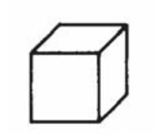
b. In terms of M_1 and M_2 determine the minimum value of μ_s that will prevent the blocks from moving.

The blocks are set in motion by giving M_2 a momentary downward push. In terms of M_1, M_2, μ_k, and g, determine each of the following:

c. The magnitude of the acceleration of M_1
d. The tension in the string.

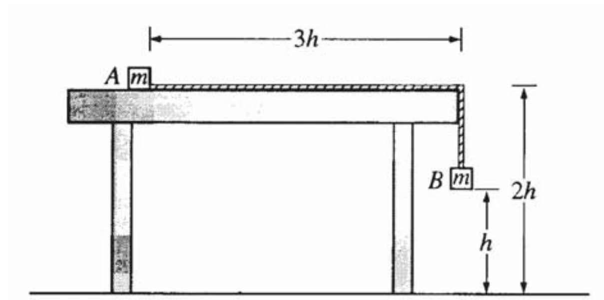
1988B1. A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of 5.2 m/s^2. Neglect air resistance on the package.

a. On the diagram below, draw and label all of the forces acting on the package.



b. Determine the tension in the rope.

c. When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at 5.2 m/s^2. Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.



1998B1 Two small blocks, each of mass m, are connected by a string of constant length 4h and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance 2h above the floor. Block B is then released from rest at a distance h above the floor at time t = 0. Express all algebraic answers in terms of h, m, and g.

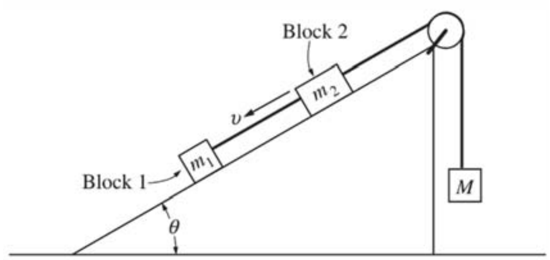
a. Determine the acceleration of block B as it descends.

b. Block B strikes the floor and does not bounce. Determine the time $t = t_1$ at which block B strikes the floor.

c. Describe the motion of block A from time t = 0 to the time when block B strikes the floor.

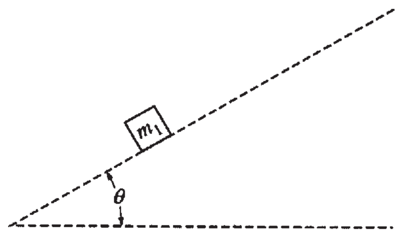
d. Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.

e. Determine the distance between the landing points of the two blocks.



2000B2. Blocks 1 and 2 of masses m_1 and m_2, respectively, are connected by a light string, as shown above. These blocks are further connected to a block of mass M by another light string that passes over a pulley of negligible mass and friction. Blocks l and 2 move with a constant velocity v down the inclined plane, which makes an angle θ with the horizontal. The kinetic frictional force on block 1 is f and that on block 2 is 2f.

a. On the figure below, draw and label all the forces on block m_1.

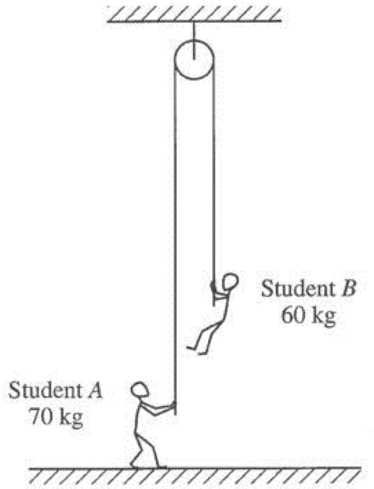


Express your answers to each of the following in terms of m_1, m_2, g, θ, and f.

b. Determine the coefficient of kinetic friction between the inclined plane and block 1.

c. Determine the value of the suspended mass M that allows blocks 1 and 2 to move with constant velocity down the plane.

d. The string between blocks 1 and 2 is now cut. Determine the acceleration of block 1 while it is on the inclined plane.



2003B1 A rope of negligible mass passes over a pulley of negligible mass attached to the ceiling, as shown above. One end of the rope is held by Student A of mass 70 kg, who is at rest on the floor. The opposite end of the rope is held by Student B of mass 60 kg, who is suspended at rest above the floor. Use $g = 10\ m/s^2$.

a. On the dots below that represent the students, draw and label free-body diagrams showing the forces on Student A and on Student B.

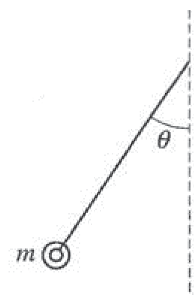
b. Calculate the magnitude of the force exerted by the floor on Student A.

Student B now climbs up the rope at a constant acceleration of $0.25\ m/s^2$ with respect to the floor.

c. Calculate the tension in the rope while Student B is accelerating.

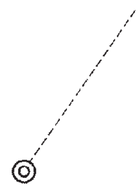
d. As Student B is accelerating, is Student A pulled upward off the floor? Justify your answer.

e. With what minimum acceleration must Student B climb up the rope to lift Student A upward off the floor?



2003Bb1 (modified) An airplane accelerates uniformly from rest. A physicist passenger holds up a thin string of negligible mass to which she has tied her ring, which has a mass *m*. She notices that as the plane accelerates down the runway, the string makes an angle θ with the vertical as shown above.

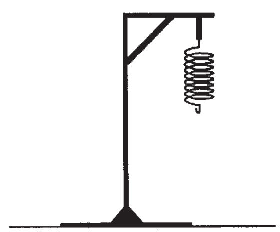
a. In the space below, draw a free-body diagram of the ring, showing and labeling all the forces present.



The plane reaches a takeoff speed of 65 m/s after accelerating for a total of 30 s.

b. Determine the minimum length of the runway needed.

c. Determine the angle θ that the string makes with the vertical during the acceleration of the plane before it leaves the ground.

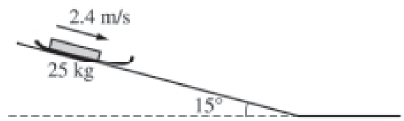


*1996B2 (modified) A spring that can be assumed to be ideal hangs from a stand, as shown above. You wish to determine experimentally the spring constant k of the spring.

a. i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?

Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.

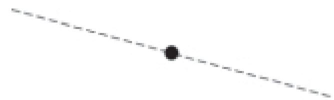
b. i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



B2007B1. An empty sled of mass 25 kg slides down a muddy hill with a constant speed of 2.4 m/s. The slope of the hill is inclined at an angle of 15° with the horizontal as shown in the figure above.

a. Calculate the time it takes the sled to go 21 m down the slope.

b. On the dot below that represents the sled, draw/label a free-body diagram for the sled as it slides down the slope



c. Calculate the frictional force on the sled as it slides down the slope.

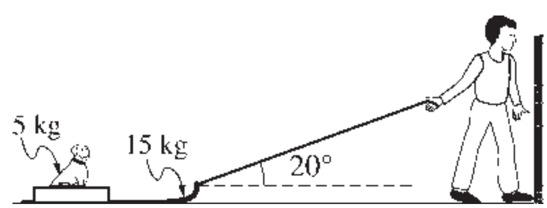
d. Calculate the coefficient of friction between the sled and the muddy surface of the slope.

e. The sled reaches the bottom of the slope and continues on the horizontal ground. Assume the same coefficient of friction.

i. In terms of velocity and acceleration, describe the motion of the sled as it travels on the horizontal ground.

ii. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel down the slope and across the horizontal ground. Clearly indicate with the symbol t_ℓ the time at which the sled leaves the slope.

B2007b1 (modified) A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20° above the horizontal, as shown in the figure. The coefficient of friction between the sled and the surface is 0.22.

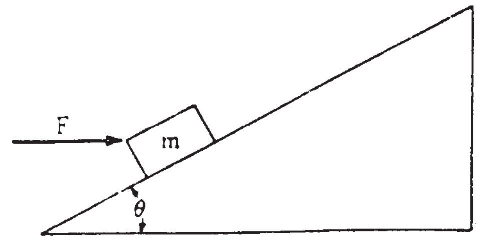


a. On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.

b. Calculate the normal force of the surface on the system.

c. Calculate the acceleration of the system.

d. At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed v versus time t for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_r the time at which the dog rolls off.

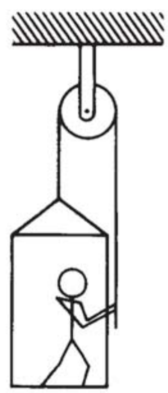


1981M1. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.

a. On the diagram of the block below, draw and label all the forces that act on the block as it slides up the plane.

b. Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.

c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?



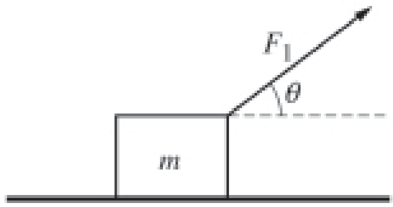
1986M1. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.

a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

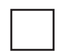
b. What is the tension in the rope under these new conditions?

c. Under these conditions, what is the force exerted by the platform on the person?



2007M1. A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1. Express all algebraic answers in terms of m, F_1, θ , a_1, and fundamental constants.

a. On the figure below, draw and label a free-body diagram showing all the forces on the block.



b. Derive an expression for the normal force exerted by the surface on the block.

c. Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

d. On the axes below, sketch graphs of the speed v and displacement x of the block as functions of time t if the block started from rest at $x = 0$ and $t = 0$.

e. If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.

1996M2. A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5 m/s^2

a. Determine the upward force exerted by the horizontal platform on the box as it is lowered.

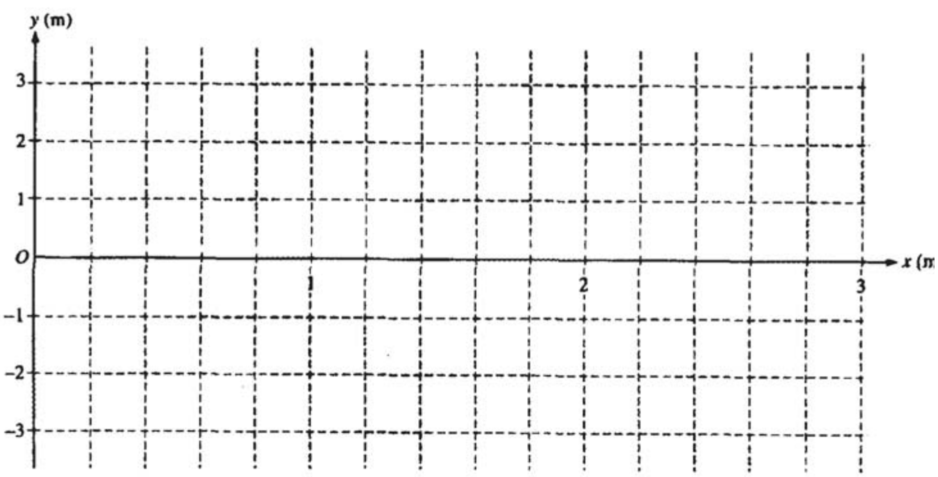
At time t = 0, the forklift also begins to move forward with an acceleration of 2 m/s^2 while lowering the box as described above. The box does not slip or tip over.

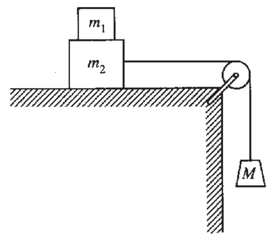
b. Determine the frictional force on the box.

c. Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.

d. Determine an equation for the path of the box that expresses y as a function of x (and <u>not</u> of t), assuming that, at time t = 0, the box has a horizontal position x = 0 and a vertical position y = 2 m above the ground, with zero velocity.

e. On the axes below sketch the path taken by the box





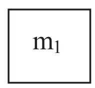
1998M3. Block 1 of mass m_1 is placed on block 2 of mass m_2 which is then placed on a table. A string connecting block 2 to a hanging mass M passes over a pulley attached to one end of the table, as shown above. The mass and friction of the pulley are negligible. The coefficients of friction between blocks 1 and 2 and between block 2 and the tabletop are nonzero and are given in the following table.

	Coefficient Between Blocks 1 and 2	Coefficient Between Block 2 and the Tabletop
Static	μ_{s1}	μ_{s2}
Kinetic	μ_{k1}	μ_{k2}

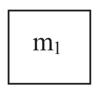
Express your answers in terms of the masses, coefficients of friction, and g, the acceleration due to gravity.

a. Suppose that the value of M is small enough that the blocks remain at rest when released. For each of the following forces, determine the magnitude of the force and draw a vector on the block provided to indicate the direction of the force if it is nonzero.

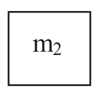
i. The normal force N_1 exerted on block 1 by block 2



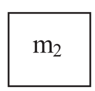
ii. The friction force f_1 exerted on block 1 by block 2



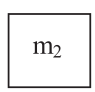
iii. The force T exerted on block 2 by the string



iv. The normal force N_2 exerted on block 2 by the tabletop



v. The friction force f_2 exerted on block 2 by the tabletop



b. Determine the largest value of M for which the blocks can remain at rest.

c. Now suppose that M is large enough that the hanging block descends when the blocks are released. Assume that blocks 1 and 2 are moving as a unit (no slippage). Determine the magnitude a of their acceleration.

d. Now suppose that M is large enough that as the hanging block descends, block 1 is slipping on block 2. Determine each of the following.

i. The magnitude a_1 of the acceleration of block 1

ii. The magnitude a_2 of the acceleration of block 2

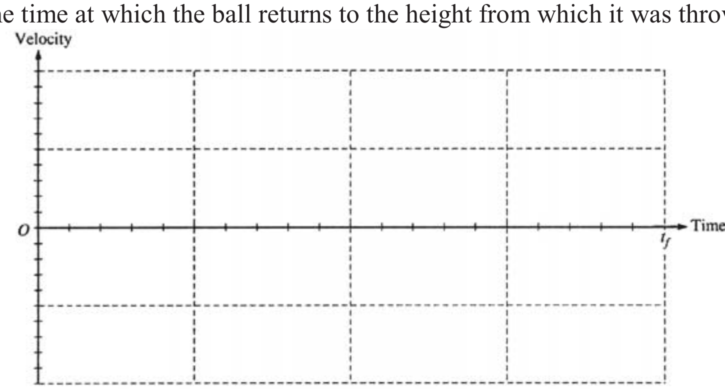
*2005M1 (modified) A ball of mass M is thrown vertically upward with an initial speed of v_o. It experiences a force of air resistance given by $F = -kv$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_o, and fundamental constants.

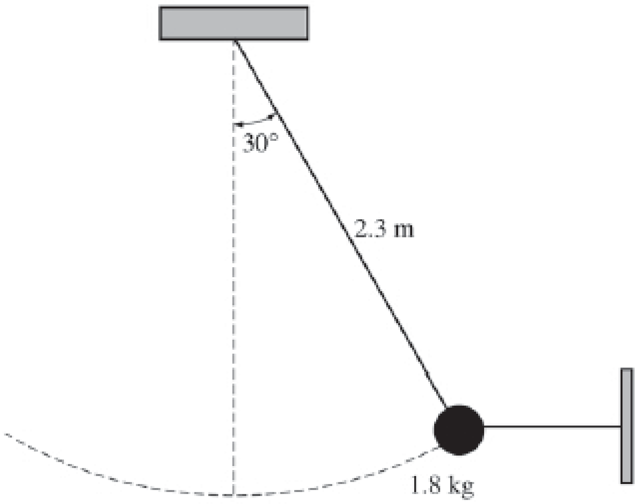
a. Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?
____ increases ____ decreases ____ remains the same
Justify your answer.

b. Determine the terminal speed of the ball as it moves downward.

c. Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?
____ longer to rise ____ longer to fall
Justify your answer.

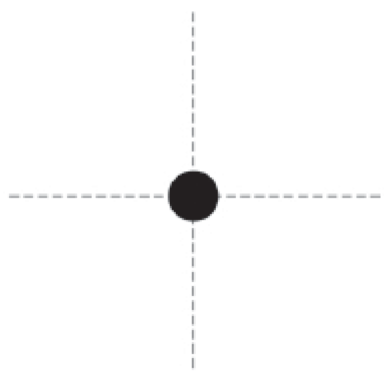
d. On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



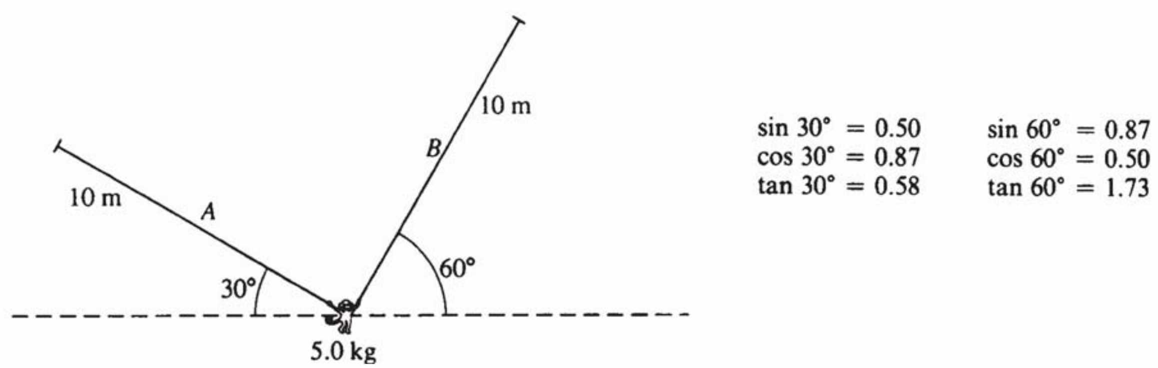


2005B2. A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



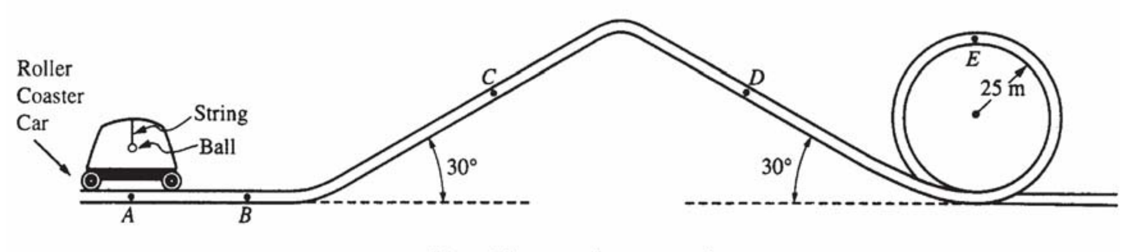
(b) Calculate the tension in the horizontal string.



1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.



Note: Figure not drawn to scale.

1995B3. Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

a. Initially, the car is at rest at point A.

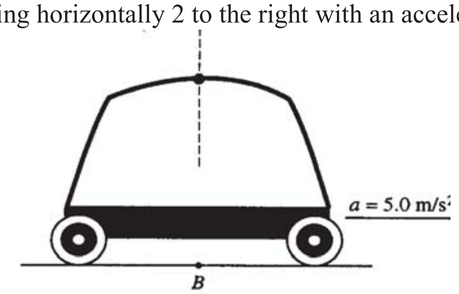
i. On the diagram below, draw and label all the forces acting on the 0.10-kilogram ball.

ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a 30° incline, goes down a 30° incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. In each situation, assume that the ball has stopped swinging back and forth.

1) Determine the horizontal component T_h of the tension in the string in newtons and record your answer in the space provided.
2)Determine the vertical component T_v of the tension in the string in newtons and record your answer in the space provided.
3)Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.

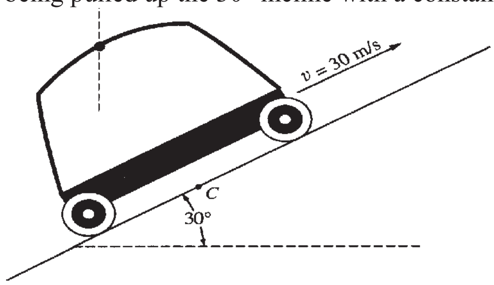
b. The car is at point B moving horizontally 2 to the right with an acceleration of 5.0 m/s .



T_h =___________________________

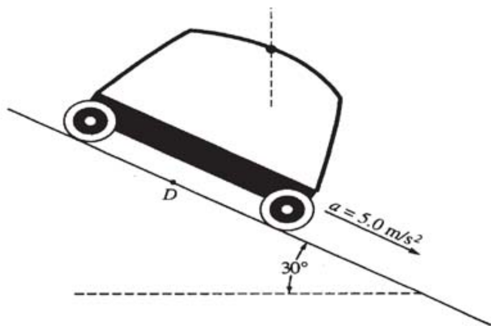
T_v = ___________________________

c. The car is at point C and is being pulled up the 30° incline with a constant speed of 30 m/s.



T_h = ___________________________

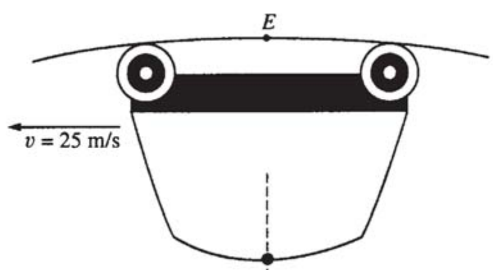
T_v = ___________________________



d. The car is at point D moving down the incline with an acceleration of 5.0 m/s^2 .

T_h =___________________________

T_v = ___________________________

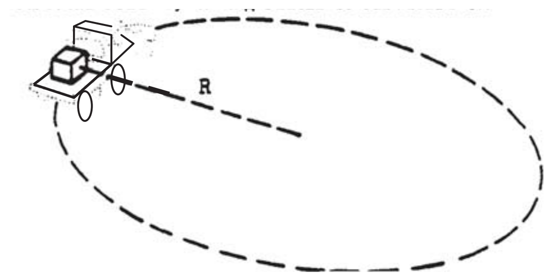


e. The car is at point E moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.

T_h =___________________________

T_v = ___________________________

SECTION B – Circular Motion



1977 B2. A box of mass M, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed v. The truck is on an unbanked circular roadway having radius of curvature R.

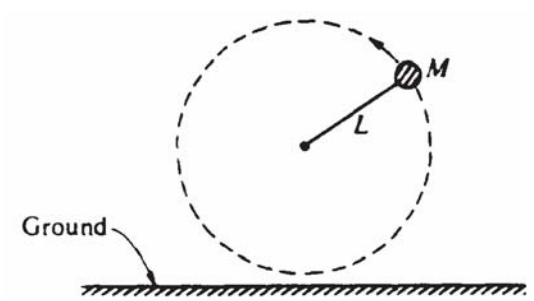
a. On the diagram provided above, indicate and clearly label all the force vectors acting on the box.

b. Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v, R, and g.

If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

c. On the diagram above indicate and clearly label the two forces acting on the box under these conditions

d. Which, if either, of the two forces acting on the box is greater in magnitude?



1984B1. A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

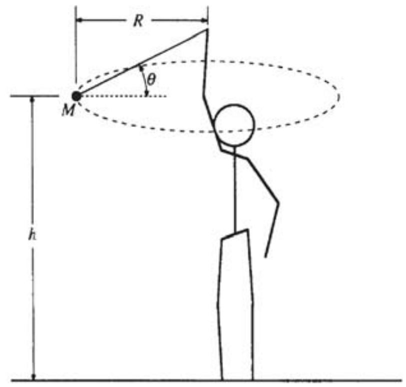
a. Determine the magnitude and direction of the net force on the ball when it is at the top.

b. Determine the speed v_o of the ball at the top.

The string is then cut when the ball is at the top.

c. Determine the time it takes the ball to reach the ground.

d. Determine the horizontal distance the ball travels before hitting the ground.

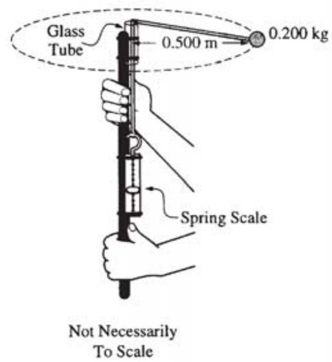


1989B1. An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_o and the circular path has radius R and is a height h above the ground. Neglect air friction.

a. Determine the following, expressing all answers in terms of h, v_o, and g.
   i. The time required for the object to hit the ground after the string breaks
   ii. The horizontal distance the object travels from the time the string breaks until it hits the ground
   iii. The speed of the object just before it hits the ground

b. On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.

c. Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_o, & g.



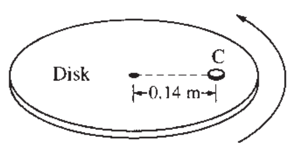
1997B2 (modified) To study circular motion, two students use the hand-held device shown above, which consists of a rod on which a spring scale is attached. A polished glass tube attached at the top serves as a guide for a light cord attached the spring scale. A ball of mass 0.200 kg is attached to the other end of the cord. One student swings the teal around at constant speed in a horizontal circle with a radius of 0.500 m. Assume friction and air resistance are negligible.

a. Explain how the students, by using a timer and the information given above, can determine the speed of the ball as it is revolving.
b. The speed of the ball is determined to be 3.7 m/s. Assuming that the cord is horizontal as it swings, calculate the expected tension in the cord.
c. The actual tension in the cord as measured by the spring scale is 5.8 N. What is the percent difference between this measured value of the tension and the value calculated in part b.?

The students find that, despite their best efforts, they cannot swing the ball so that the cord remains exactly horizontal.

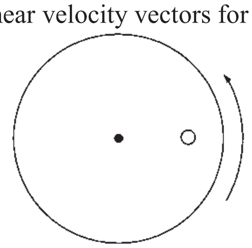
d. i. On the picture of the ball below, draw vectors to represent the forces acting on the ball and identify the force that each vector represents.

ii. Explain why it is not possible for the ball to swing so that the cord remains exactly horizontal.
iii. Calculate the angle that the cord makes with the horizontal.

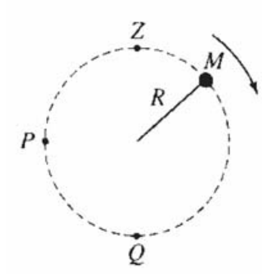


1999B5 A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.



b. Determine the linear speed of the coin.
c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.
d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c)? Explain your reasoning.

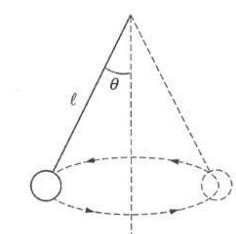


Side View

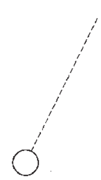
2001B1. A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. On the figures below, draw and label all the forces exerted on the ball when it is at points P and Q, respectively.

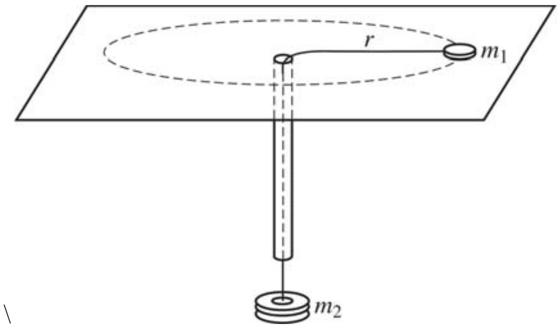
b. Derive an expression for v_{min} the minimum speed the ball can have at point Z without leaving the circular path.
c. The maximum tension the string can have without breaking is T_{max} Derive an expression for v_{max}, the maximum speed the ball can have at point Q without breaking the string.
d. Suppose that the string breaks at the instant the ball is at point P. Describe the motion of the ball immediately after the string breaks.



2002B2B A ball attached to a string of length *l* swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and *T* is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.
a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.
c. Determine the speed of the ball.
d. Determine the frequency of revolution of the ball.
e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.



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2009Bb1 An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.

a. Derive the equation $P = 2\pi\sqrt{\dfrac{m_1 r}{m_2 g}}$ that relates P and m_2.

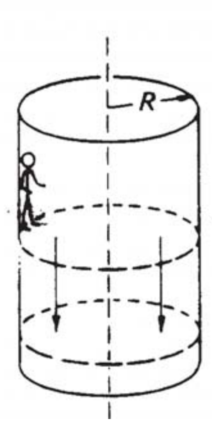
The procedure is repeated, and the period P is determined for four different values of m_2, where $m_1 = 0.012$ kg and $r = 0.80$ m. The data, which are presented below, can be used to compute an experimental value for g.

m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75

b. What quantities should be graphed to yield a straight line with a slope that could be used to determine g?

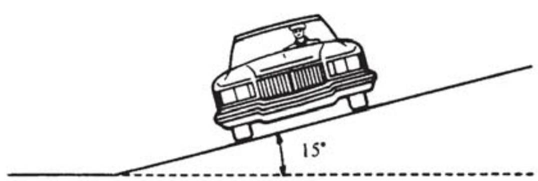
c. On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.

d. Use your graph to calculate the experimental value of g.



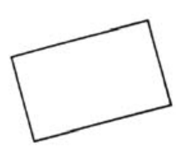
*1984M1 (modified) An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, The radius R of the cylinder is 5 meters, the frequency of the cylinder when rotating is $1/\pi$ revolutions per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

a. On the diagram above, draw and identify the forces on the rider when the system is rotating and the floor has dropped down.
b. Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.
c. Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.
d. At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.

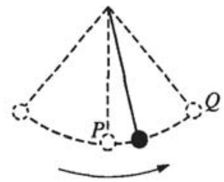


1988M1. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.
a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.

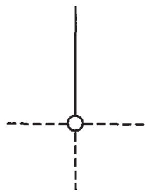


b. On the diagram above, in which the block represents the automobile, draw and label all of the forces on the automobile.
c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.

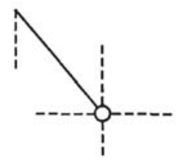


1998B6 A heavy ball swings at the end of a string as shown above, with negligible air resistance. Point P is the lowest point reached by the ball in its motion, and point Q is one of the two highest points.

a. On the following diagrams draw and label vectors that could represent the velocity and acceleration of the ball at points P and Q. If a vector is zero, explicitly state this fact. The dashed lines indicate horizontal and vertical directions.

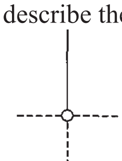


i. Point P

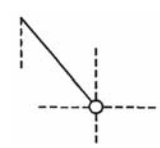


ii. Point Q

b. After several swings, the string breaks. The mass of the string and air resistance are negligible. On the following diagrams, sketch the path of the ball if the break occurs when the ball is at point P or point Q. In each case, briefly describe the motion of the ball after the break.



i. Point P



ii. Point Q

ANSWERS - AP Physics Multiple Choice Practice – Dynamics

SECTION A – Linear Dynamics

	Solution	Answer
1.	As T_2 is more vertical, it is supporting more of the weight of the ball. The horizontal components of T_1 and T_2 are equal.	C
2.	Normal force is perpendicular to the incline, friction acts up, parallel to the incline (opposite the motion of the block), gravity acts straight down.	D
3.	The component of the weight down the plane is 20 N sin θ. The net force is 4 N, so the friction force up the plane must be 4 N less than 20 N.	D
4.	The force between objects is the applied force times the ratio of the mass behind the rope to the total mass being pulled. This can be derived from $a = F/m_{total}$ and $F_T = m_{\text{behind the rope}}a$	D
5.	Since the ball's speed is increasing from rest, the retarding force F is also increasing. The net force, which is the weight of the ball minus *F*, is thus decreasing. So the acceleration also must decrease. Time $t/2$ is before the constant-speed motion begins, so the acceleration has not yet decreased to zero.	B
6.	For vertical equilibrium, the weight equals the normal force plus the vertical component of F. This leads to the normal force being W – something. The block remains in contact with the surface, so the normal force does not reach zero.	B
7.	The bottom of the rope supports the box, while the top of the rope must support the rope itself and the box.	D
8.	The vertical components of the tension in the rope are two equal upward components of $T\cos\theta$, which support the weight. $\Sigma F_y = 0 = 2T\cos\theta - W$	D
9.	$\Sigma F_{external} = m_{total}a$; mg is the only force acting from outside the system of masses so we have mg = (4m)a	A
10.	The weight component perpendicular to the plane is 20 N sin 37°. To get equilibrium perpendicular to the plane, the normal force must equal this weight component, which must be less than 20 N.	B
11.	(A) is the definition of translational equilibrium. Equilibrium means no net force and no acceleration, so (D) is also correct.	A,D
12.	Motion at constant speed includes, for example, motion in a circle, in which the direction of the velocity changes and thus acceleration exists. Constant momentum for a single object means , that the velocity doesn't change.	D
13.	$\Sigma F = ma$; $F_T - mg = ma$; Let $F_T = 50$ N (the maximum possible tension) and m = W/g = 3 kg	A
14.	The sum of the tensions in the chains (250 N + T_{left}) must support the weight of the board and the person (125 N + 500 N)	C
15.	The board itself provides the same torque about the attachment point of both chains, but since the left chain provides a bigger force on the board, the person must be closer to the left chain in order to provide an equivalent torque on both chains by $\tau = Fd$.	A
16.	The horizontal component of the 30 N force is 15 N left. So the net force is 5 N left. So the acceleration is left. This could mean either A or D – when acceleration is opposite velocity, an object slows down.	A,D

17. Consider that no part of the system is in motion, this means at each end of the rope, a person pulling with 100 N of force is reacted to with a tension in the rope of 100 N. C

18. As v is proportional to t^2 and a is proportional to $\Delta v/t$, this means a should be proportional to t A

19. The direction of the force is the same as the direction of the acceleration, which is proportional to $\Delta v = v_f + (-v_i)$ B

20. A force diagram will show that the forces provided by each spring add up to 12 N: $F_1 + F_2 = 12$ N. Each force is kx; each spring is stretched the same amount x = 24 cm. So $k_1x + k_2x = 12$ N; dividing both sides by x shows that $k_1 + k_2 = 0.5$ N/cm. D

21. Net force is the gravitational force which acts downward D

22. $\Sigma F = ma = F\cos\phi - f$ D

23. The string pulling all three masses (total $6m$) must have the largest tension. String A is only pulling the block of mass $3m$ and string B is pulling a total mass of $5m$. C

24. At t = 2 s the force is 4 N. F = ma A

25. Since P is at an upward angle, the normal force is decreased as P supports some of the weight. Since a component of P balances the frictional force, P itself must be larger than f. A

26. The force of friction = $\mu F_N = 0.2 \times 10 \text{ kg} \times 9.8 \text{ m/s}^2 = 19.6$ N, which is greater than the applied force, which means the object is accelerating to the left, or slowing down A

27. The upward component of the tension is $T_{up} = T\sin\theta$, where θ is the angle to the horizontal. This gives $T = T_{up}/\sin\theta$. Since the upward components are all equal to one half the weight, the rope at the smallest angle (and the smallest value of sinθ) will have the greatest tension, and most likely break B

28. From the 1 kg block: F = ma giving a = 2 m/s^2. For the system: F = (4 kg)(2 m/s^2) D

29. Elevator physics: F_N represents the scale reading. $\Sigma F = ma$; $F_N - mg = ma$, or $F_N = m(g + a)$. The velocity of the elevator is irrelevant. B

30. Newton's third law C

31. The normal force is mgcosθ. For a horizontal surface, $F_N = mg$. At any angle $F_N < mg$ and F_f is proportional to F_N. C

32. Slope = $\Delta y/\Delta x$ = Weight/mass = acceleration due to gravity C

33. Newton's second law applied to m_1: $T = m_1a$, or $a = T/m_1$, substitute this into Newton's second law for the hanging mass: $m_2g - T = m_2a$ D

34. Elevator physics: R represents the scale reading. $\Sigma F = ma$; $R - mg = ma$, or $R = m(g + a)$. This ranks the value of R from largest to smallest as accelerating upward, constant speed, accelerating downward A

35. $\Sigma F = ma$ for the whole system gives $F - \mu(3m)g = (3m)a$ and solving for a gives $a = (F - 3\mu mg)/3m$. For the top block, $F_m = ma = m[(F - 3\mu mg)/3m]$ A

36. The normal force comes from the perpendicular component of the applied force which is Fcosθ = 50 N. The maximum value of static friction is then $\mu F_N = 25$ N. The upward component of the applied force is Fsinθ = 87 N. $\Sigma F_y = F_{up} - mg = 87 \text{ N} - 60 \text{ N} > 25$ N. Since the net force on the block is great than static friction can hold, the block will begin moving up the wall. Since it is in motion, kinetic friction is acting opposite the direction of the block's motion C

37. Since P is at a downward angle, the normal force is increased. Since a component of P balances the frictional force, P itself must be larger than f. A

38. Since the force is applied horizontally, the mass has no effect. C

39. The only force in the direction of the crate's acceleration is the force of friction from the sleigh B

40. Given that the box accelerates toward Ted, Ted's force must be greater than Mario's force plus the force of friction. Since Mario's force is ½ of Ted's force, the force of frction must be less than half of Ted's force. A

41. $\Sigma F = ma$; $F - mg = m(5g)$ or $F = 6mg$ A

42. Between the lower block and the tabletop, there is a force of friction to the left of maximum magnitude $\mu(2W)$ as both blocks are pushing down on the tabletop. There is also a force of friction acting to the left on the upper surface of the lower block due to the upper block of maximum magnitude μW. The total maximum static frictional force holding the lower block in place is therefore $\mu(2W) + \mu W$ C

43. The normal force on the block can be found from $\Sigma F_y = 0 = F_N - mg\cos\theta - F$. The force of friction necessary to hold the block in place is $mg\sin\theta$. Setting the force of friction equal to $mg\sin\theta$ gives $\mu F_N = mg\sin\theta = \mu(F + mg\cos\theta)$ D

44. This is a tricky one. In order to move the car forward, the rear tires roll back against the ground, the force of friction pushing forward on the rear tires. The front tires, however, are not trying to roll on their own, rather they begin rolling due to the friction acting backward, increasing their rate of rotation A

45. The external forces acting on the system of masses are the weights of block 1 (pulling the system to the left), the weight of block 3 (pulling the system to the right) and the force of friction on block 2 (pulling the system to the left with a magnitude $\mu F_N = \mu m_2 g$) $\Sigma F_{external} = m_{total}a$ gives $(m_1 g - \mu m_2 g - m_3 g) = (m_1 + m_2 + m_3)a$ D

46. $F = ma$ gives 30 N = (12 kg)a or an acceleration of 2.5 m/s^2. The 5 kg block is accelerating due to the tension in the rope $F_T = ma = (5\text{ kg})(2.5\text{ m/s}^2) = 12.5$ N. C

47. As they are all at the same position after 8 seconds, they all have the same average velocity D

48. Car A decelerates with the same magnitude that C accelerates. Car B is moving at constant speed, which means $F_B = 0$. B

49. When falling with terminal velocity, the force of air resistance equals your weight, regardless of the speed. D

50. For each case, $\Sigma F_{external} = m_{total}a$ gives $Mg - mg = (M + m)a$, or $a = \dfrac{M - m}{M + m}g$. A

51. The two ends of the light string must have the same tension, eliminating choices A and C. If choice D was correct, both masses would be accelerating downward and T_A must be greater than the weight of block A. B

52. $\Sigma F_{external} = m_{total}a$ gives $(M + m)g - Mg = (2M + m)a$ A

53. As the entire system moves as one, $F = (3m)a$, or $a = F/(3m)$. The force of friction acting on block 1 is the force moving block 1 and we have $\mu mg = m(F/(3m))$ D

54. Since the system is moving at constant velocity, m_1 is pushing m_2 and m_3 with a force equal to the force of friction acting on those two blocks, which is $\mu(F_{N2} + F_{N3})$ A

SECTION B – Circular Motion

1.	Newton's third law and friction force	B,D
2.	$F = mv^2/r$; $v = \sqrt{\frac{Fr}{m}}$; all other variables being constant, if r is quadrupled, v is doubled	D
3.	With acceleration south the car is at the top (north side) of the track as the acceleration points toward the center of the circular track. Moving east indicates the car is travelling clockwise. The magnitude of the acceleration is found from $a = v^2/r$	A
4.	The frictional force acts as the centripetal force (toward the center)	C
5.	Acceleration occurs when an object is changing speed and/or direction	B,C
6.	Velocity is tangential, acceleration points toward the center of the circular path	B
7.	To move in a circle, a force directed toward the center of the circle is required. While the package slides to the right in the car, it is actually moving in its original straight line path while the car turns from under it.	D
8.	$a = v^2/r$ and $v = 2\pi r/T$ giving $a = 4\pi^2 r/T^2$	B
9.	Once projected, the ball is no longer subject to a force and will travel in a straight line with a component of its velocity tangent to the circular path and a component outward due to the spring	D
10.	The net force is inward. The normal force is counteracted by gravity.	C
11.	$a = v^2/r$ where $v = 2\pi rf$ and $f = 2.0$ rev/sec	D
12.	At Q the ball is in circular motion and the acceleration should point to the center of the circle. At R, the ball comes to rest and is subject to gravity as in free-fall.	C
13.	The net force and the acceleration must point in the same direction. Velocity points tangent to the objects path.	D
14.	The centripetal force is provided by the spring where $F_C = F_s = kx$	B
15.	In the straight sections there is no acceleration, in the circular sections, there is a centripetal acceleration	B
16.	Once the stone is stuck, it is moving in circular motion. At the bottom of the circle, the acceleration points toward the center of the circle at that point.	A
17.	Feeling weightless is when the normal force goes to zero, which in only possible going over the top of the hill where mg (inward) – F_N (outward) = mv^2/R. Setting F_N to zero gives a maximum speed of $\sqrt{gR}$	A
18.	Centripetal force points toward the center of the circle	B
19.	While speed may be constant, the changing direction means velocity cannot be constant as velocity is a vector. In uniform circular motion, acceleration is constant.	A,C
20.	$F = mv^2/r$. $F_{new} = (2m)(2v)^2/(2r) = 4(mv^2/r) = 4F$	C
21.	Assuming the track is circular at the bottom, the acceleration points toward the center of the circular path	A
22.	Average speed = (total *distance*)/(total time). Lowest average speed is the car that covered the	C

least distance

23. As all the cars are changing direction, there must be a net force to change the direction of their velocity vectors D

24. $F = mv^2/r$; $v^2 = rF/m$, if r decreases, v will decrease with the same applied force. Also, $v = 2\pi r f$ so $4\pi^2 r^2 f = rF/m$, or $f = F/(4\pi^2 rm)$ and as r decreases, f increases. D

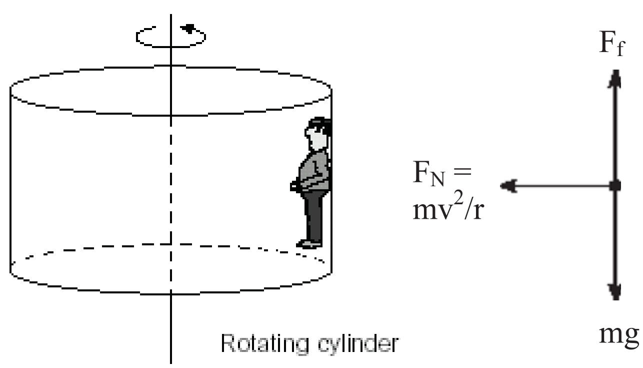
25. There is a force acting downward (gravity) and a centripetal force acting toward the center of the circle (up and to the right). Adding these vectors cannot produce resultants in the directions of B, C or D A

26. $\Sigma F = ma$; $mg + F_T = mv^2/r$ giving $F_T = mv^2/r - mg$ B

27. At the top of the circle, $\Sigma F = F_T + mg = mv^2/R$, giving $F_T = mv^2/R - mg$. At the bottom of the circle, $\Sigma F = F_T - mg = mv^2/R$, giving $F_T = mv^2/R + mg$ The difference is $(mv^2/R + mg) - (mv^2/R - mg)$ B

28. At the bottom of the swing, $\Sigma F = F_T - mg = ma_c$; since the tension is 1.5 times the weight of the object we can write $1.5mg - mg = ma_c$, giving $0.5mg = ma_c$ A

29. B

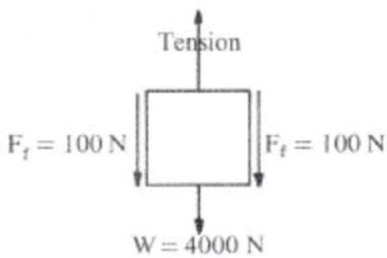


$F_f = mg$ to balance
$\mu F_N = \mu mv^2/r = mg$, where $v = 2\pi rf$ which gives $\mu = g/(4\pi^2 rf^2)$
Be careful! f is given in rev/min (45 rev/min = 0.75 rev/sec) and 8.0 m is the ride's diameter

SECTION A – Linear Dynamics

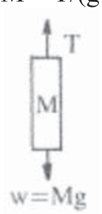
1976B1

a.



b. $\Sigma F = ma$; $T - W - 2F_f = 800$ N; T = 5000 N

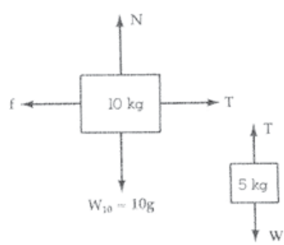
c. Looking at the FBD for the counterweight we have $\Sigma F = ma$; $Mg - T = Ma$
$M = T/(g - a)$ where T = 5000 N gives M = 625 kg



1979B2

a. $\Sigma F = ma$; 50 N – f = ma where f = μN and N = mg gives 50 N – μmg = ma; a = 3 m/s^2

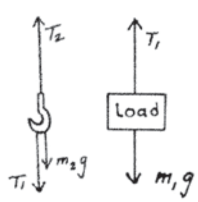
b.



c. $\Sigma F = ma$ for each block gives $W_5 - T = m_5a$ and $T - f = m_{10}a$. Adding the two equations gives $W_5 - f = (m_5 + m_{10})a$, or a = 2 m/s^2

1982B2

a.

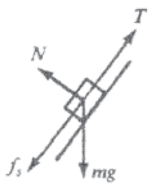


b. T_1 is in internal system force and will cancel in combined equations. Using $\Sigma F_{external} = m_{total}a$ gives $T_2 - m_1g - m_2g = (m_1 + m_2)a$, solving yields T_2 = 6600 N. Now using $\Sigma F = ma$ for the load gives $T_1 - m_1g = m_1a$ and T_1 = 6000 N

1985B2

a. Note that the system is at rest. The only forces on the hanging block are gravity and the tension in the rope, which means the tension must equal the weight of the hanging block, or 100 N. You cannot use the block on the incline because friction is acting on that block and the amount of friction is unknown.

b.

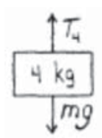


c. $\Sigma F = 0$; $f_s + mg\sin\theta - T = 0$ gives $f_s = 13$ N

1986B1

a. $\Sigma F_{external} = m_{total}a$; $m_4g - m_1g - m_2g = (m_4 + m_2 + m_1)a$ gives $a = 1.4$ m/s^2

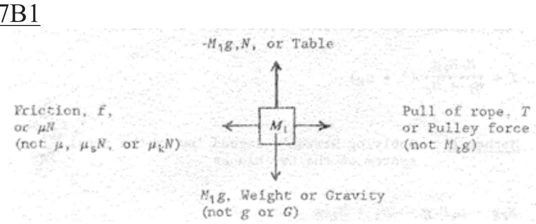
b. For the 4 kg block:
$\Sigma F = ma$
$mg - T_4 = ma$ gives
$T_4 = 33.6$ N



c. Similarly for the 1 kg block: $T_1 - mg = ma$ gives $T_1 = 11.2$ N

1987B1

a.

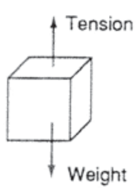


b. $\Sigma F_{ext} = m_{tot}a$; Where the maximum force of static friction on mass M_1 is μ_sN and $N = M_1g$; $M2g - \mu_sM_1g = 0$ gives $\mu_s = M_2/M_1$

c/d. $\Sigma F_{ext} = m_{tot}a$ where we now have kinetic friction acting gives $M_2g - \mu_kM_1g = (M_1 + M_2)a$
so $a = (M_2g - \mu_kM_1g)/(M_1 + M_2)$
$\Sigma F = ma$ for the hanging block gives $M_2g - T = M_2a$ and substituting a from above gives $T = \frac{M_1M_2g}{M_1+M_2}(1 + \mu_k)$

1988B1

a.



b. $\Sigma F = ma$ gives $T - mg = ma$ and $T = 1050$ N

c. The helicopter and the package have the same initial velocity, 30 m/s upward. Use $d = v_it + \frac{1}{2}at^2$
$d_h = (+30 \text{ m/s})t + \frac{1}{2}(+5.2 \text{ m/s}^2)t^2$ and $d_p = (+30 \text{ m/s})t + \frac{1}{2}(-9.8 \text{ m/s}^2)t^2$.
The difference between d_h and d_p is 30 m, but they began 5 m apart so the total distance is 35 m.

1998B1

a. $\Sigma F_{ext} = m_{tot}a$ gives $mg = 2ma$, or $a = g/2$

b. $d = v_0t + \frac{1}{2}at^2$; $h = 0 + \frac{1}{2}(g/2)t^2$ gives $t = 2\sqrt{\frac{h}{g}}$

c. Block A accelerates across the table with an acceleration equal to block B (g/2).

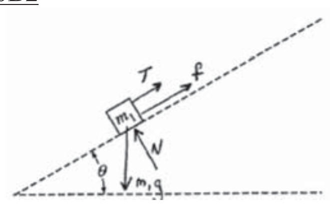
d. Block A is still in motion, but with no more applied force, Block A will move at constant speed across the table.

e. Since block B falls straight to the floor and stops, the distance between the landing points is equal to the horizontal distance block A lands from the edge of the table. The speed with which block A leaves the tabletop is the speed with which block B landed, which is found from $v = v_0 + at = \frac{g}{2}\left(2\sqrt{\frac{h}{g}}\right) = \sqrt{hg}$ and the time for block A to reach the floor is found from $2h = \frac{1}{2}gt^2$, which gives $t = 2\sqrt{\frac{h}{g}}$.

The distance is now $d = vt = \sqrt{hg} \times 2\sqrt{\frac{h}{g}} = 2h$

2000B2

a.



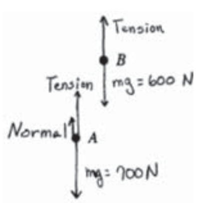
b. $f = \mu N$ where $N = m_1 g \cos\theta$ gives $\mu = \frac{f}{m_1 g \cos\theta}$

c. constant velocity means $\Sigma F = 0$ where $\Sigma F_{external} = m_1 g \sin\theta + m_2 g \sin\theta - f - 2f - Mg = 0$
solving for M gives $M = (m_1 + m_2)\sin\theta - (3f)/g$

d. Applying Newton's second law to block 1 gives $\Sigma F = m_1 g \sin\theta - f = m_1 a$ which gives $a = g\sin\theta - f/m_1$

2003B1

a.



b. The tension in the rope is equal to the weight of student B: $T = m_Bg = 600$ N
$\Sigma F_A = T + N - m_Ag = 0$ gives $N = 100$ N

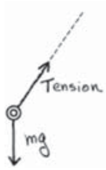
c. For the climbing student $\Sigma F = ma$; $T - m_Bg = m_Ba$ gives $T = 615$ N

d. For student A to be pulled off the floor, the tension must exceed the weight of the student, 700 N. No, the student is not pulled off the floor.

e. Applying Newton's second law to student B with a tension of 700 N gives $\Sigma F = T - m_Bg = m_Ba$ and solving gives $a = 1.67$ m/s^2

2003Bb1

a.



b. We can find the acceleration from $a = \Delta v/t = 2.17\ m/s^2$ and use $d = ½\ at^2$ to find d = 975 m

c. The x and y components of the tension are $T_x = T \sin \theta$ and $T_y = T \cos \theta$ (this is using the angle to the vertical) Relating these to the other variables gives $T \sin \theta = ma$ and $T \cos \theta = mg$.
Dividing the two equations gives $\tan \theta = a/g = (2.17\ m/s^2)/(9.8\ m/s^2)$ and $\theta = 12.5°$

1996B2

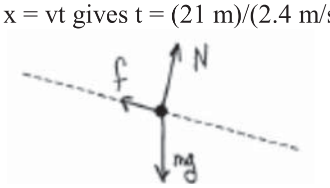
a. There are other methods, but answers are restricted to those taught to this point in the year.
   i. A device to measure distance and a calibrated mass or force scale or sensor
   ii. Hang the mass from the bottom of the spring and measure the spring extension (Δx) or pull on the spring with a known force and measure the resulting extension.
   iii. Use hooke's law with the known force or weight of the known mass $F = k\Delta x$ or $mg = k\Delta x$ and solve for k

b. Many methods are correct, for example, place the object held by the scale on an inclined plane and find the weight using $W\sin\theta = k\Delta x$. One could similarly use a pulley system to reduce the effort applied by the spring scale.

2007B1

a. x = vt gives t = (21 m)/(2.4 m/s) = 8.75 s

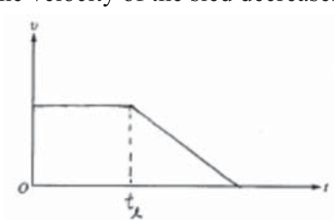
b.



c. $\Sigma F = 0$ if the sled moves at constant speed. This gives $mg \sin \theta - f = 0$, or $f = mg \sin \theta = 63.4$ N

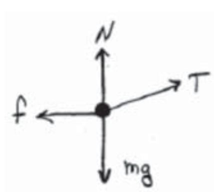
d. $f = \mu N$ where $N = mg \cos \theta$ so $\mu = f/N = (mg \sin \theta)/(mg \cos \theta) = \tan \theta = 0.27$

e. i. The velocity of the sled decreases while its acceleration remains constant
   ii.



2007B1B

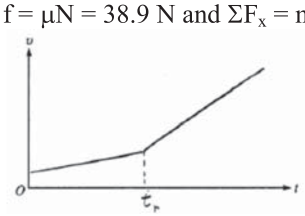
a.



b. $\Sigma F_y = 0$; $N + T \sin \theta - mg = 0$ gives $N = mg - T \sin \theta = 177$ N

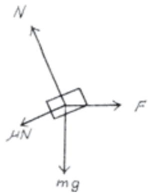
c. $f = \mu N = 38.9$ N and $\Sigma F_x = ma$; $T \cos \theta - f = ma$ yields $a = 0.64\ m/s^2$

d.



1981M1

a.



b. F can be resolved into two components: F sin θ acting into the incline and F cos θ acting up the incline. The normal force is then calculated with ΣF = 0; N – F sin θ – mg cos θ = 0 and f = μN
Putting this together gives ΣF = ma; F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = ma, solve for a

c. for constant velocity, a = 0 in the above equation becomes F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = 0 solving for F gives $F = mg\left(\frac{\mu\cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right)$ In order that F remain positive (acting to the right), the denominator must remain positive. That is cos θ > μ sin θ, or tan θ < 1/μ

1986M1

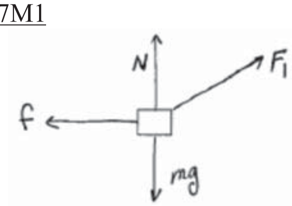
a. Combining the person and the platform into one object, held up by two sides of the rope we have ΣF = ma; 2T = (80 kg + 20 kg)g giving T = 500 N

b. Similarly, ΣF = ma; 2T – 1000 N = (100 kg)(2 m/s^2) giving T = 600 N

c. For the person only: ΣF = ma; N + 600 N – mg = ma gives N = 360 N

2007M1

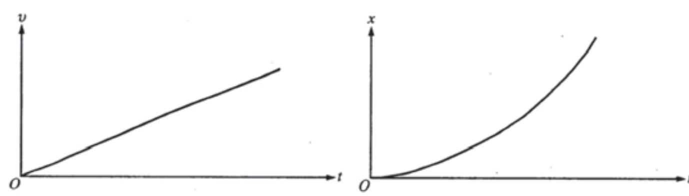
a.



b. $\Sigma F_y = 0$; $N + F_1 \sin\theta - mg = 0$ gives $N = mg - F_1 \sin\theta$

c. $\Sigma F_x = ma$; $F_1 \cos\theta - \mu N = ma_1$. Substituting N from above gives $\mu = (F_1 \cos\theta - ma_1)/(mg - F_1 \sin\theta)$

d.



e. The condition for the block losing contact is when the normal force goes to zero, which means friction is zero as well. $\Sigma F_x = F_{max} \cos\theta = ma_{max}$ and $\Sigma F_y = F_{max} \sin\theta - mg = 0$ giving $F_{max} = mg/(\sin\theta)$ and $a_{max} = (F_{max} \cos\theta)/m$ which results in $a_{max} = g \cot\theta$

1996M2

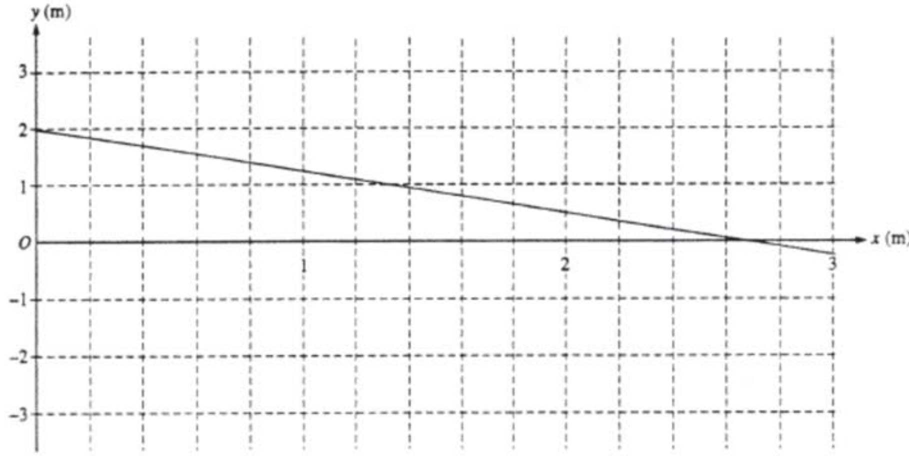
a. $\Sigma F = ma$; using downward as the positive direction, $mg - N = ma_y$ gives $N = m(g - a_y) = 2490$ N

b. Friction is the only horizontal force exerted; $\Sigma F = f = ma_x = 600$ N

c. At the minimum coefficient of friction, static friction will be at its maximum value $f = \mu N$, giving $\mu = f/N = (600\text{ N})/(2490\text{ N}) = 0.24$

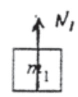
d. $y = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 = 2\text{ m} + \frac{1}{2}(-1.5\text{ m/s}^2)t^2$ and $x = x_0 + v_{0x}t + \frac{1}{2} a_x t^2 = \frac{1}{2}(2\text{ m/s}^2)t^2$, solving for t^2 in the x equation gives $t^2 = x$. Substituting into the y equation gives y as a function of x: $y = 2 - 0.75x$

e.



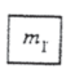
1998M3

a. i.



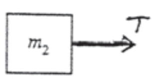
$N_1 = m_1 g$

ii.



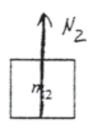
$f_1 = 0$

iii.



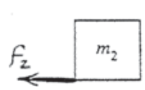
$T = Mg$

iv.



$N_2 = (m_1 + m_2)g$

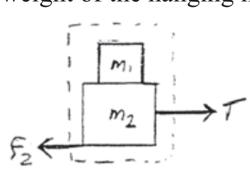
v.

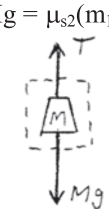


$f_2 = Mg$

b. The maximum friction force on the blocks on the table is $f_{2max} = \mu_{s2}N_2 = \mu_{s2}(m_1 + m_2)g$ which is balanced by the weight of the hanging mass: $Mg = \mu_{s2}(m_1 + m_2)g$ giving $M = \mu_{s2}(m_1 + m_2)$

c.



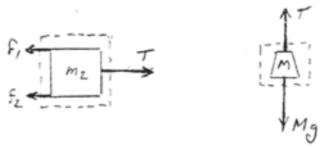


For the hanging block: $Mg - T = Ma$; For the two blocks on the plane: $T - f_2 = (m_1 + m_2)a$

Combining these equations (by adding them to eliminate T) and solving for a gives $a = \left|\frac{M - \mu_{k2}(m_1 + m_2)}{M + m_1 + m_2}\right| g$

d. i. $f_1 = \mu_{k1}m_1g = m_1a_1$ giving $a_1 = \mu_{k1}g$

ii.



For the two blocks: $Mg - T = Ma_2$ and $T - f_1 - f_2 = m_2a_2$. Eliminating T and substituting values for friction gives $a_2 = \left|\frac{M - \mu_{k1}m_1 - \mu_{k2}(m_1+m_2)}{M+m_2}\right| g$

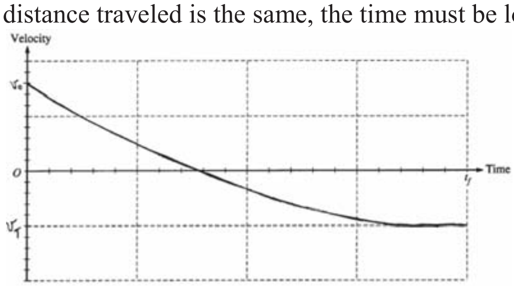
2005M1

a. The magnitude of the acceleration decreases as the ball moves upward. Since the velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

b. At terminal speed $\Sigma F = 0$. $\Sigma F = -Mg + kv_T$ giving $v_T = Mg/k$

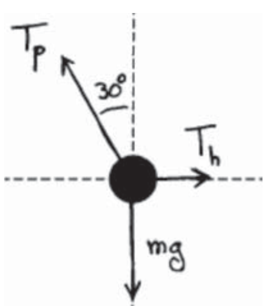
c. It takes longer for the ball to fall. Friction is acting on the ball on the way up and on the way down, where it begins from rest. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.

d.



2005B2.

(a)

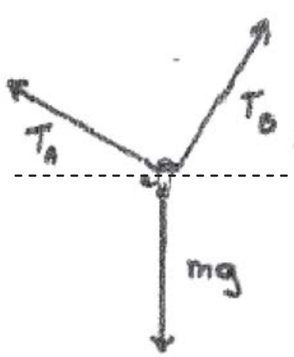


(b) Apply

$F_{net(X)} = 0$	$F_{net(Y)} = 0$
$T_P \cos 30 = mg$	$T_P \sin 30 = T_H$
$T_P = 20.37$ N	$T_H = 10.18$ N

1991B1.

a)



(b) SIMULTANEOUS EQUATIONS

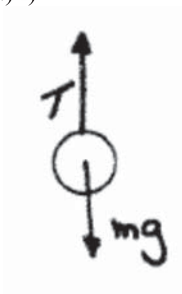
$F_{net(X)} = 0$	$F_{net(Y)} = 0$
$T_a \cos 30 = T_b \cos 60$	$T_a \sin 30 + T_b \sin 60 - mg = 0$

…. Solve above for T_b and plug into Fnet(y) eqn and solve

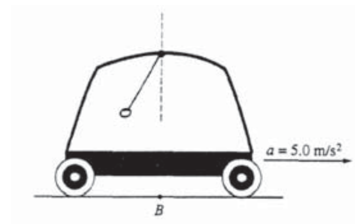
$T_a = 24$ N $\quad T_b = 42$ N

1995B3

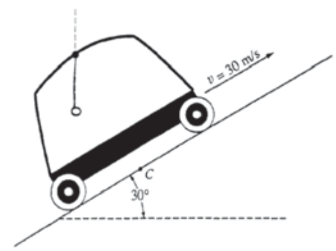
a) i)



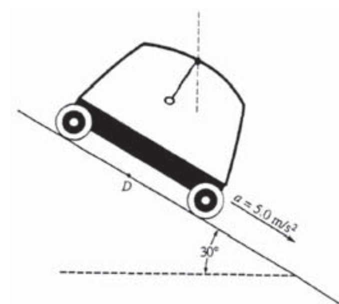
ii) T = mg = 1 N



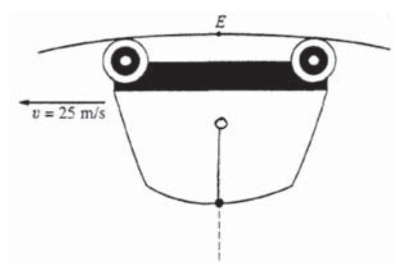
b) The horizontal component of the tension supplies the horizontal acceleration.
$T_h = ma$ = 0.5 N
The vertical component of the tension is equal to the weight of the ball, as in (a) ii. T_v = 1 N



c) Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball. T_h =zero, T_v = 1N



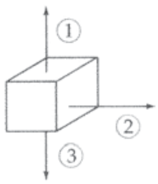
d) The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:
$T_h = ma \cos\theta$, where θ is the angle between the acceleration and horizontal
$T_h = (0.10 \text{ kg})(5.0 \text{ m/s}^2) \cos 30°$, T_h = 0.43 N
The vertical component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration. Applying Newton's second law. $Tv = mg - ma \sin\theta$
$Tv = (0.10 \text{ kg})(10 \text{ m/s}^2) - (0.10 \text{ kg})(5.0 \text{ m/s}^2) \sin 30°$, T_v =0.75 N



e) Since there is no horizontal acceleration, there is no horizontal component of the tension. T_h = zero
Assuming for the moment that the string is hanging downward, the centripetal is the difference between the gravitational force and the tension. Applying Newton's second law.
$mv^2/r = mg - T_v$, Solving for the vertical component of tension:
$T_v = -1.5$ N i.e. the string is actually pulling down on the ball.

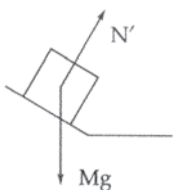
SECTION B – Circular Motion

1977B2

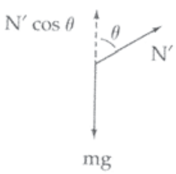
a. 

1 = normal force; 2 = friction; 3 = weight

b. Friction, $f \le \mu N$ where $N = Mg$. Friction provides the necessary centripetal force so we have $f = Mv^2/R$
$Mv^2/R \le \mu Mg$, or $\mu \ge v^2/Rg$

c. 

d. from the diagram below, a component of the normal force N′ balances gravity so N′ must be greater than mg



1984B1

a. At the top of the path, tension and gravity apply forces downward, toward the center of the circle.
$\Sigma F = T + mg = 2Mg + Mg = 3Mg$

b. In the circular path, $F = mv^2/r$ which gives $3Mg = mv_0^2/L$ and $v_0 = \sqrt{3Lg}$

c. The ball is moving horizontally ($v_{0y} = 0$) from a height of 2L so this gives $2L = \frac{1}{2} gt^2$ or $t = 2\sqrt{L/g}$

d. $x = v_0 t = \sqrt{3Lg} \times 2\sqrt{L/g} = 2\sqrt{3}L$

1989B1

a. i. $v_{iy} = 0$ so we have $h = \frac{1}{2} gt^2$ which gives $t = \sqrt{\frac{2h}{g}}$

ii. $x = v_0 t = v_0 \sqrt{\frac{2h}{g}}$

iii. $v_x = v_0$; $v_y = v_{iy} + gt = \sqrt{2gh}$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + 2gh}$$

b. 

c. Horizontal forces: $T \cos \theta = Mv_0^2/R$; Vertical forces: $T \sin \theta = Mg$. Squaring and adding the equations gives

$$T = M\sqrt{g^2 + \frac{v_0^4}{R^2}}$$

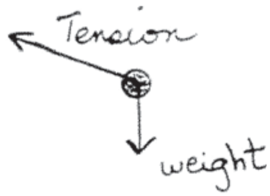
1997B2

a. The circumference of the path, d, can be calculated from the given radius. Use the timer to obtain the period of revolution, t, by timing a number of revolutions and dividing the total time by that number of revolutions. Calculate the speed using v = d/t.

b. If the cord is horizontal, $T = mv^2/r = 5.5$ N

c. (5.5 N – 5.8 N)/(5.8 N) × 100 = –5.2%

d. i.

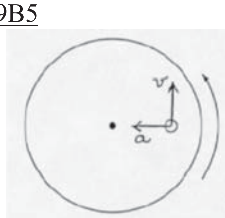


ii. The cord cannot be horizontal because the tension must have a vertical component to balance the weight of the ball.

iii. Resolving tension into components gives $T \sin\theta = mg$ and $T \cos\theta = mv^2/r$ which gives $\theta = 21^o$

1999B5

a.



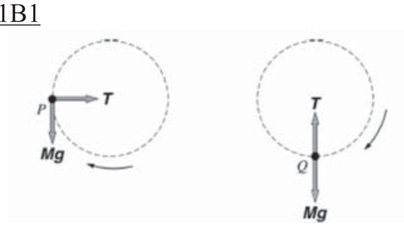
b. v = circumference/period = $2\pi R/T = 2\pi(0.14 \text{ m})/(1.5 \text{ s}) = 0.6$ m/s

c. The coin will slip when static friction has reached its maximum value of $\mu_s N = \mu_s mg = mv^2/r$ which gives $v = \sqrt{\mu_s g r} = 0.83$ m/s

d. It would not affect the answer to part (c) as the mass cancelled out of the equation for the speed of the coin.

2001B1

a.



b. The minimum speed occurs when gravity alone supplies the necessary centripetal force at the top of the circle (i.e. tension is zero and is not required). Therefore we have $Mg = Mv_{min}^2/R$ which gives $v_{min} = \sqrt{Rg}$

c. At the bottom of the swing $\Sigma F = ma$ becomes $T - Mg = Mv^2/R$ which gives $T_{max} - Mg = Mv_{max}^2/R$ and solving for v_{max} gives $v_{max} = \sqrt{\frac{R}{M}(T_{max} - Mg)}$

d. At point P the ball is moving straight up. If the string breaks at that point, the ball would continue to move straight up, slowing down until it reaches a maximum height and fall straight back to the ground.

2002B2B

a.

b. $\Sigma F_y = 0$; $T \cos \theta - mg = 0$ gives $m = (T \cos \theta)/g$

c. The centripetal force is supplied by the horizontal component of the tension: $F_C = T \sin \theta = mv^2/r$. Substituting the value of m found in part b. and the radius as ($l \sin \theta$) gives $v = \sqrt{gl \sin\theta \tan\theta}$

d. substituting the answer above into $v = 2\pi r f$ gives $f = \frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}$

e. The initial velocity of the ball is horizontal and the subsequent trajectory is parabolic.

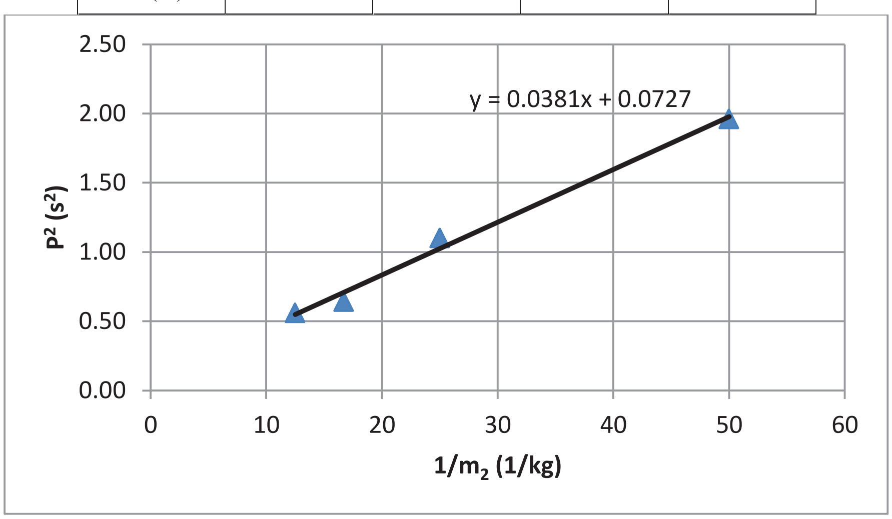
2009B1B

a. The centripetal force is provided by the weight of the hanging mass: $F_C = m_2 g = m_1 v^2/r$ and v is related to the period of the motion $v = 2\pi r/P$. This gives $m_2 g = \frac{m_1 v^2}{r} = \frac{m_1}{r}\frac{4\pi^2 r^2}{P^2}$ and thus $P^2 = 4\pi^2\left(\frac{m_1 r}{m_2 g}\right)$

b. The quantities that may be graphed to give a straight line are P^2 and $1/m_2$, which will yield a straight line with a slope of $4\pi^2\left(\frac{m_1 r}{g}\right)$

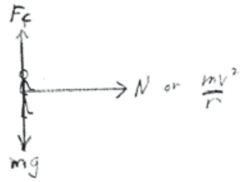
c.

$1/m_2$ (kg^{-1})	50	25	16.7	12.5
m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75
P^2 (s^2)	1.96	1.10	0.64	0.56



d. Using the slope of the line (0.038 kg/s^2) in the equation from part b. gives $g = 9.97$ m/s^2

1984M1

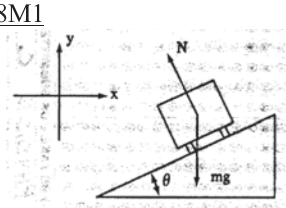
a. 

b. $F = mv^2/r$ where $v = 2\pi rf = 2\pi r(1/\pi) = 2r = 10$ m/s giving F = 1000 N provided by the normal force

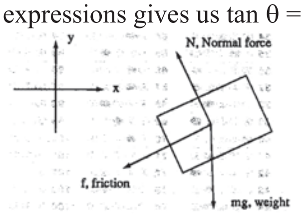
c. $\Sigma F_y = 0$ so the upward force provided by friction equals the weight of the rider = mg = 490 N

d. Since the frictional force is proportional to the normal force and equal to the weight of the rider, m will cancel from the equation, meaning a rider with twice the mass, or any different mass, will not slide down the wall as mass is irrelevant for this condition.

1988M1

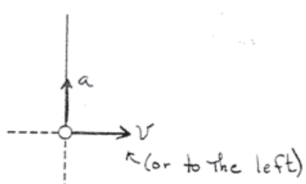
a. 

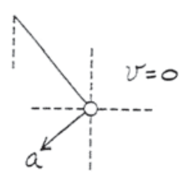
Toward the center of the turn we have $\Sigma F = N \sin\theta = mv^2/r$ and vertically $N \cos\theta = mg$. Dividing the two expressions gives us $\tan\theta = v^2/rg$ and v = 16 m/s

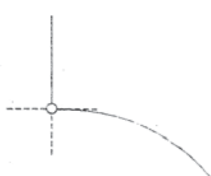
b. 

c. $\Sigma F_y = N \cos\theta - f \sin\theta - mg = 0$ and $\Sigma F_x = N \sin\theta + f \cos\theta = mv^2/r$ solve for N and f and substitute into $f = \mu N$ gives $\mu_{min} = 0.32$

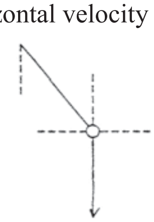
1998B6

a. i. 

ii. 

b. i. 

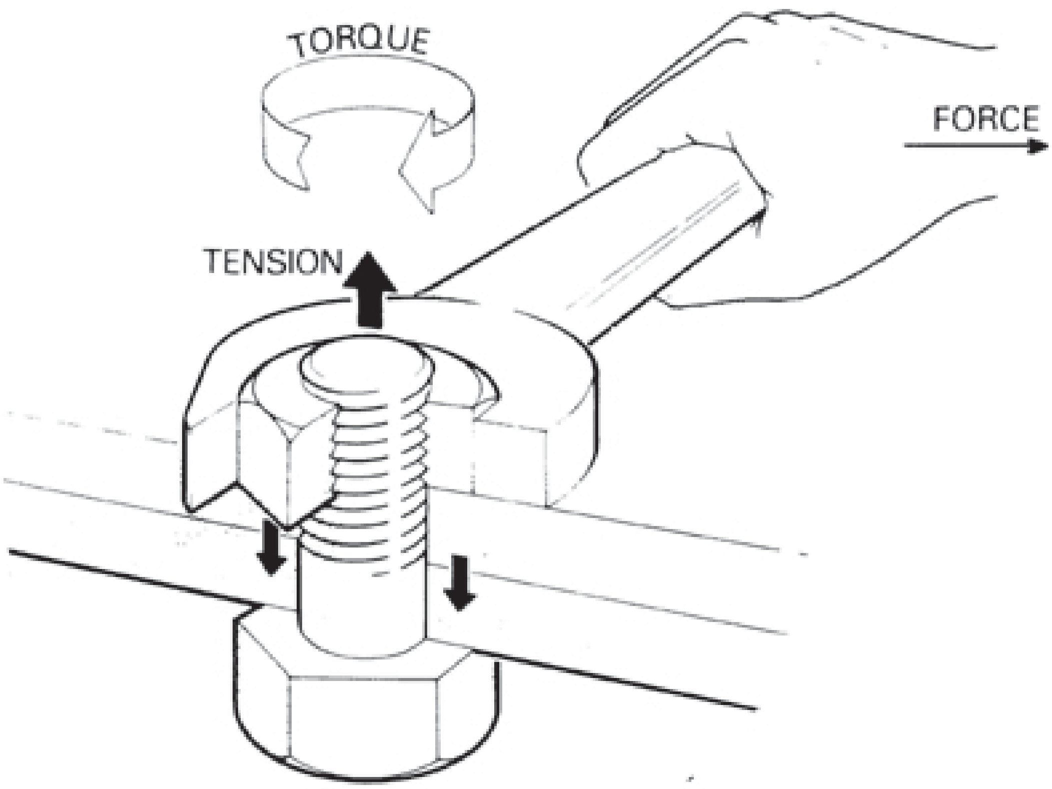
The horizontal velocity is constant, the vertical motion is in free fall and the path is parabolic

ii. 

The ball falls straight down in free fall

Chapter 3

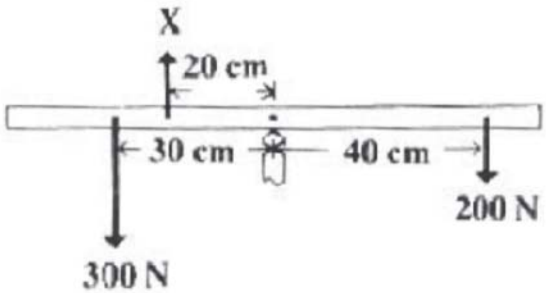
Torque



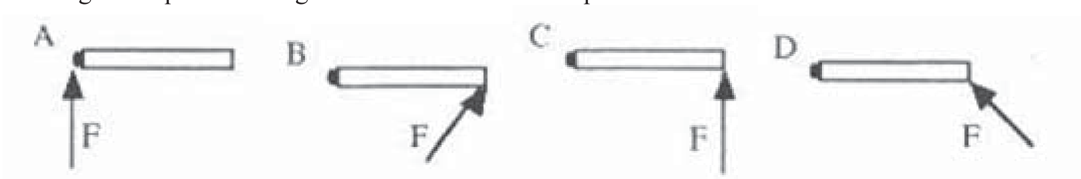
AP Physics Multiple Choice Practice – Torque

1. A uniform meterstick of mass 0.20 kg is pivoted at the 40 cm mark. Where should one hang a mass of 0.50 kg to balance the stick?
(A) 16 cm (B) 36 cm (C) 44 cm (D) 46 cm

2. A uniform meterstick is balanced at its midpoint with several forces applied as shown below. If the stick is in equilibrium, the magnitude of the force X in newtons (N) is
(A) 50 N (B) 100 N (C) 200 N (D) 300 N

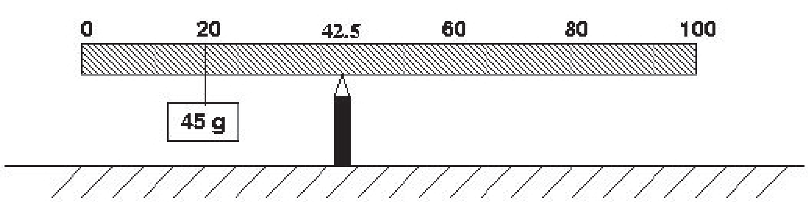


3. A door (seen from above in the figures below) has hinges on the left hand side. Which force produces the largest torque? The magnitudes of all forces are equal.



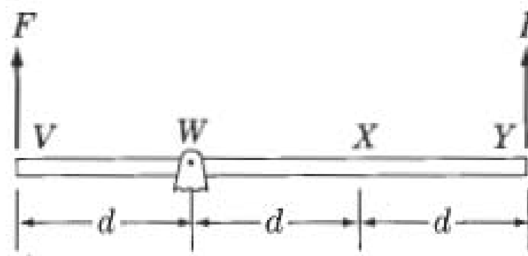
4. A meterstick is supported at each side by a spring scale. A heavy mass is then hung on the meterstick so that the spring scale on the left hand side reads four times the value of the spring scale on the right hand side. If the mass of the meterstick is negligible compared to the hanging mass, how far from the right hand side is the large mass hanging.
(A) 25 cm (B) 67 cm (C) 75 cm (D) 80 cm

5. A uniform meter stick has a 45.0 *g* mass placed at the 20 *cm* mark as shown in the figure. If a pivot is placed at the 42.5 *cm* mark and the meter stick remains horizontal in static equilibrium, what is the mass of the meter stick?



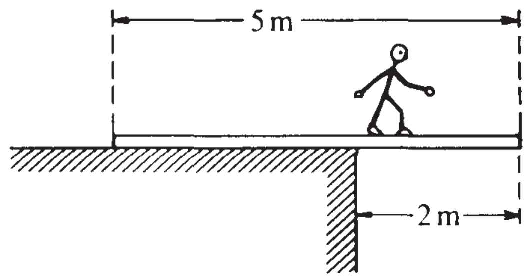
(A) 45.0 *g* (B) 72.0 *g* (C) 120.0 *g* (D) 135.0 *g*

6. A massless rigid rod of length $3d$ is pivoted at a fixed point W, and two forces each of magnitude F are applied vertically upward as shown. A third vertical force of magnitude F may be applied, either upward or downward, at one of the labeled points. With the proper choice of direction at each point, the rod can be in equilibrium if the third force of magnitude F is applied at point

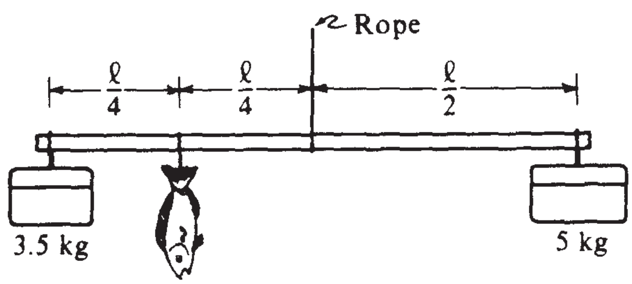


(A) Y only (B) V or X only (C) V or Y only (D) V, W, or X

7. A 5-meter uniform plank of mass 100 kilograms rests on the top of a building with 2 meters extended over the edge as shown. How far can a 50-kilogram person venture past the edge of the building on the plank before the plank just begins to tip?
(A) 0.5 m (B) 1 m (C) 1.5 m (D) 2 m

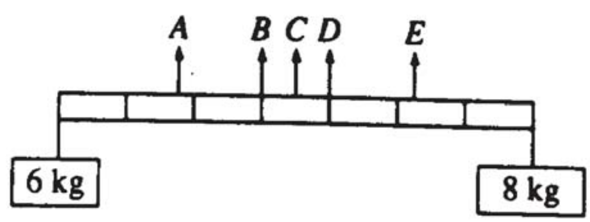


8. To weigh a fish, a person hangs a tackle box of mass 3.5 kilograms and a cooler of mass 5 kilograms from the ends of a uniform rigid pole that is suspended by a rope attached to its center. The system balances when the fish hangs at a point 1/4 of the rod's length from the tackle box. What is the mass of the fish?

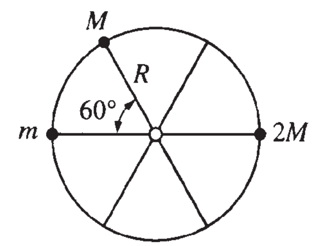


(A) 1.5 kg (B) 2 kg (C) 3 kg (D) 6 kg

9. Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 cm long and has marks every 10 cm, as shown. If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?
(A) A (B) B (C) C (D) D

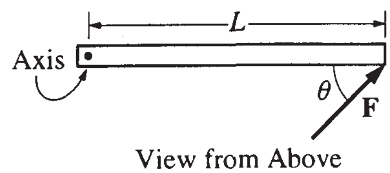


10. A wheel of radius R and negligible mass is mounted on a horizontal frictionless axle so that the wheel is in a vertical plane. Three small objects having masses m, M, and $2M$, respectively, are mounted on the rim of the wheel, as shown. If the system is in static equilibrium, what is the value of m in terms of M?

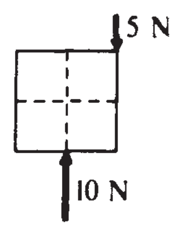


(A) $M/2$ (B) M (C) $3M/2$ (D) $5M/2$

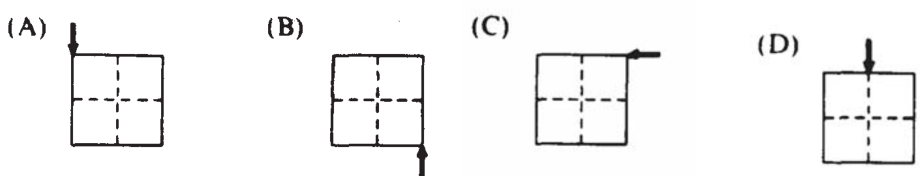
11. A rod on a horizontal tabletop is pivoted at one end and is free to rotate without friction about a vertical axis, as shown. A force **F** is applied at the other end, at an angle θ to the rod. If **F** were to be applied perpendicular to the rod, at what distance from the axis should it be applied in order to produce the same torque?

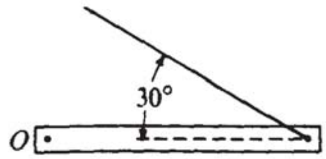


(A) L sin θ (B) L cos θ (C) L (D) L tan θ

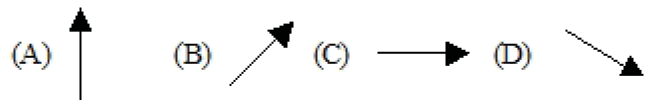


12. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?

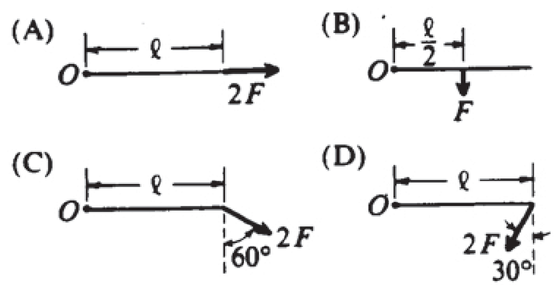




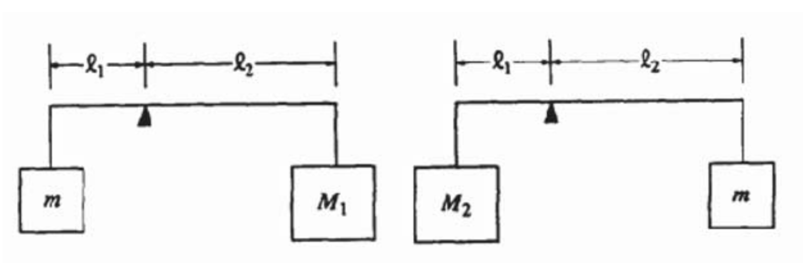
13. A uniform rigid bar of weight W is supported in a horizontal orientation as shown by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?



14. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram? (All forces lie in the plane of the paper.)

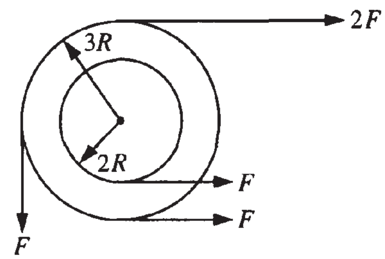


15. A rod of length L and of negligible mass is pivoted at a point that is off-center with lengths shown in the figure below. The figures show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

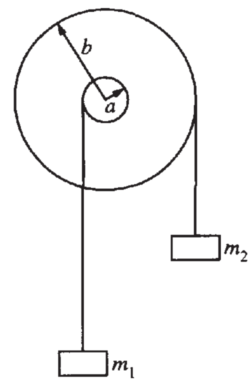


(A) $\sqrt{M_1 M_2}$ (B) ½(M_1 + M_2) (C) M_1 M_2 (D) ½M_1M_2

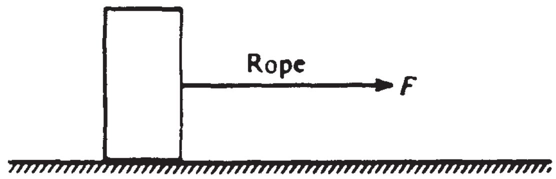
16. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown. The magnitude of the net torque on the system about the axis is
(A) zero (B) 2FR (C) 5FR (D) 14FR



17. For the wheel-and-axle system shown, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$
(B) $am_1 = bm_2$
(C) $am_2 = bm_1$
(D) $a^2m_1 = b^2m_2$

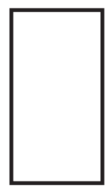


18. A meterstick of negligible mass is placed on a fulcrum at the 0.60 m mark, with a 2.0 kg mass hung at the 0 m mark and a 1.0 kg mass hung at the 1.0 m mark. The meterstick is released from rest in a horizontal position. Immediately after release, the magnitude of the net torque on the meterstick about the fulcrum is most nearly
(A) 2.0 N•m (B) 8.0 N•m (C) 10 N•m (D) 16 N•m

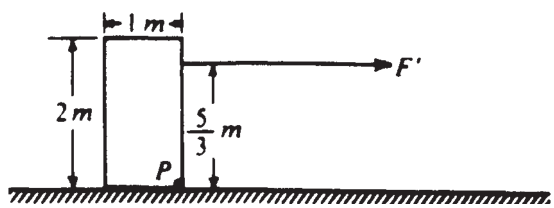


1983B1. A box of uniform density weighing 100 newtons moves in a straight line with constant speed along a horizontal surface. The coefficient of sliding friction is 0.4 and a rope exerts a force F in the direction of motion as shown above.

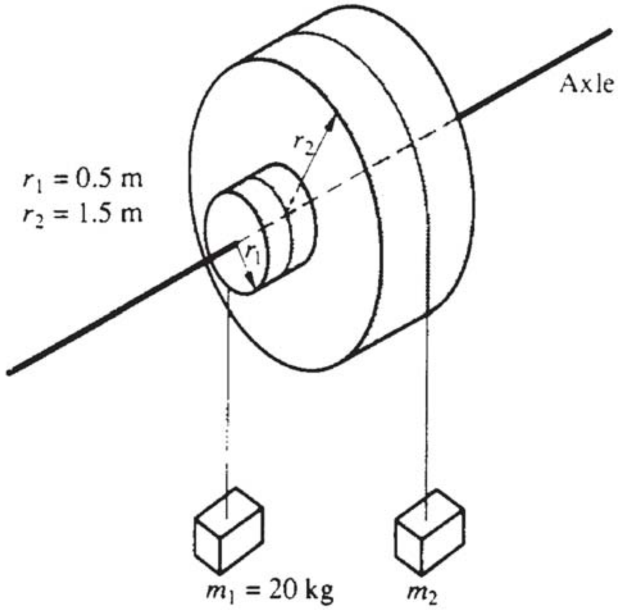
a. On the diagram below, draw and identify all the forces on the box.



b. Calculate the force F exerted by the rope that keeps the box moving with constant speed.

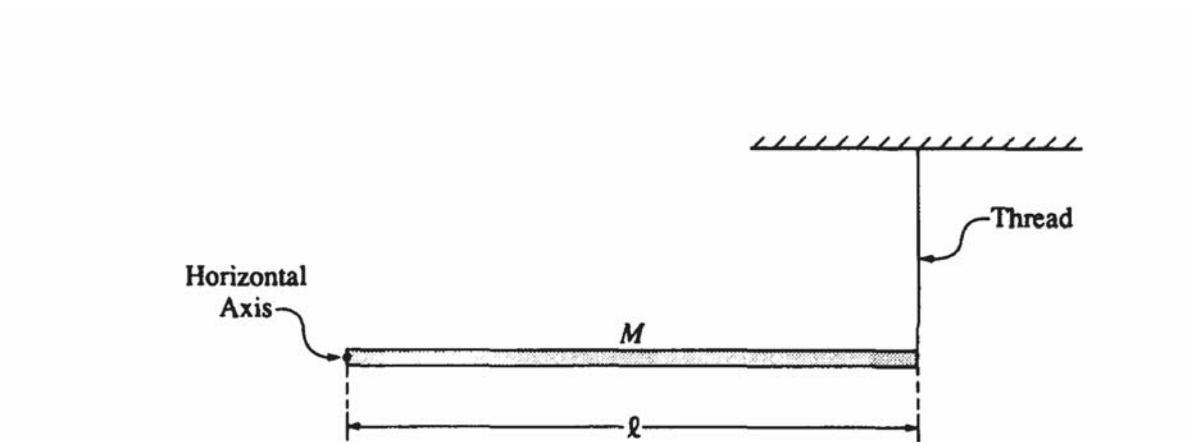


c. A horizontal force F', applied at a height 5/3 meters above the surface as shown in the diagram above, is just sufficient to cause the box to begin to tip forward about an axis through point P. The box is 1 meter wide and 2 meters high. Calculate the force F'.



C1991M2. Two masses. m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above with $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

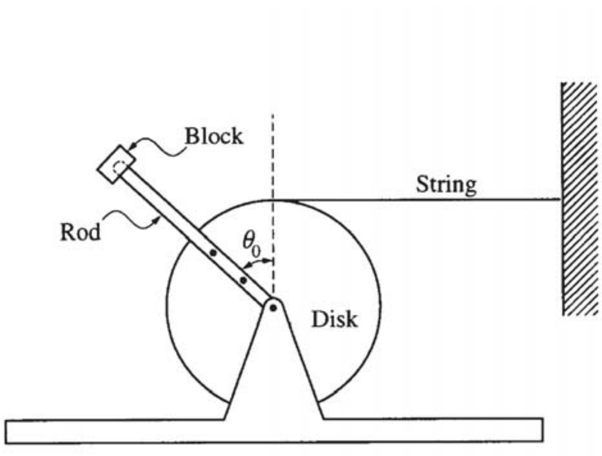
a. Determine m_2 such that the system will remain in equilibrium.



C1993M3. A long, uniform rod of mass M and length *l* is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. Express the answers to all parts of this question in terms of M, L and g.

a. Determine the magnitude and direction of the force exerted on the rod by the axis.

b. If the breaking strength of the thread is 2Mg, determine the maximum distance, r, measured from the hinge axis, that a box of mass 4M could be placed without breaking the thread

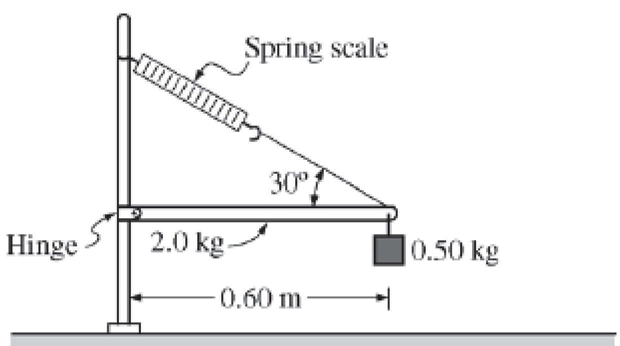


C1999M3. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk. A block is attached to the end of the rod. Properties of the rod, and block are as follows.

Rod: mass = m, length = 2R
Block: mass = 2m
Disk: radius = R

The system is held in equilibrium with the rod at an angle θ_o to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Determine the tension in the string in terms of m, θ_o, and g.

C2008M2.

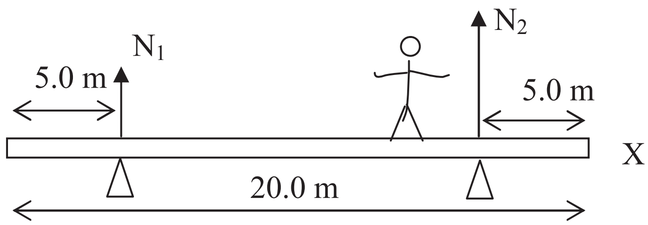


The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.
(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

(b) Calculate the reading on the spring scale.
(c) Calculate the magnitude of the force exerted by the hinge on the rod

Supplemental Problem

The diagram below shows a beam of length 20.0 m and mass 40.0 kg resting on two supports placed at 5.0 m from each end.



A person of mass 50.0 kg stands on the beam between the supports. The reaction forces at the supports are shown.

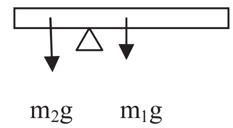
(a) State the value of $N_1 + N_2$

(b) The person now moves toward the X end of the beam to the position where the beam just begins to tip and reaction force N_1 becomes zero as the beam starts to leave the left support. Determine the distance of the girl from the end X when the beam is about to tip.

ANSWERS - AP Physics Multiple Choice Practice – Torque

Solution		Answer

1. Mass of stick m_1=0.20 kg at midpoint, Total length L=1.0 m, Pivot at 0.40 m, attached mass m_2=0.50 kg. **B**

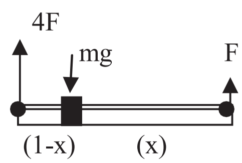
   

   Applying rotational equilibrium $\tau_{net}=0$
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (0.2) (0.1 m) = (0.5)(x)
   x = 0.04 m (measured away from 40 cm mark)
   → gives a position on the stick of 36 cm

2. As above, apply rotational equilibrium **A**
   + (300) (30cm) – X (20 cm) – (200) (40 cm) = 0

3. Torque = $(Fr)_\perp$ Choices A and E make zero torque, Of the remaining choices, each has moment arm = r but choice C has the full value of F to create torque (perpendicular) while the others would only use a component of F to make less torque **C**

4. Applying rotational equilibrium, using location of unknown mass as pivot … **D**

   4F (1 – x) = (F) (x)
   4F = 5Fx
   x = 4/5 = 0.80 m measured from right side

5. Applying rotational equilibrium ("g" cancels on each side) **D**
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (45) (22.5 cm) = (m) (7.5 cm) → m = 135 g

6. On the left of the pivot τ=Fd, on the right side of the pivot τ = F(2d). So we either have to add 1(Fd) to the left side to balance out the torque or remove 1(Fd) on the right side to balance out torque. Putting an upwards force on the left side at V gives (2Fd) on the left to balance torques, or putting a downwards force on the right side at X give a total of Fd on the right also causing a balance **B**

7. Apply rotational equilibrium using the corner of the building as the pivot point. Weight of plank (acting at midpoint) provides torque on left and weight of man provides torque on right. **B**
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (100 kg) (0.5m) = (50 kg) (r) → r = 1m

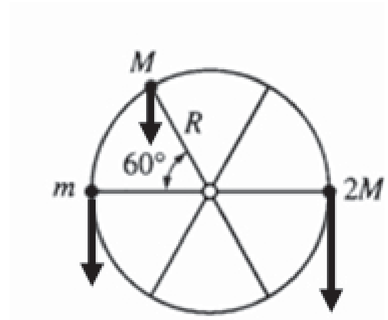
8. Apply rotational equilibrium using the rope as the pivot point. **C**
   (3.5)(9.8)(L/2) + m(9.8)(L/4) – (5)(9.8)(L/2) = 0 → m = 3 kg

9. To balance the torques on each side, we obviously need to be closer to the heavier mass. **D**
   Trying point D as a pivot point we have:
   $(m_1g) \bullet r_1$?=? $(m_2g) \bullet r_2$
   (6kg) (40 cm) ?=? (8kg) (30 cm) and we see it works.

10. Applying rotational equilibrium at the center pivot we get: C

$+mg(R) + Mg(R\cos60°) – 2Mg(R) = 0$.

Using $\cos60° = ½$ we arrive at the answer 3M/2



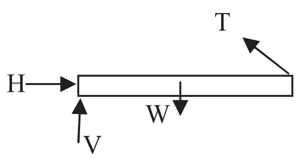
11. Finding the torque in the current configuration we have: A

$(F\sin\theta)(L) = FL \sin \theta$.

To get the same torque with F applied perpendicular we would have to change the L to get $F (L\sin\theta)$

12. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed. A

13. FBD B



Since the rope is at an angle it has x and y components of force.
Therefore, H would have to exist to counteract Tx.
Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar

14. In the given diagram the torque is = FL. C

Finding the torque of all the choices reveals C as correct.

$(2F\sin60°) (L) = 2F \cdot ½ L = FL$

15. Applying rotational equilibrium to each diagram gives A

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$

$L_1 = M_1(L_2) /m$

(sub this L_1) into the Diagram 2 eqn, and solve.

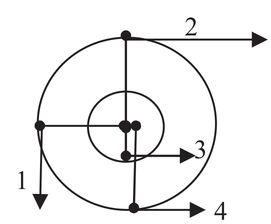
DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$

$M_2 (L_1) = m (L_2)$

16. Find the torques of each using proper signs and add up. B

$+ (1) – (2) + (3) + (4)$

$+F(3R) – (2F)(3R) + F(2R) +F(3R) = 2FR$



17. Simply apply rotational equilibrium B

$(m_1g) \bullet r_1 = (m_2g) \bullet r_2$

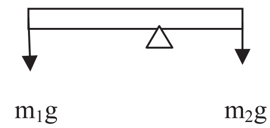
$m_1a = m_2b$

18. Question says meterstick has no mass, so ignore that force. Pivot placed at 0.60 m. Based on the applied masses, this meterstick would have a net torque and rotate. Find the net Torque as follows

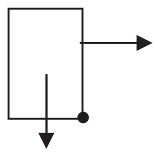
B

$\tau_{net} = + (m_1 g) \bullet r_1 - (m_2 g) \bullet r_2$

$+ (2)(10\ m/s^2)(0.6\ m) - (1)(10\ m/s^2)(0.4\ m)$



1983B1.
a) FBD. F_n pointing up, F_g pointing down, f_k applied to base of box pointing left
b) Constant speed → a=0.
Fnet = 0 F – fk = 0 F – uFn = 0 F – (0.4)(100)=0 F = 40 N
c) The force F' occurs at the limit point of tipping which is when the torque trying to tip it (caused by F') is equal to the torque trying to stop it from tipping (from the weight) using the tipping pivot point of the bottom right corner of the box.



(F')(5/3 m) = Fg (0.5m)
(F') (5/3) = (100)(0.5)
F' = 30 N

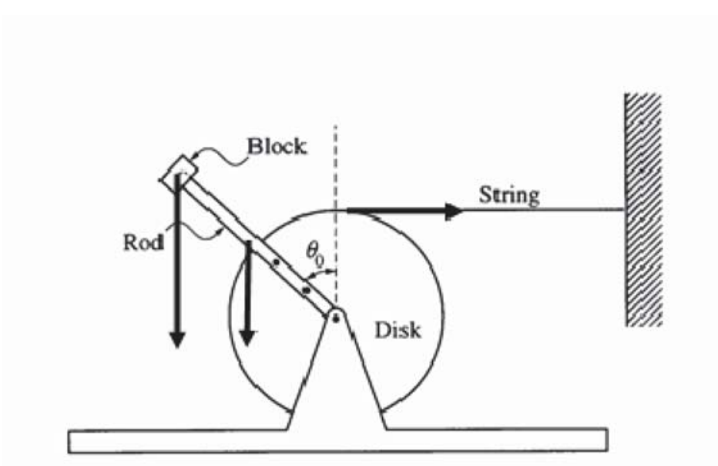
C1991M2.
Apply rotational equilibrium with the center as the pivot
$(m_1g) \bullet r_1 = (m_2g) \bullet r_2$ $(20)(9.8)(0.5) = m_2(9.8)(1.5)$ $m_2 = 6.67$ kg

C1993M3
(a) There is no H support force at the hinge since there are no other horizontal forces acting, so there is only vertical support for V. The tension in the thread T acts upwards and the weight of the rod acts at the midpoint. Apply rotational equilibrium using the hinge axis as the pivot:
+(T)(L) – (Mg)(L/2) = 0 T = Mg/2

Then using Fnet(y) = 0 V + T – Mg = 0 V + Mg/2 – Mg =0 V = Mg / 2

(b) Apply rotational equilibrium using the hinge axis as the pivot and "r" as the unknown distance of the box
Thread torque – Box torque - Rod Torque= 0
(2Mg)(L) – (4Mg)(r) – (Mg)(L/2) = 0
2L – 4r – L/2 = 0 r = 3/8 L

C1999M3



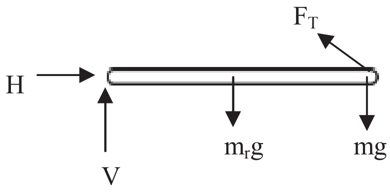
Apply rotational equilibrium using the center of the disk as the pivot

$(m_bg)(2R\sin\theta_o) + (m_rg)(R\sin\theta_o) - T(R) = 0$
$(2mg)(2R\sin\theta_o) + (mg)(R\sin\theta_o) - T(R) = 0$

$T = 5mg(\sin\theta_o)$

C2008M2

a) FBD



b) Apply rotational equilibrium using the hinge as the pivot

$+(F_T\sin 30)(0.6) - (mg)(0.6) - (m_rg)(0.3) = 0$

$+(F_T\sin 30)(0.6) - (0.5)(9.8)(0.6) - (2)(9.8)(0.3)=0$ $F_T = 29.4$ N

c) Apply Fnet(x), Fnet(y) = 0 to find H and V V=9.8N, H=25.46N

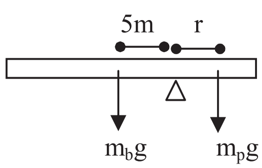
combining H and V $F_{hinge} = 27.28$ N

Supplemental

(a) Simple application of Fnet(y) = 0 $N_1 + N_2 - m_bg - m_pg = 0$

$N_1 + N_2 = (40)(9.8) + (50)(9.8)$ = 882 N

(b)



Apply rotational equilibrium

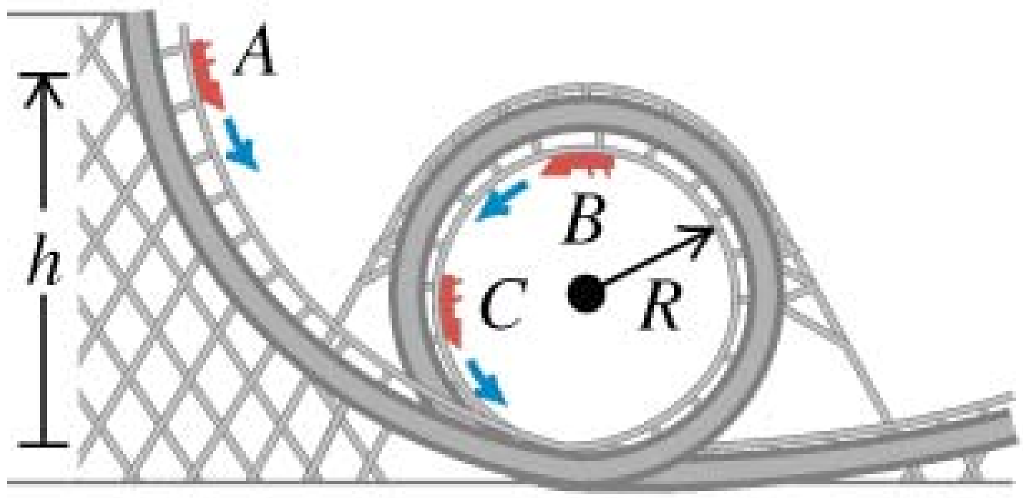
$(m_bg) \cdot r_1 = (m_pg) \cdot r_2$

(40) (5m) = (50) (r) r = 4m from hinge

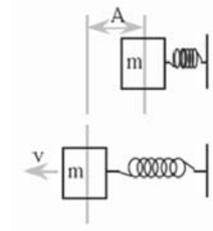
➔ 1 m from point X

Chapter 4

Work, Power and Energy



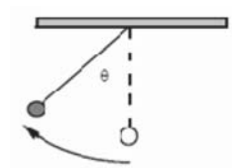
AP Physics Multiple Choice Practice – Work-Energy



1. A mass m attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the masses speed as it passes through its equilibrium position?
   (A) 0 (B) $A\sqrt{\frac{k}{m}}$ (C) $A\sqrt{\frac{m}{k}}$ (D) $\frac{1}{A}\sqrt{\frac{k}{m}}$

2. A force F at an angle θ above the horizontal is used to pull a heavy suitcase of weight mg a distance d along a level floor at constant velocity. The coefficient of friction between the floor and the suitcase is μ. The work done by the frictional force is:
   (A) –Fd cos θ (B) – μ Fd cos θ (C) -μmgd (D) -μ mgd cos θ

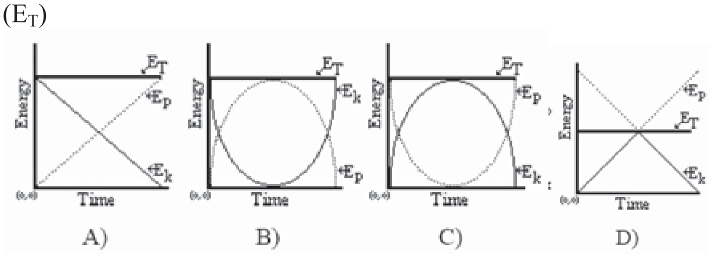
3. A 2 kg ball is attached to a 0.80 m string and whirled in a horizontal circle at a constant speed of 6 m/s. The work done on the ball during each revolution is:
   (A) 90 J (B) 72 J (D) 16 J (D) zero



4. A pendulum bob of mass m on a cord of length L is pulled sideways until the cord makes an angle θ with the vertical as shown in the figure to the right. The change in potential energy of the bob during the displacement is:
   (A) mgL (1–cos θ) (B) mgL (1–sin θ) (C) mgL sin θ
   (D) mgL cos θ

5. A softball player catches a ball of mass m, which is moving towards her with horizontal speed V. While bringing the ball to rest, her hand moved back a distance d. Assuming constant deceleration, the horizontal force exerted on the ball by the hand is
   (A) $mV^2/(2d)$ (B) mV^2/d (C) 2mV/d (D) mV/d

6. A pendulum is pulled to one side and released. It swings freely to the opposite side and stops. Which of the following might best represent graphs of kinetic energy (E_k), potential energy (E_p) and total mechanical energy (E_T)



Questions 7-8: A car of mass m slides across a patch of ice at a speed v with its brakes locked. It the hits dry pavement and skids to a stop in a distance d. The coefficient of kinetic friction between the tires and the dry road is μ.

7. If the car has a mass of 2m, it would have skidded a distance of
   (A) 0.5 d (B) d (C) 1.41 d (D) 2 d

8. If the car has a speed of 2v, it would have skidded a distance of
   (A) d (B) 1.41 d (C) 2 d (D) 4 d

9. A ball is thrown vertically upwards with a velocity v and an initial kinetic energy E_k. When half way to the top of its flight, it has a velocity and kinetic energy respectively of
   (A) $\frac{v}{2}, \frac{E_k}{2}$ (B) $\frac{v}{\sqrt{2}}, \frac{E_k}{2}$ (C) $\frac{v}{4}, \frac{E_k}{2}$ (D) $\frac{v}{2}, \frac{E_k}{\sqrt{2}}$

10. A football is kicked off the ground a distance of 50 yards downfield. Neglecting air resistance, which of the following statements would be INCORRECT when the football reaches the highest point?
(A) all of the balls original kinetic energy has been changed into potential energy
(B) the ball's horizontal velocity is the same as when it left the kickers foot
(C) the ball will have been in the air one-half of its total flight time
(D) the vertical component of the velocity is equal to zero

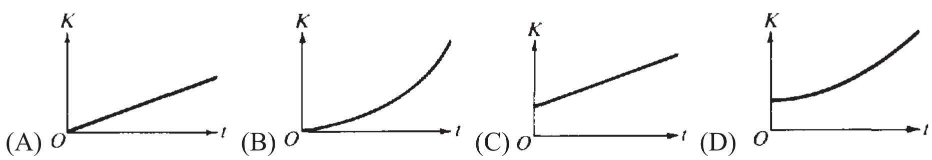
11. A mass m is attached to a spring with a spring constant k. If the mass is set into motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to equilibrium position?
(A) $v = \sqrt{\frac{kd}{m}}$ (B) $v = d\sqrt{\frac{k}{m}}$ (C) $v = \frac{kd}{mg}$ (D) $v^2 = \frac{mgd}{k}$

12. A fan blows the air and gives it kinetic energy. An hour after the fan has been turned off, what has happened to the kinetic energy of the air?
(A) it disappears (B) it turns into potential energy (C) it turns into thermal energy
(D) it turns into sound energy

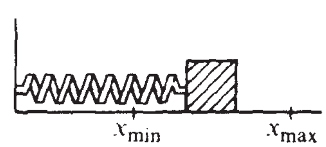
13. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) they strike the ground more than half a second apart.
(D) they strike the ground with the same kinetic energy.

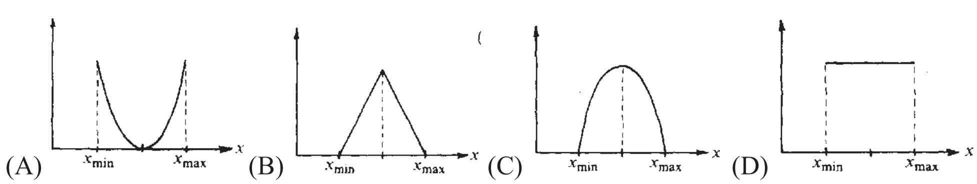
14. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(C) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
(D) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position

15. From the top of a high cliff, a ball is thrown horizontally with initial speed v_o. Which of the following graphs best represents the ball's kinetic energy K as a function of time t ?



Questions 16-17: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



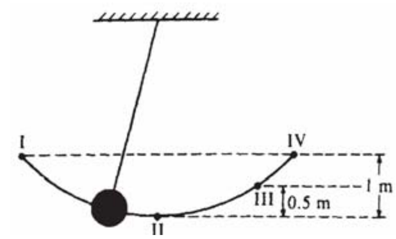


16. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C (D) D

17. Which graph can represent the kinetic energy of the block as a function of x ?
(A) A (B) B (C) C (D) D

<u>Questions 18-19</u>

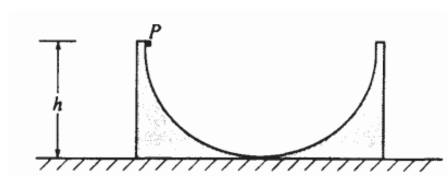
A ball swings freely back and forth in an arc from point I to point IV, as shown. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is I meter above II. Air resistance is negligible.



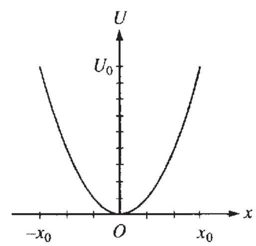
18. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?
(A) At point II (B) At some point between II and III
(C) At point III (D) At some point between III and IV

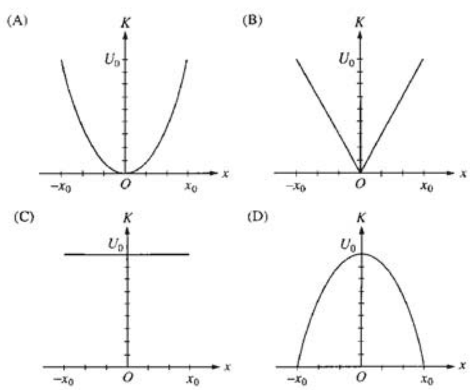
19. The speed of the ball at point II is most nearly
(A) 3.0 m/s (B) 4.5 m/s (C) 9.8 m/s (D) 14 m/s

20. The figure shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?
(A) It is equal to h/4 (B) It is equal to h/2
(C) It is equal to h
(D) It is between zero and h; the exact height depends on how much energy is lost to friction.



21. The graph shown represents the potential energy *U* as a function of displacement *x* for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy *K* of the object as a function of displacement *x* ?

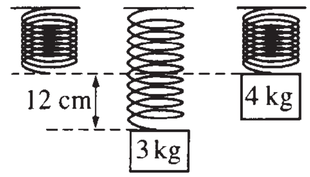




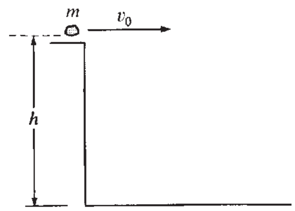
22. A child pushes horizontally on a box of mass *m* which moves with constant speed *v* across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

(A) μmgv (B) mgv (C) $\mu mg/v$ (D) $\mu mg/v$

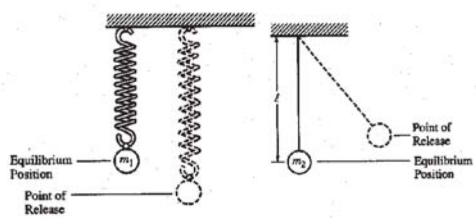
23. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?
(A) 18 cm (B) 24 cm
(C) 32 cm (D) 48 cm



24. What is the kinetic energy of a satellite of mass m that orbits the Earth, of mass M, in a circular orbit of radius R?
(A) $\frac{1}{2}\frac{GMm}{R}$ (B) $\frac{1}{4}\frac{GMm}{R}$ (C) $\frac{1}{2}\frac{GMm}{R^2}$ (D) $\frac{GMm}{R^2}$



25. A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0. What is the kinetic energy of the rock just before it hits the ground?
(A) mgh (B) $\frac{1}{2} mv_0^2$ (C) $\frac{1}{2} mv_0^2 - mgh$ (D) $\frac{1}{2} mv_0^2 + mgh$

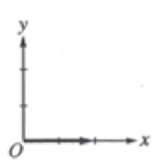
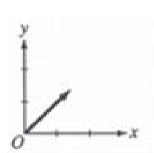
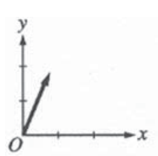
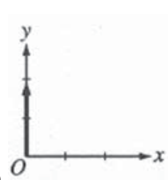


26. A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion Which of the following is true for both spheres?
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position.
(B) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(C) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(D) The maximum total energy is attained only as the sphere passes through its equilibrium position.

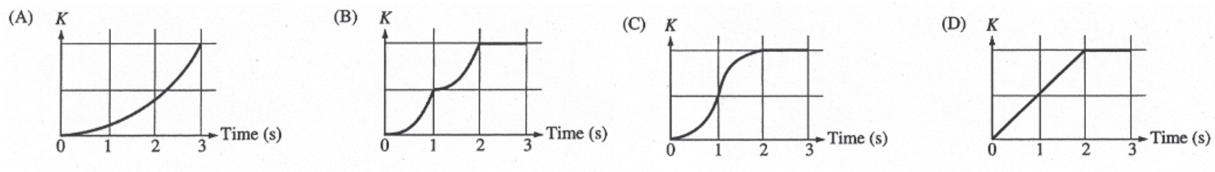
Questions 27-28

An object of mass m is initially at rest and free to move without friction in any direction in the xy-plane. A constant net force of magnitude F directed in the +x direction acts on the object for 1 s. Immediately thereafter a constant net force of the same magnitude F directed in the +y direction acts on the object for 1 s. After this, no forces act on the object.

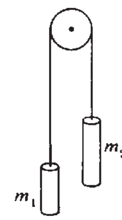
27. Which of the following vectors could represent the velocity of the object at the end of 3 s, assuming the scales on the x and y axes are equal?

(A)  (B)  (C)  (D) 

28. Which of the following graphs best represents the kinetic energy K of the object as a function of time?

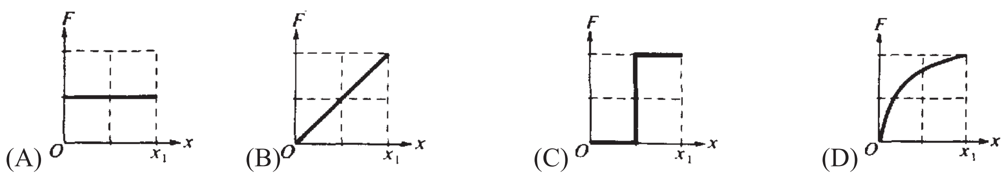


29. A system consists of two objects having masses m_l and m_2 ($m_l < m_2$). The objects are connected by a massless string, hung over a pulley as shown, and then released. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by

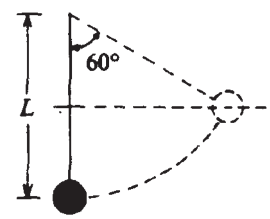


(A) $(m_2 - m_l)gh$ (B) m_2gh (C) $(m_l + m_2)gh$ (D) $\frac{1}{2}(m_l + m_2)gh$

30. The following graphs, all drawn to the same scale, represent the net force F as a function of displacement x for an object that moves along a straight line. Which graph represents the force that will cause the greatest change in the kinetic energy of the object from x = 0 to $x = x_1$?



31. A pendulum consists of a ball of mass m suspended at the end of a massless cord of length L as shown. The pendulum is drawn aside through an angle of 60° with the vertical and released. At the low point of its swing, the speed of the pendulum ball is

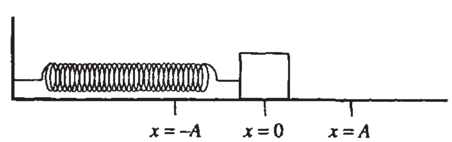


(A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) ½gL (D) gL

32. A rock is lifted for a certain time by a force F that is greater in magnitude than the rock's weight W. The change in kinetic energy of the rock during this time is equal to the

(A) work done by the net force (F - W)
(B) work done by F alone
(C) work done by W alone
(D) difference in the potential energy of the rock before and after this time.

33. A block on a horizontal frictionless plane is attached to a spring, as shown. The block oscillates along the x-axis with amplitude A. Which of the following statements about energy is correct?

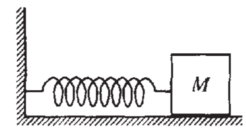


(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.

34. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

(A) h/4 (B) $\frac{h}{2\sqrt{2}}$ (C) h/2 (D) $\frac{h}{\sqrt{2}}$

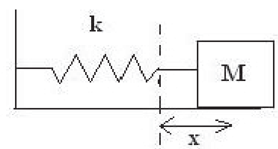
35. An ideal massless spring is fixed to the wall at one end, as shown. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is



(A) $\frac{Mgv_m}{2A}$ (B) $\frac{Mv_m^2}{2A}$ (C) $\frac{Mv_m^2}{A^2}$ (D) $\frac{Mv_m^2}{2A^2}$

36. A person pushes a block of mass M = 6.0 kg with a constant speed of 5.0 m/s straight up a flat surface inclined 30.0° above the horizontal. The coefficient of kinetic friction between the block and the surface is μ = 0.40. What is the net force acting on the block?
(A) 0 N (B) 21 N (C) 30 N (D) 51 N

37. A block of mass M on a horizontal surface is connected to the end of a massless spring of spring constant k . The block is pulled a distance x from equilibrium and when released from rest, the block moves toward equilibrium. What coefficient of kinetic friction between the surface and the block would allow the block to return to equilibrium and stop?



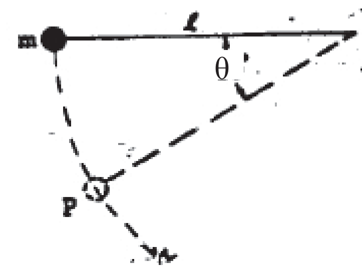
(A) $\frac{kx^2}{2Mg}$ (B) $\frac{kx}{Mg}$ (C) $\frac{kx}{2Mg}$ (D) $\frac{Mg}{2kx}$

38. An object is dropped from rest from a certain height. Air resistance is negligible. After falling a distance d, the object's kinetic energy is proportional to which of the following?
(A) $1 / d^2$ (B) 1 / d (C) √d (D) d

39. An object is projected vertically upward from ground level. It rises to a maximum height H. If air resistance is negligible, which of the following must be true for the object when it is at a height H/2 ?
(A) Its speed is half of its initial speed.
(B) Its kinetic energy is half of its initial kinetic energy.
(C) Its potential energy is half of its initial potential energy.
(D) Its total mechanical energy is half of its initial value.

AP Physics Free Response Practice – Work Power Energy

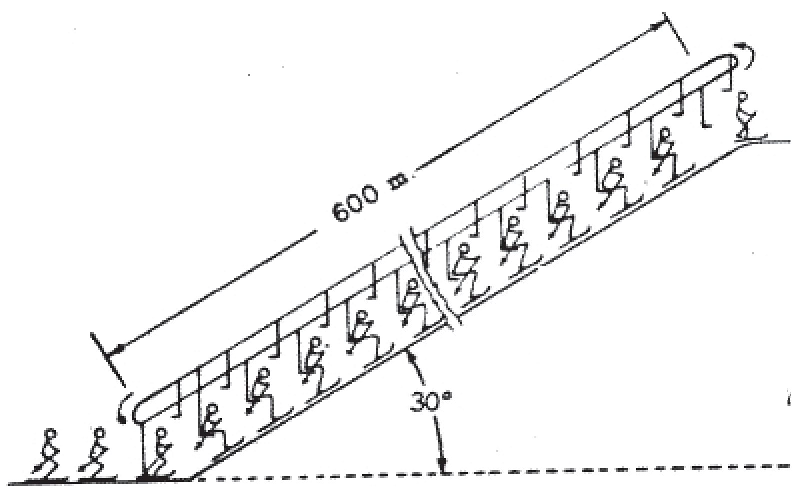
1974B1. A pendulum consisting of a small heavy ball of mass m at the end of a string of length L is released from a horizontal position. When the ball is at point P, the string forms an angle of θ with the horizontal as shown.

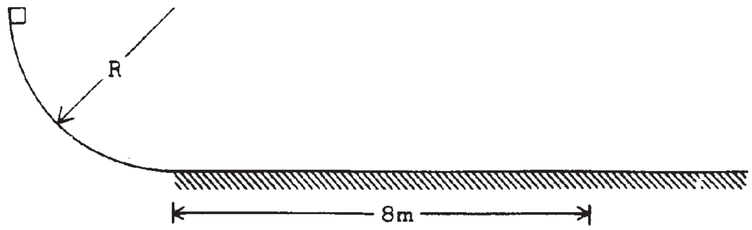


(a) In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

(b) Determine the speed of the ball at P.
(c) Determine the tension in the string when the ball is at P.

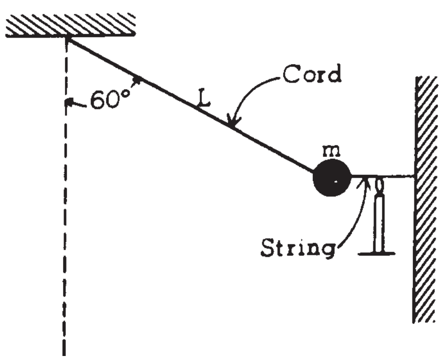
1974B7. A ski lift carries skiers along a 600 meter slope inclined at 30°. To lift a single rider, it is necessary to move 70 kg of mass to the top of the lift. Under maximum load conditions, six riders per minute arrive at the top. If 60 percent of the energy supplied by the motor goes to overcoming friction, what average power must the motor supply?





1975B1. A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume g = 10 meters per second^2.

a. Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.
b. How much time elapses while the block is sliding horizontally?
c. Calculate the radius of the incline in meters.



1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.
c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

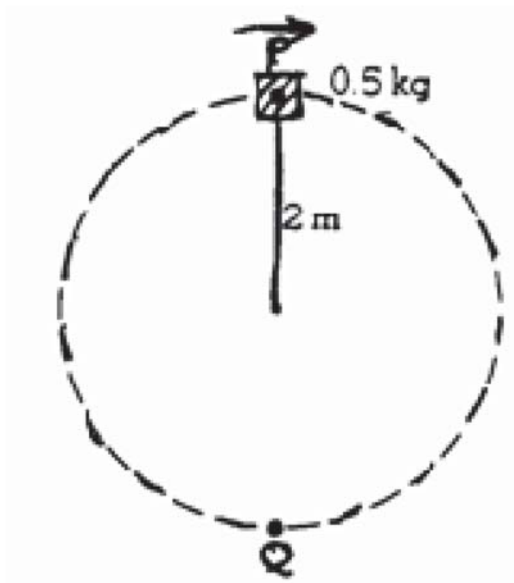
1977 B1. A block of mass 4 kilograms, which has an initial speed of 6 meters per second at time t = 0, slides on a horizontal surface.

a. Calculate the work W that must be done on the block to bring it to rest.

If a constant friction force of magnitude 8 newtons is exerted on the block by the surface, determine the following:

b. The speed v of the block as a function of the time t.

c. The distance x that the block slides as it comes to rest



1978B1. A 0.5 kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume g = 10 meters per second squared.

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P.

(b) Calculate the speed of the object at point P.

(c) Calculate the increase in kinetic energy of the object as it moves from point P to point Q.

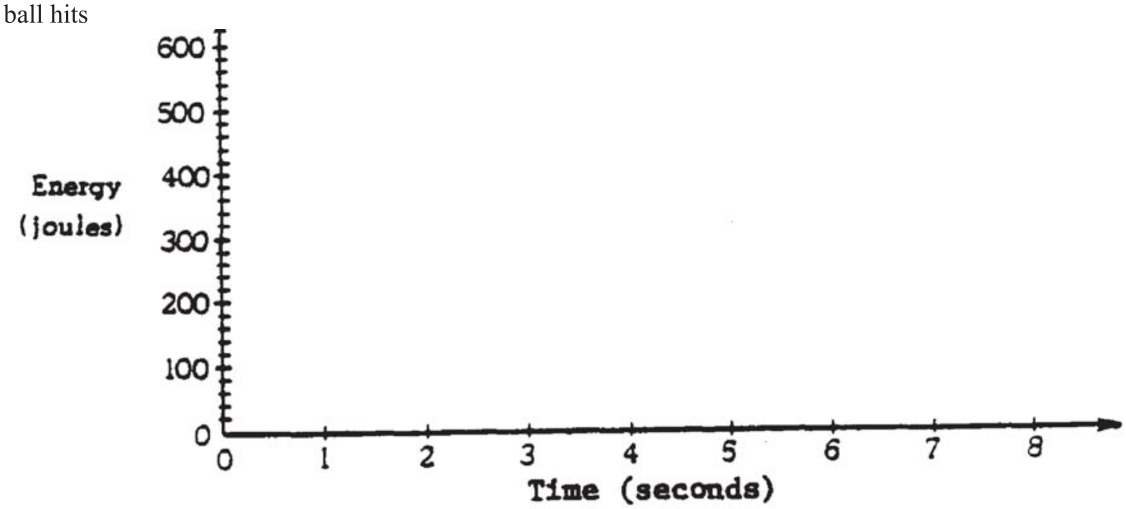
(d) Calculate the tension in the string as the object passes through point Q.

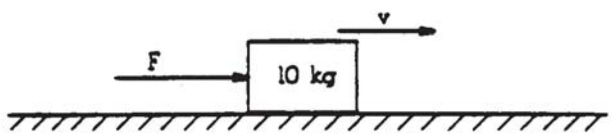
1979B1. From the top of a cliff 80 meters high, a ball of mass 0.4 kilogram is launched horizontally with a velocity of 30 meters per second at time t = 0 as shown above. The potential energy of the ball is zero at the bottom of the cliff. Use g = 10 meters per second squared.

a. Calculate the potential, kinetic, and total energies of the ball at time t = 0.

b. On the axes below, sketch and label graphs of the potential, kinetic, and total energies of the ball as functions of the distance fallen from the top of the cliff

c. On the axes below sketch and label the kinetic and potential energies of the ball as functions of time until the ball hits





1981B1. A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time t = 0, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume g = 10 meters per second squared.

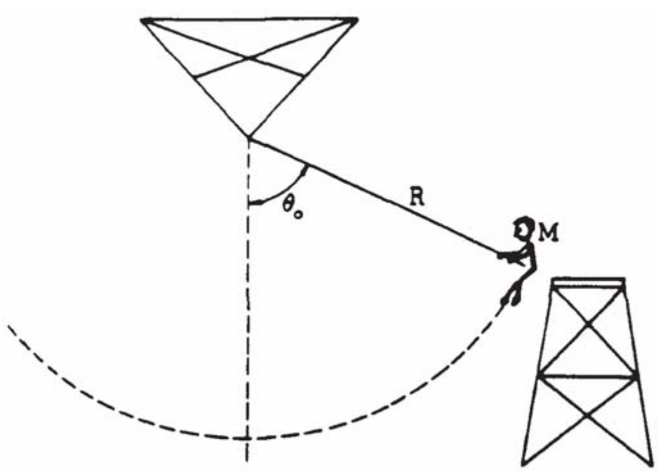
a. Calculate the force F necessary to keep the velocity constant.

The force is now changed to a larger constant value F'. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.

b. Calculate the force F'.

c. Calculate the acceleration of the block.

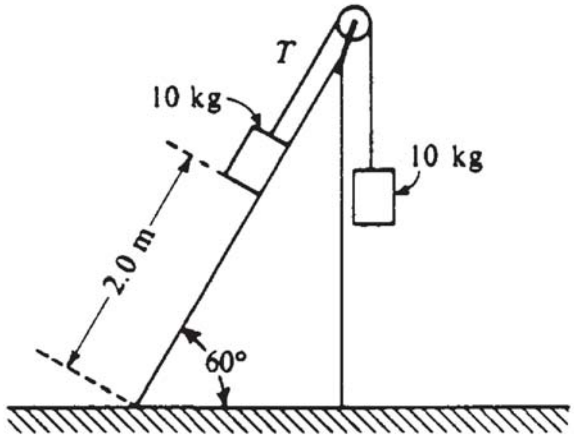
1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second. Determine the minimum work needed to compress the spring in this experiment.



1982B3. A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

a. Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and cos θ_o

b. The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of cos θ_o.



1985B2. Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, $\sin 60° = 0.87$, and $\cos 60° = 0.50$.

a. What is the tension T in the string?

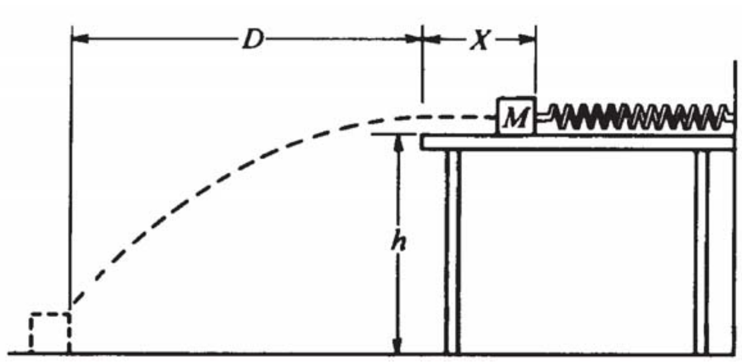
b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

c. Determine the magnitude of the frictional force acting on the box on the plane.

The string is then cut and the left-hand box slides down the inclined plane.

d. Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

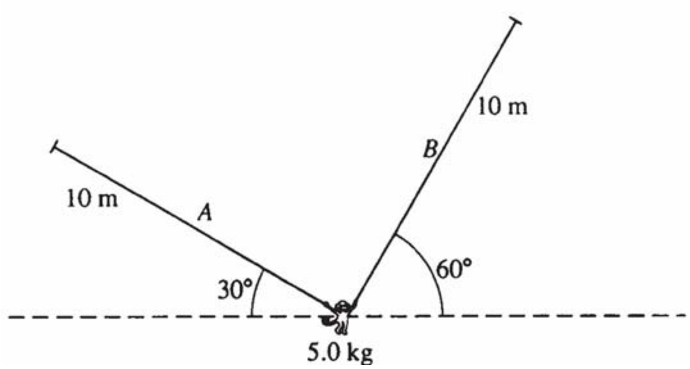
e. Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.



1986B2. One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance *h* above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible.

Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

a. The time elapsed from the instant the block leaves the table to the instant it strikes the floor
b. The horizontal component of the velocity of the block just before it hits the floor
c. The work done on the block by the spring
d. The spring constant



sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

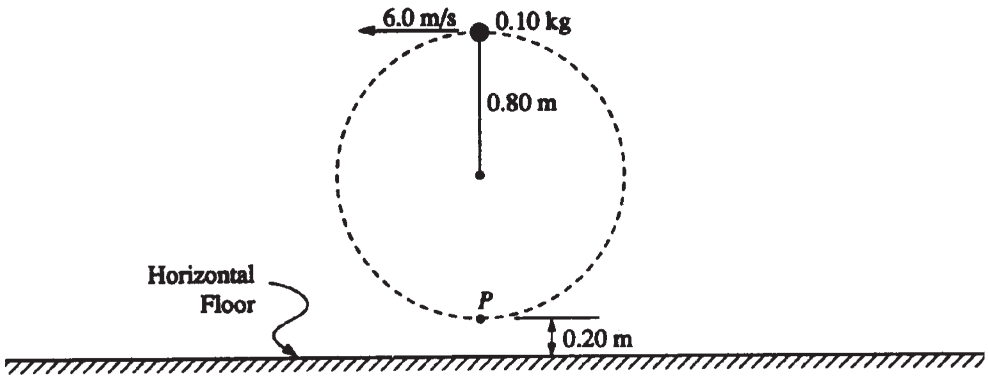
1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.

The monkey releases vine A and swings on vine B. Neglect air resistance.

c. Determine the speed of the monkey as it passes through the lowest point of its first swing.
d. Determine the tension in vine B as the monkey passes through the lowest point of its first swing.

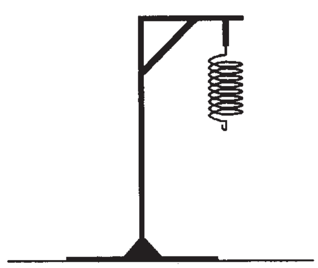


1992B1. A 0.10-kilogram solid rubber ball is attached to the end of an 0.80 meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

a. Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.
b. Determine the speed of the ball at point P, the lowest point of the circle.
c. Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.

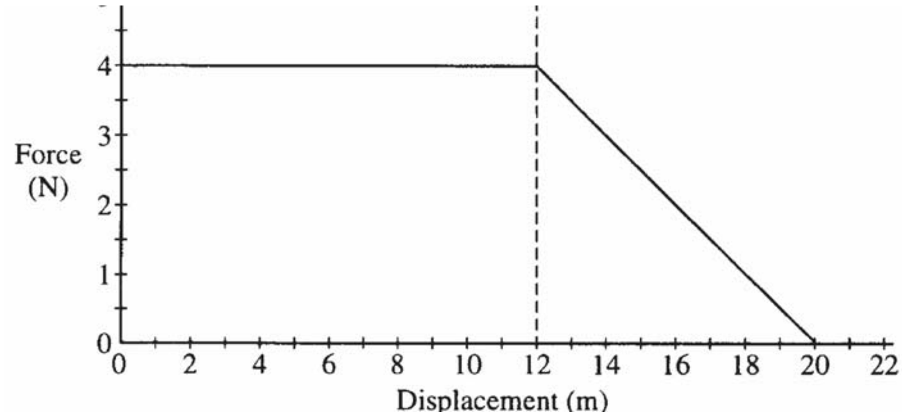
The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

d. Determine the horizontal distance that the ball travels before hitting the floor.



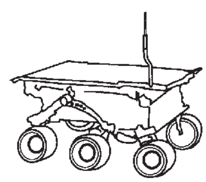
1996B2 (15 points) A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?
b. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

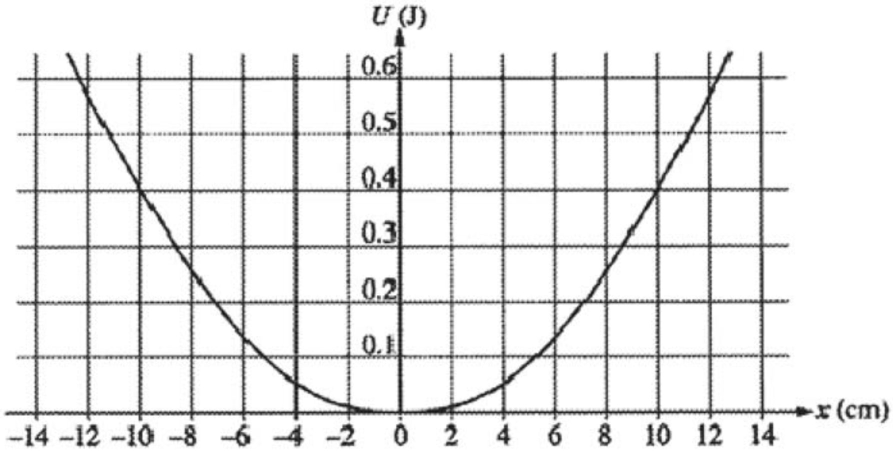
a. The acceleration of the particle when its displacement x is 6 m.
b. The time taken for the object to be displaced the first 12 m.
c. The amount of work done by the net force in displacing the object the first 12 m.
d. The speed of the object at displacement x = 12 m.
e. The final speed of the object at displacement x = 20 m.



1999B1. The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data	
Radius: 0.53 x Earth's radius	Mass of Sojourner vehicle:	11.5 kg
Mass: 0.11 x Earth's mass	Wheel diameter: 0.13 m	
	Stored energy available:	5.4×10^5 J
	Power required for driving under average conditions:	10 W
	Land speed:	6.7×10^{-3} m/s

a. Determine the acceleration due to gravity at the surface of Mars in terms of g, the acceleration due to gravity at the surface of Earth.
b. Calculate Sojourner's weight on the surface of Mars.
c. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
d. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
e. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
f. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.



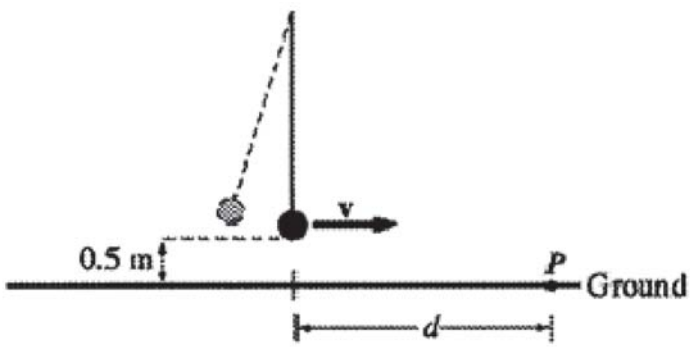
2002B2. A 3.0 kg object subject to a restoring force F is undergoing simple harmonic motion with a small amplitude. The potential energy U of the object as a function of distance x from its equilibrium position is shown above. This particular object has a total energy E: of 0.4 J.

(a) What is the object's potential energy when its displacement is +4 cm from its equilibrium position?

(b) What is the farthest the object moves along the x axis in the positive direction? Explain your reasoning.

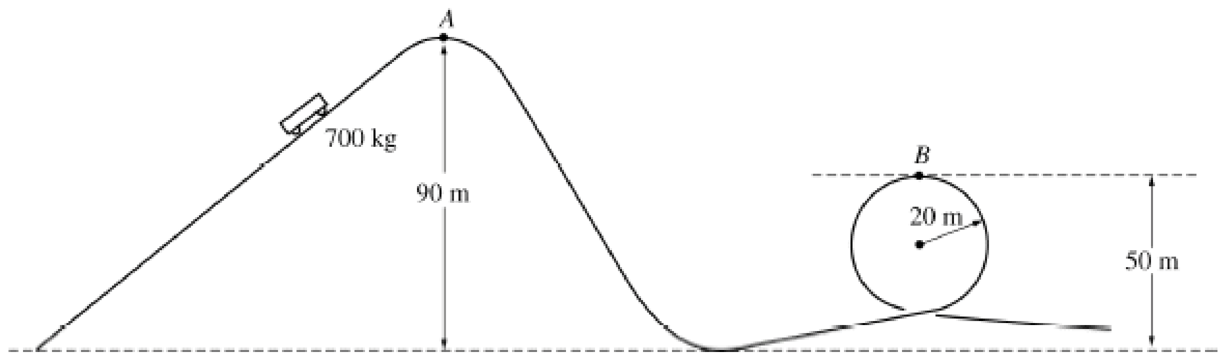
(c) Determine the object's kinetic energy when its displacement is – 7 cm.

(d) What is the object's speed at x = 0 ?



Note: Figure not drawn to scale.

(e) Suppose the object undergoes this motion because it is the bob of a simple pendulum as shown above. If the object breaks loose from the string at the instant the pendulum reaches its lowest point and hits the ground at point P shown, what is the horizontal distance d that it travels?



2004B1.
A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

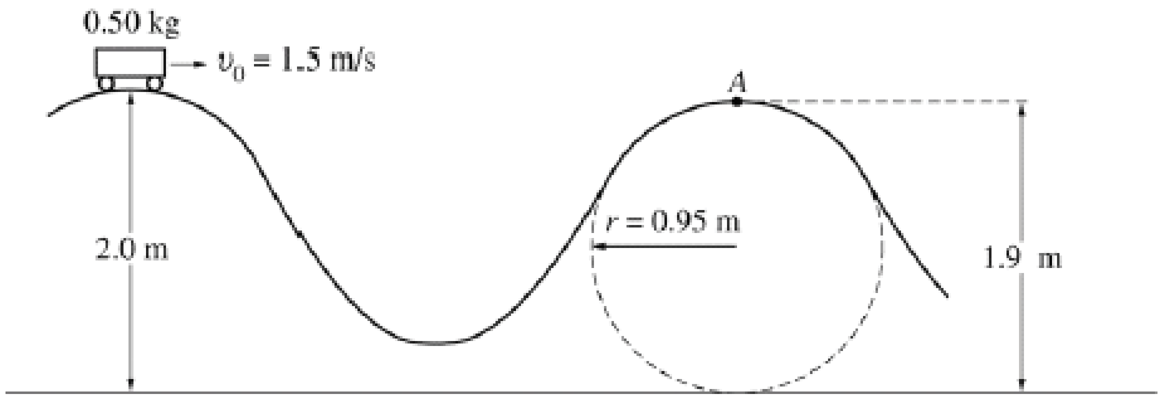
(a)
i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.
ii. Calculate the value v_{msx} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.

ii. Calculate the magnitude of all the forces identified in (c)

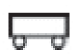
(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.



B2004B1.
A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_o = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

(a) Calculate the speed of the car at point *A*.

(b) On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.



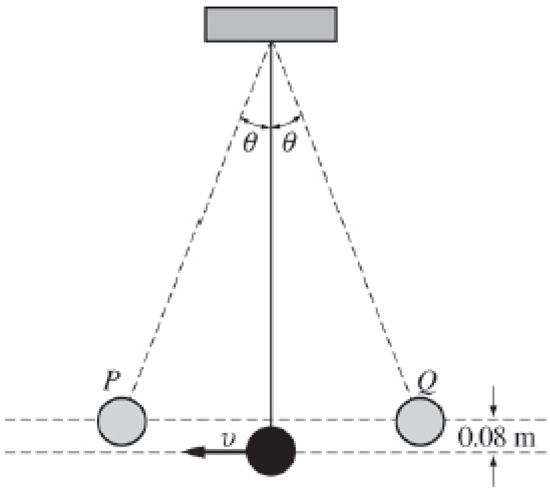
(c) Calculate the magnitude of the force of the track on the car at point *A*.

(d) In order to stop the car at point *A*, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

(e) Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

B2005B2

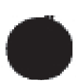
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



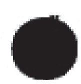
Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P



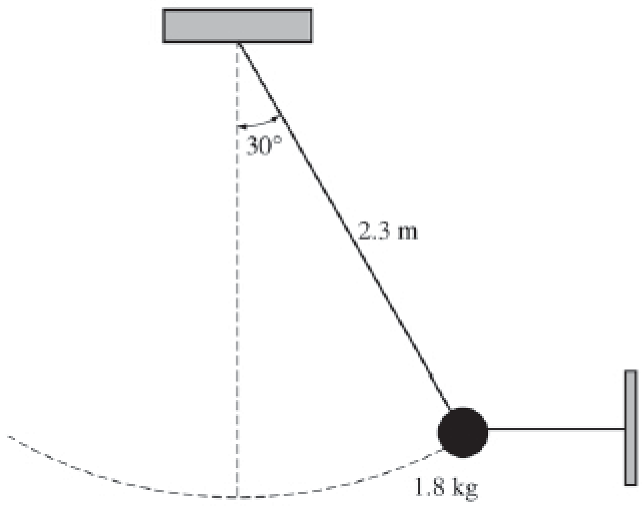
ii. When it is in motion at its lowest position



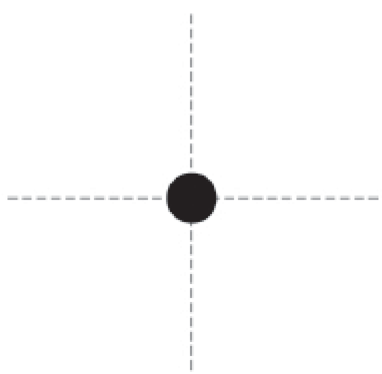
(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

2005B2.

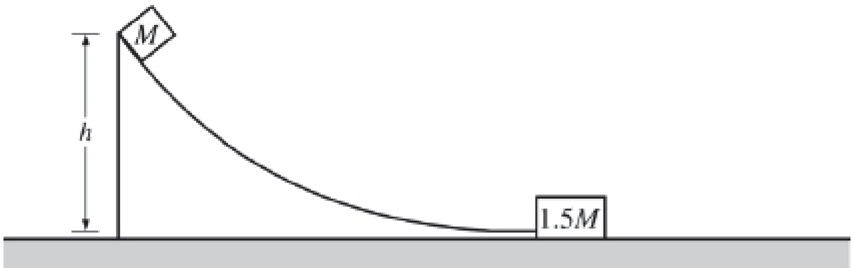


2. (10 points)
A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

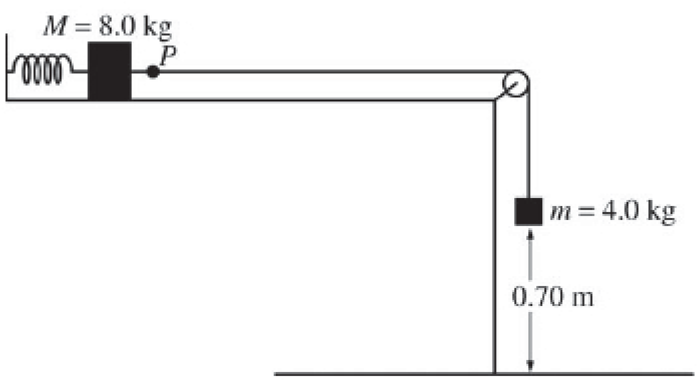
B2006B2



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.

Express your answers to the following questions in terms of the given quantities and fundamental constants.
(a) Determine the height h of the ramp from which the small block was released.
(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

2006B1

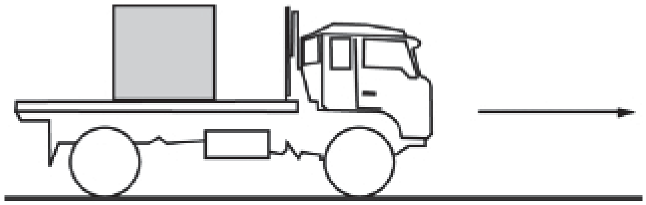


An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass $M = 8.0$ kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass $m = 4.0$ kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.
(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.
$M = 8.0$ kg $\qquad\qquad$ $m = 4.0$ kg

(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

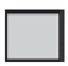
The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the maximum speed attained by the 8.0 kg block as it oscillates back and forth

B2008B2



A 4700 kg truck carrying a 900 kg crate is traveling at 25 m/s to the right along a straight, level highway, as shown above. The truck driver then applies the brakes, and as it slows down, the truck travels 55 m in the next 3.0 s. The crate does not slide on the back of the truck.

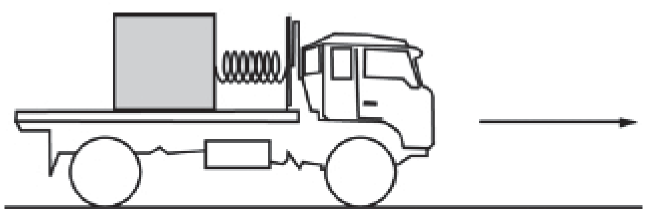
(a) Calculate the magnitude of the acceleration of the truck, assuming it is constant.
(b) On the diagram below, draw and label all the forces acting on the crate during braking.



(c)
i. Calculate the minimum coefficient of friction between the crate and truck that prevents the crate from sliding.

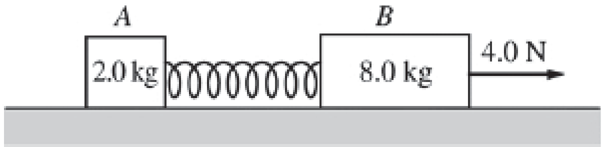
ii. Indicate whether this friction is static or kinetic.
____ Static ____Kinetic

Now assume the bed of the truck is frictionless, but there is a spring of spring constant 9200 N/m attaching the crate to the truck, as shown below. The truck is initially at rest.



(d) If the truck and crate have the same acceleration, calculate the extension of the spring as the truck accelerates from rest to 25 m s in 10 s.
(e) At some later time, the truck is moving at a constant speed of 25 m/s and the crate is in equilibrium. Indicate whether the extension of the spring is greater than, less than, or the same as in part (d) when the truck was accelerating.
___ Greater ___ Less ___ The same
Explain your reasoning.

2008B2

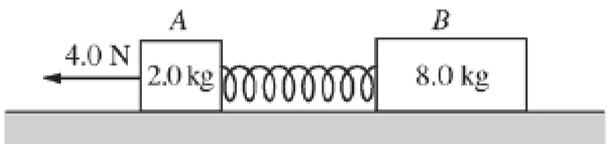


Block *A* of mass 2.0 kg and block *B* of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.

(b) Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c) Is the magnitude of the acceleration greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

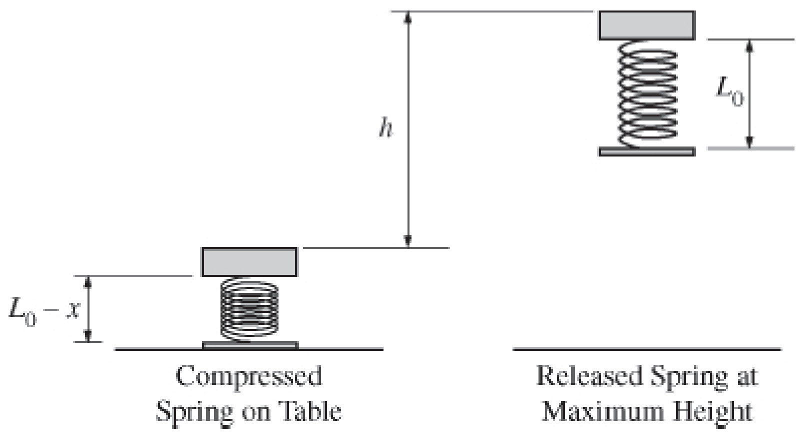
(d) Is the amount the spring has stretched greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m s to the left. Block *A* then hits and sticks to a wall. Calculate the maximum compression of the spring.

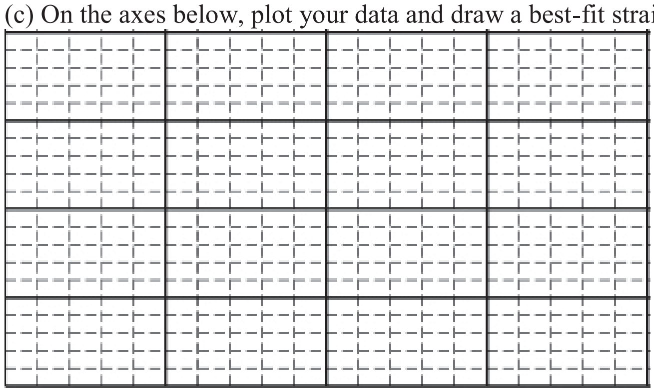
2009B1



In an experiment, students are to calculate the spring constant k of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance x from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height h above the point of maximum compression. The students repeat the experiment several times, measuring h with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is m. The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped to it.
(a) Derive an expression for the height h in terms of m, x, k, and fundamental constants.
With the spring compressed a distance x = 0.020 m in each trial, the students obtained the following data for different values of m.

	m (kg)	h (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

(b)
i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant k ?
ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.
(c) On the axes below, plot your data and draw a best-fit straight line. Label the axes and indicate the scale.

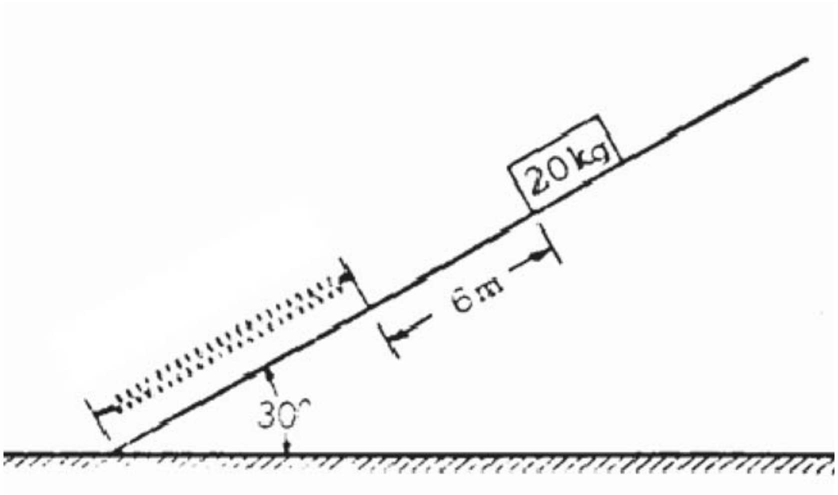


(d) Using your best-fit line, calculate the numerical value of the spring constant.
(e) Describe a procedure for measuring the height h in the experiment, given that the toy is only momentarily at that maximum height.

C1973M2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.

a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.

b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?

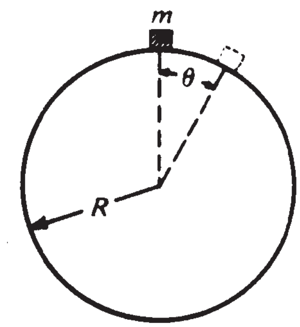


C1982M1. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30º with the horizontal and strikes a spring of unknown spring constant as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved.

a. Determine the speed of the block just before it hits the spring.

b. Determine the spring constant given that the distance the spring compresses along the incline is 3m when the block comes to rest.

c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

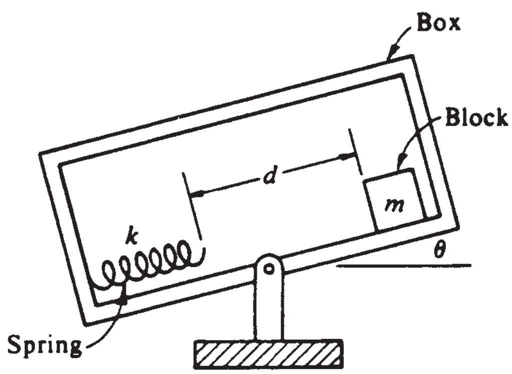


C1983M3. A particle of mass m slides down a fixed, frictionless sphere of radius R. starting from rest at the top.

a. In terms of m, g, R. and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass



C1985M2. An apparatus to determine coefficients of friction is shown above. At the angle θ shown with the horizontal, the block of mass m just starts to slide. The box then continues to slide a distance d at which point it hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities and fundamental constants, derive an expression for each of the following.

a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

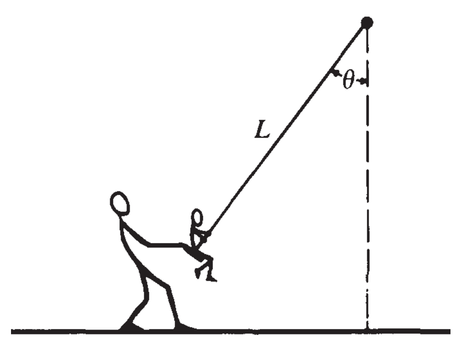
c. μ_k, the coefficient of kinetic friction.

C1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

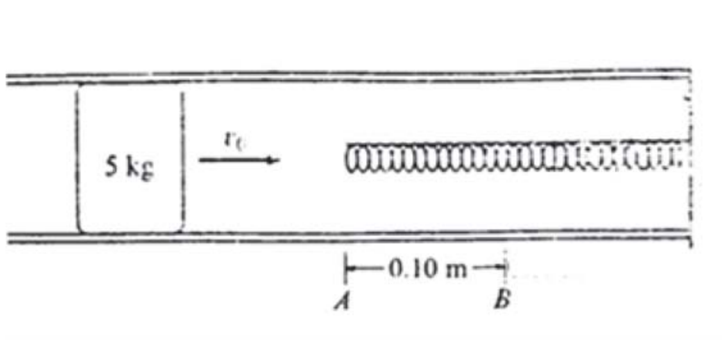
a) The tension in the rope

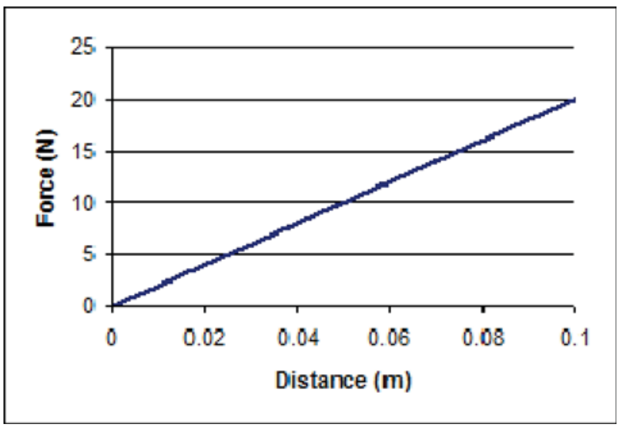
b) The horizontal force exerted by the adult.

c) The adult releases the swing from rest. In terms of W and θ determine the tension in the rope as the swing passes through its lowest point



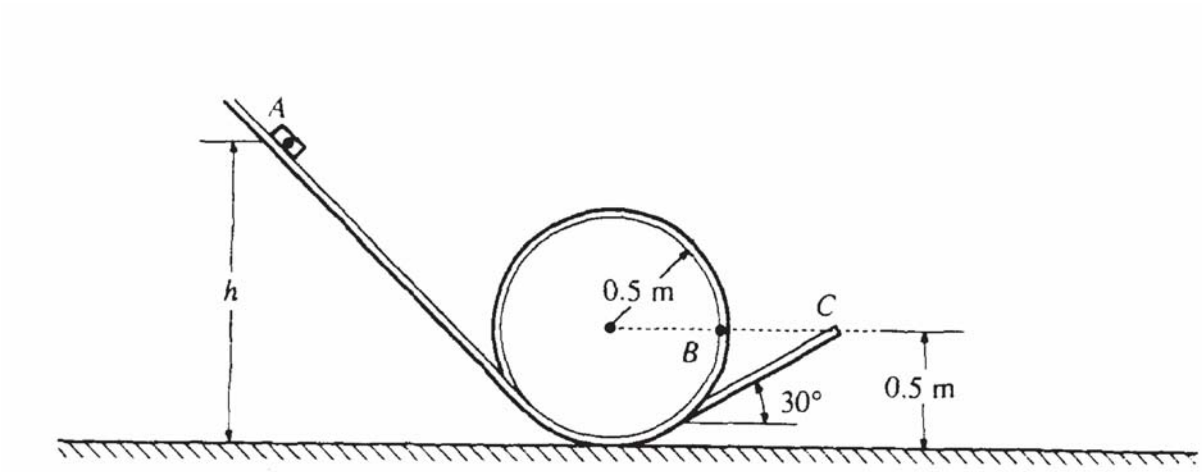
C1988M2.





A 5-kilogram object initially slides with speed v_o in a hollow frictionless pipe. The end of the pipe contains a spring as shown. The object makes contact with the spring at point A and moves 0.1 meter before coming to rest at point B. The graph shows the magnitude of the force exerted on the object by the spring as a function of the objects distance from point A.

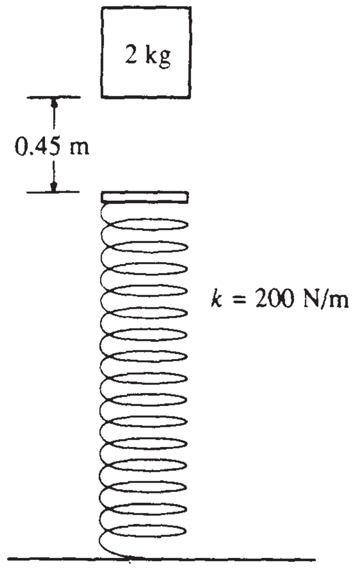
a. Calculate the spring constant for the spring.
b. Calculate the decrease in kinetic energy of the object as it moves from point A to point B.
c. Calculate the initial speed v_o of the object



C1989M1. A 0.1 kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 m/s at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

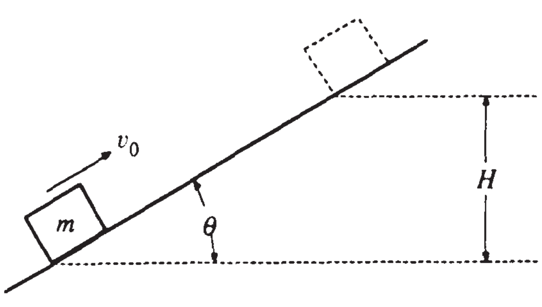
a. Determine the height h .
b. On the figure below, draw and label all the forces acting on the block when it is at point B. which is 0.5 meter above the ground.

c. Determine the magnitude of the velocity of the block when it is at point B.
d. Determine the magnitude of the force exerted by the track on the block when it is at point B.
e. Determine the maximum height above the ground attained by the block after it leaves the track.
f. Another track that has the same configuration, but is **NOT** frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 m/s, the height h must be 2 meters. Determine the work done by the frictional force.



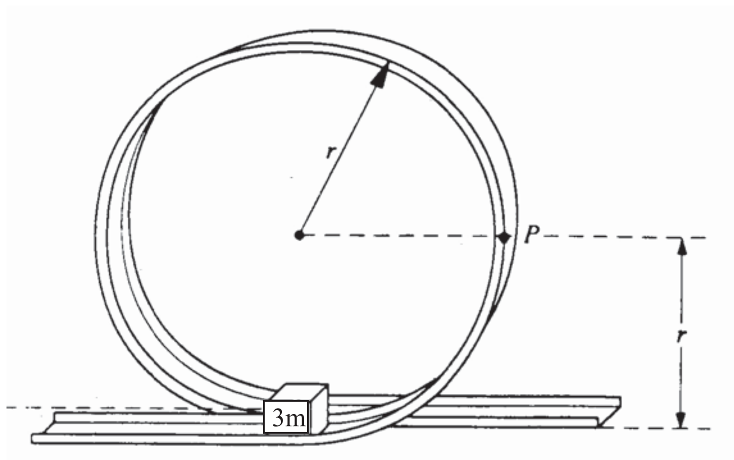
C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Is the speed of the block a maximum at the equilibrium position, explain.



C1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, the box slides a distance $d = h_2 / \sin\theta$ along the length of the ramp as it reaches a new maximum height h_2. Determine the new maximum height h_2 in terms of the given quantities.

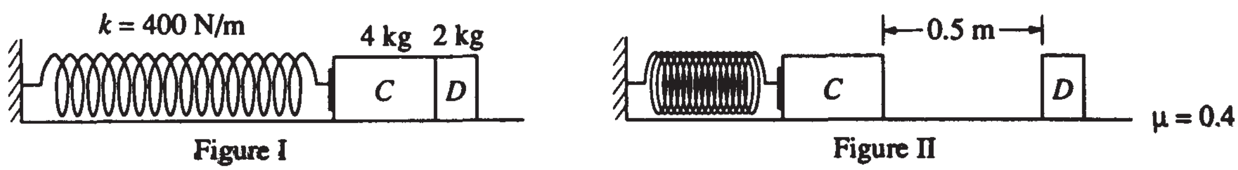


C1991M1. A small block of mass 3m moving at speed $v_0 / 3$ enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o, r, and g.

a. The kinetic energy of the block and bullet when they reach point P on the loop

b. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times

c The new required entry speed v_o' at the bottom of the loop such that the conditions in part b apply.



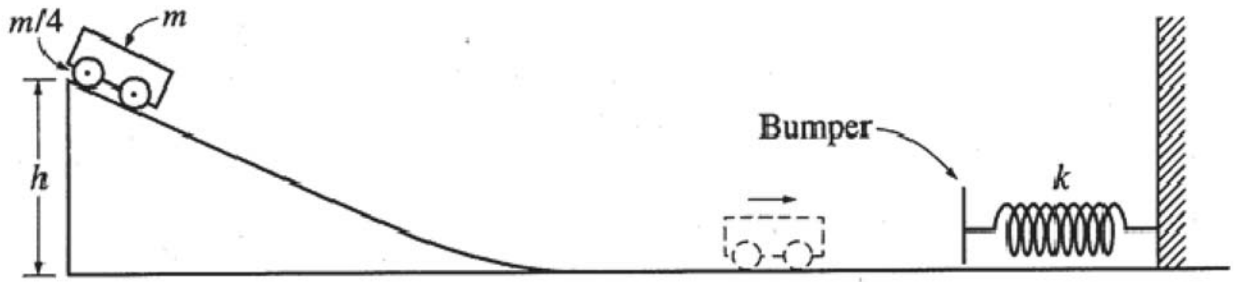
C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a rough horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use $g = 10\ m/s^2$.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D*. The surface is rough and the coefficient of friction between each block and the surface is $\mu = 0.4$. The two blocks collide instantaneously, stick together, and move to the right at 3 m/s. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

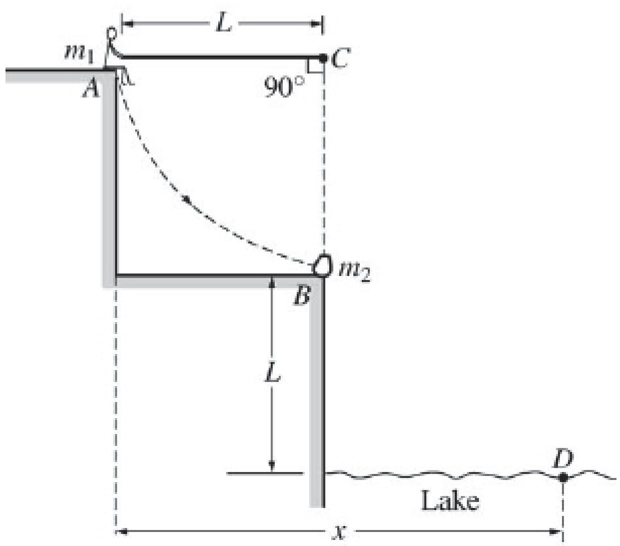
c. The horizontal distance the combined blocks move after leaving the spring before coming to rest



C2002M2. The cart shown above has a mass 2m. The cart is released from rest and slides from the top of an inclined frictionless plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the speed of the cart when it reaches the bottom of the incline.
b. After sliding down the incline and across the frictionless horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

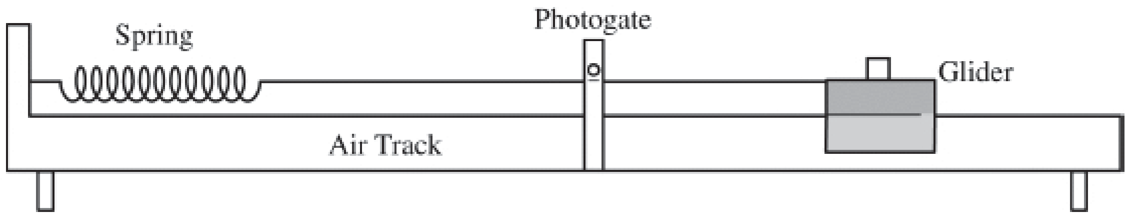
C2004M1



A rope of length *L* is attached to a support at point *C*. A person of mass m_1 sits on a ledge at position *A* holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position *B* on a lower ledge where an object of mass m_2 is at rest. At position *B* the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point *D*, which is a vertical distance *L* below position *B*. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, *L*, and *g*.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) After the person hits and grabs the rock, the speed of the combined masses is determined to be v'. In terms of v' and the given quantities, determine the total horizontal displacement *x* of the person from position *A* until the person and object land in the water at point *D*.

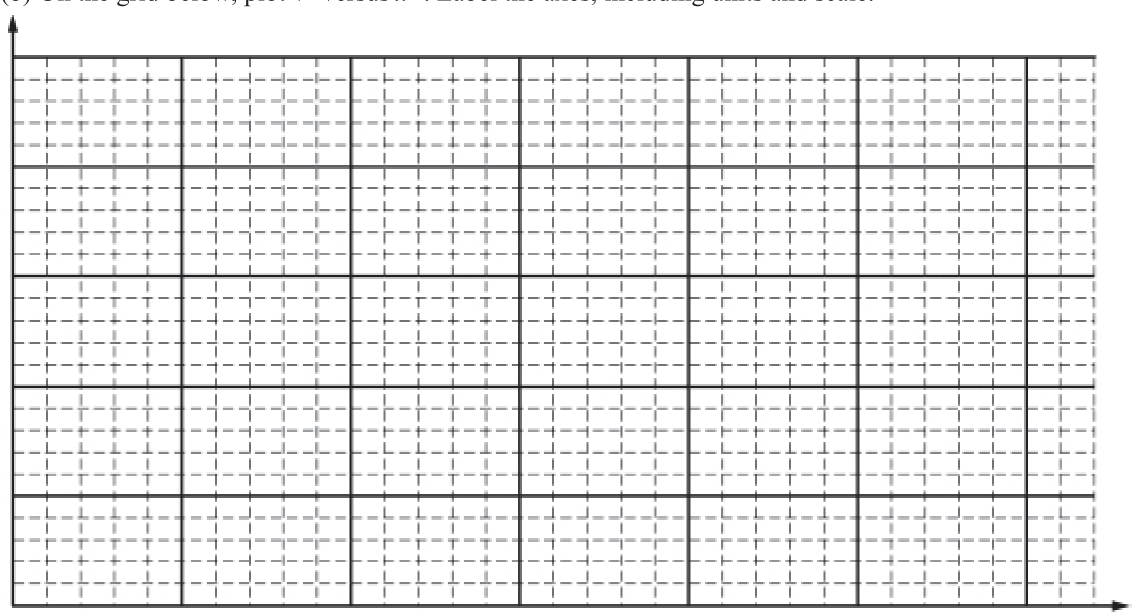
C2007M3.



The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass *m*. The glider is pulled to stretch the spring an amount *x* from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time *t* that it takes the small block on top of the glider to pass through. Information about the distance *x* and the speed *v* of the glider as it passes through the photogate are given below.

Trial #	Extension of the Spring x (m)	Speed of Glider v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

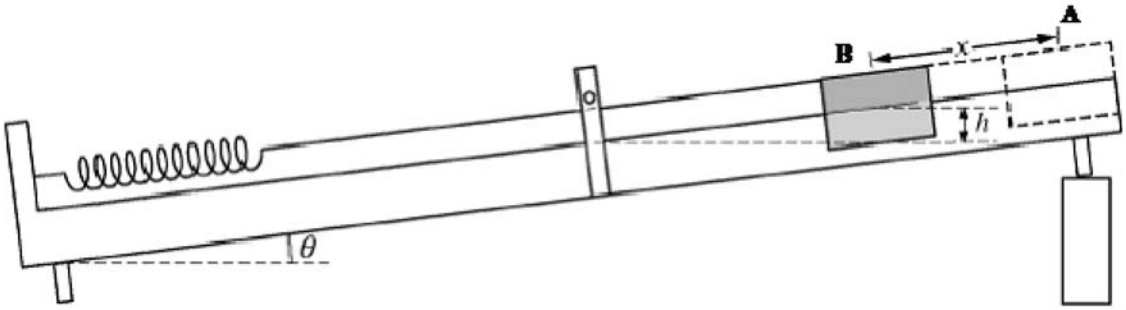
(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.
(b) On the grid below, plot v^2 versus x^2 . Label the axes, including units and scale.



(c)
i. Draw a best-fit straight line through the data.

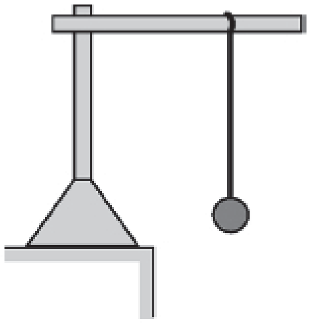
ii. Use the best-fit line to obtain the mass m of the glider.

(d) The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height h above the photogate. The experiment is repeated with a variety of values of x.



Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation starting from position A and ending at position B

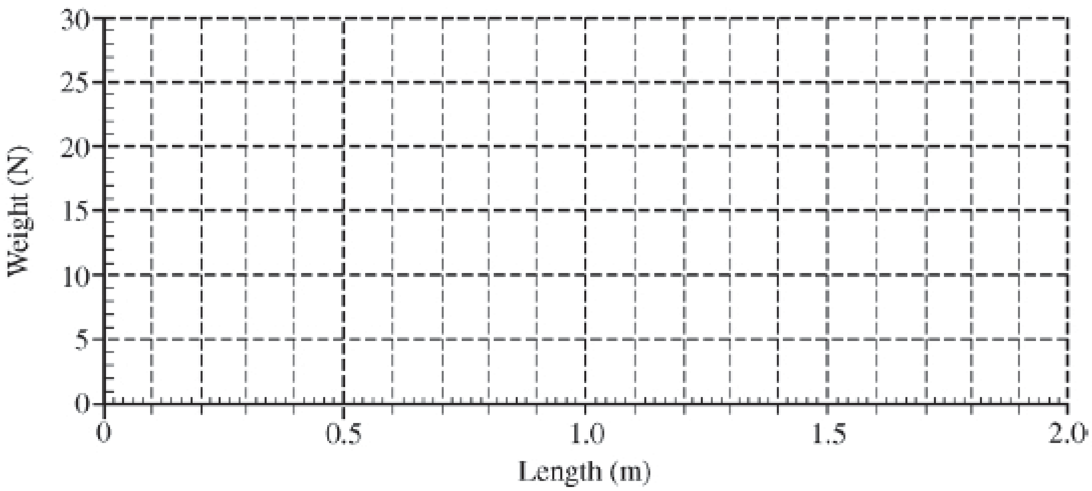
C2008M3



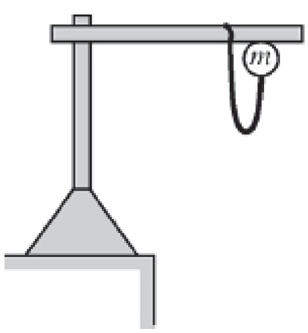
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



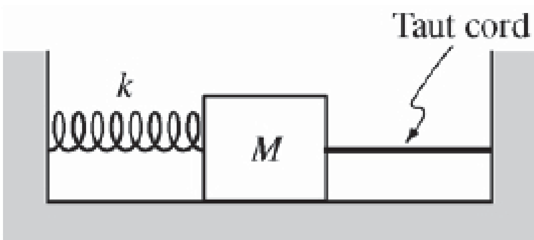
The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass m of the object.

(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

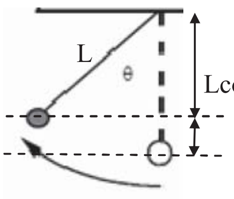
(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Work-Energy

	Solution	Answer
1.	Conservation of Energy, $U_{sp} = K$, $\frac{1}{2} kA^2 = \frac{1}{2} mv^2$ solve for v	B
2.	Constant velocity → $F_{net}=0$, $f_k = Fx = F\cos\theta$ $W_{fk} = -f_k d = -F\cos\theta\, d$	A
3.	In a circle moving at a constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle	D
4.	 The potential energy at the first position will be the amount "lost" as the ball falls and this will be the change in potential. $U=mgh = mg(L-L\cos\theta)$	A
5.	The work done by the stopping force equals the loss of kinetic energy. $-W=\Delta K$ $-Fd = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$ $v_f = 0$ $F = mv^2/2d$	A
6.	This is a conservative situation so the total energy should stay same the whole time. It should also start with max potential and min kinetic, which only occurs in choice C	C
7.	Stopping distance is a work-energy relationship. Work done by friction to stop = loss of kinetic $-f_k d = -\frac{1}{2} mv_i^2$ $\mu_k mg = \frac{1}{2} mv_i^2$ The mass cancels in the relationship above so changing mass doesn't change the distance	B
8.	Same relationship as above … double the v gives 4x the distance	D
9.	Half way up you have gained half of the height so you gained ½ of potential energy. Therefore you must have lost ½ of the initial kinetic energy so $E_2 = (E_k/2)$. Subbing into this relationship $E_2 = (E_k/2)$ $\frac{1}{2} mv_2^2 = \frac{1}{2} m v^2 / 2$ $v_2^2 = v^2 / 2$ …. Sqrt both sides gives answer	B
10.	At the top, the ball is still moving (v_x) so would still possess some kinetic energy	A
11.	Same as question #1 with different variables used	B
12.	Total energy is always conserved so as the air molecules slow and lose their kinetic energy, there is a heat flow which increases internal (or thermal) energy	C
13.	Eliminating obviously wrong choices only leaves A as an option. The answer is A because since the first ball has a head start on the second ball it is moving at a faster rate of speed at all times. When both are moving in the air together for equal time periods the first faster rock will gain more distance than the slower one which will widen the gap between them.	A
14.	For a mass on a spring, the max U occurs when the mass stops and has no K while the max K occurs when the mass is moving fast and has no U. Since energy is conserved it is transferred from one to the other so both maximums are equal	C

15. Since the ball is thrown with initial velocity it must start with some initial K. As the mass falls it gains velocity directly proportional to the time (V=Vi+at) but the K at any time is equal to 1/2 mv^2 which gives a parabolic relationship to how the K changes over time. D

16. Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates D

17. The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the K gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph. C

18. Point IV is the endpoint where the ball would stop and have all U and no K. Point II is the minimum height where the ball has all K and no U. Since point III is halfway to the max U point half the energy would be U and half would be K C

19. Apply energy conservation using points IV and II. $U_4 = K_2$ mgh = ½ mv^2 B

20. Since the track is rough there is friction and some mechanical energy will be lost as the block slides which means it cannot reach the same height on the other side. The extent of energy lost depends on the surface factors and cannot be determined without more information D

21. As the object oscillates its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D D

22. To push the box at a constant speed, the child would need to use a force equal to friction so $F=f_k=\mu mg$. The rate of work (W/t) is the power. Power is given by P=Fv → μmgv A

23. Two steps. I) use hookes law in the first situation with the 3 kg mass to find the spring constant (k). $F_{sp}=k\Delta x$, $mg=k\Delta x$, k = 30/.12 = 250. II) Now do energy conservation with the second scenario (note that the initial height of drop will be the same as the stretch Δx).
U top = U_{sp} bottom, mgh = ½ k Δx^2, (4)(10)(Δx) = ½ (250) (Δx^2) C

24. In a circular orbit, the velocity of a satellite is given by $v = \sqrt{\frac{Gm_e}{r}}$ with m_e = M. Kinetic energy of the satellite is given by K = ½ m v^2. Plug in v from above to get answer A

25. Projectile. V_x doesn't matter V_{iy} = 0. Using d = $v_{iy}t$ + ½ at^2 we get the answer D

26. A is true; both will be moving the fastest when they move through equilibrium. A

27. X and Y directions are independent and both start with the same velocity of zero in each direction. The same force is applied in each direction for the same amount of time so each should gain the same velocity in each respective direction. B

28. Kinetic energy is not a vector and the total resultant velocity should be used to determine the KE. For the 1st second the object gains speed at a uniform rate in the x direction and since KE is proportional to v^2 we should get a parabola. However, when the 2nd second starts the new gains in velocity occur only in the y direction and are at smaller values so the gains essentially start over their parabolic trend as shown in graph B B

29. As the system moves, m_2 loses energy over distance h and m_1 gains energy over the same distance h but some of this energy is converted to KE so there is a net loss of U. Simply subtract the $U_2 - U_1$ to find this loss — A

30. In a force vs. displacement graph, the area under the line gives the work done by the force and the work done will be the change in the K so the largest area is the most K change — D

31. Use energy conservation, U top = K bottom. As in problem #6 (in this document), the initial height is given by $L - L\cos\theta$, with cos 60 = .5 so the initial height is ½ L. — A

32. Use application of the net work energy theorem which says … $W_{net} = \Delta K$. The net work is the work done by the net force which gives you the answer — A

33. There is no U_{sp} at position x=0 since there is no Δx here so this is the minimum U location — A

34. Using energy conservation in the first situation presented K=U gives the initial velocity as $v = \sqrt{2gh}$. The gun will fire at this velocity regardless of the angle. In the second scenario, the ball starts with the same initial energy but at the top will have both KE and PE so will be at a lower height. The velocity at the top will be equal to the v_x at the beginning — C

35. Use energy conservation $K=U_{sp}$ $\frac{1}{2} m v_m^2 = \frac{1}{2} k \Delta x^2$, with Δx=A, solve for k — D

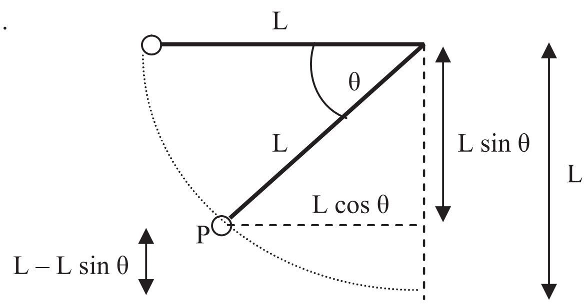
36. Based on net work version of work energy theorem. $W_{net} = \Delta K$, we see that since there is a constant speed, the ΔK would be zero, so the net work would be zero requiring the net force to also be zero. — A

37. As the block slides back to equilibrium, we want all of the initial spring energy to be dissipated by work of friction so there in no kinetic energy at equilibrium where all of the spring energy is now gone. So set work of friction = initial spring energy and solve for μ. The distance traveled while it comes to rest is the same as the initial spring stretch, d = x.
$\frac{1}{2} kx^2 = \mu mg(x)$ — C

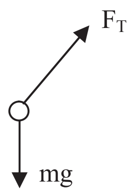
38. V at any given time is given by $v = v_i + at$, with $v_i = 0$ gives $v = at$,
V at any given distance is found by $v^2 = v_i^2 + 2ad$, with $v_i = 0$ gives $v^2 = 2ad$ — D
This question asks for the relationship to distance.
The kinetic energy is given by $K = \frac{1}{2} m v^2$ and since $v^2 = 2ad$ we see a linear direct relationship of kinetic energy to distance (2*d → 2*K)
Another way of thinking about this is in relation to energy conservation. The total of $mgh+1/2mv^2$ must remain constant so for a given change in (h) the $\frac{1}{2} mv^2$ term would have to increase or decrease directly proportionally in order to maintain energy conservation.

39. Similar to the discussion above. Energy is conserved so the term $mgh + \frac{1}{2} mv^2$ must remain constant. As the object rises it loses K and gains U. Since the height is H/2 it has gained half of the total potential energy it will end up with which means it must have lost half of its kinetic energy, so its K is half of what it was when it was first shot. — B

1974B1.



(a) FBD



(b) Apply conservation of energy from top to point P

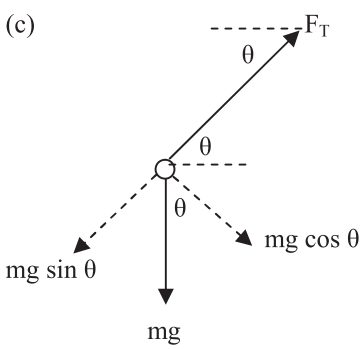
$U_{top} = U_p + K_p$

$mgh = mgh_p + ½ m v_p^2$

$gL = g(L – L\sin θ) + ½ v_p^2$

$v = \sqrt{2gL\sin\theta}$

(c)



$F_{NET(C)} = m v^2 / r$

$F_T – mg \sin θ = m v^2/ r$

$F_T – mg \sin θ = m (2gL\sin θ) / L$

$F_T = 2mg \sin θ + mg \sin θ$

$F_T = 3mg \sin θ$

1974B7.

6 riders per minute is equivalent to 6x(70kg)*9.8 = 4116 N of lifting force in 60 seconds

Work to lift riders = work to overcome gravity over the vertical displacement (600 sin 30)
Work lift = Fd = 4116N (300m) = 1.23×10^6 J

P lift = W / t = 1.23×10^6 J / 60 sec = 20580 W

But this is only 40% of the necessary power. ➔ 0.40 (total power) = 20580 W

Total power needed = 51450 W

1975B1.

(a) $F_{net} = ma$ $\quad - f_k = ma$ $\quad -8 = 2a$ $\quad a = -4\ m/s^2$

(b) $v_f^2 = v_i^2 + 2ad$ $\quad (0)^2 = v_i^2 + 2(-4)(8)$ $\quad v_i = 8\ m/s$

$v_f = v_i + at$ $\quad t = 2\ sec$

(c) Apply energy conservation top to bottom

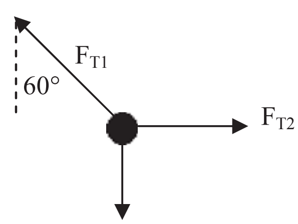
$U_{top} = K_{bot}$

$mgh = ½\ mv^2$

$(10)(R) = ½\ (8)^2$ $\quad R = 3.2\ m$

1975 B7

(a)

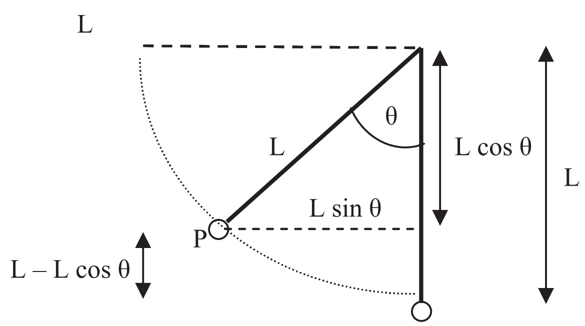


(b) $F_{NET(Y)} = 0$

$F_{T1} \cos\theta = mg$

$F_{T1} = mg / \cos(60) = 2mg$

(c) When string is cut it swing from top to bottom, similar to diagram for 1974B1 with θ moved as shown below



$U_{top} = K_{bot}$

$mgh = ½\ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

1977B1.

(a) Apply work–energy theorem

$W_{NC} = \Delta$ ME

$W_{fk} = \Delta K \quad (K_f - K_i)$

$W = -K_i$

$W = -\frac{1}{2} mv_i 2 \qquad -\frac{1}{2}(4)(6)^2 \qquad = -72$ J

(b) $F_{net} = ma$

$-f_k = m\,a \qquad v = v_i + at$

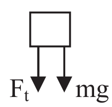
$a = -(8)/4 = -2$ m/s^2 $\qquad v = (6) + (-2)\,t$

(c) $W_{fk} = -f_k\,d$

$-72J = -(8)\,d \qquad d = 9$ m

1978B1,

(a)



(b) Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t + mg) = mv^2/r \qquad (20+0.5(10))=(0.5)v^2 / 2 \qquad v = 10$ m/s

(c) As the object moves from P to Q, it loses U and gains K. The gain in K is equal to the loss in U.

$\Delta U = mg\Delta h = (0.5)(10)(4) = 20$ J

(d) First determine the speed at the bottom using energy.

$K_{top} + K_{gain} = K_{bottom} \qquad \frac{1}{2} mv_{top}^2 + 20\text{ J} = \frac{1}{2} mv_{bot}^2 \qquad v_{bot} = 13.42$ m/s

At the bottom, Ft acts up (towards center) and mg acts down (away from center)

Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t - mg) = mv^2/r \qquad (F_t - 0.5(10)) = (0.5)(13.42)^2 / 2 \qquad F_t = 50$ N

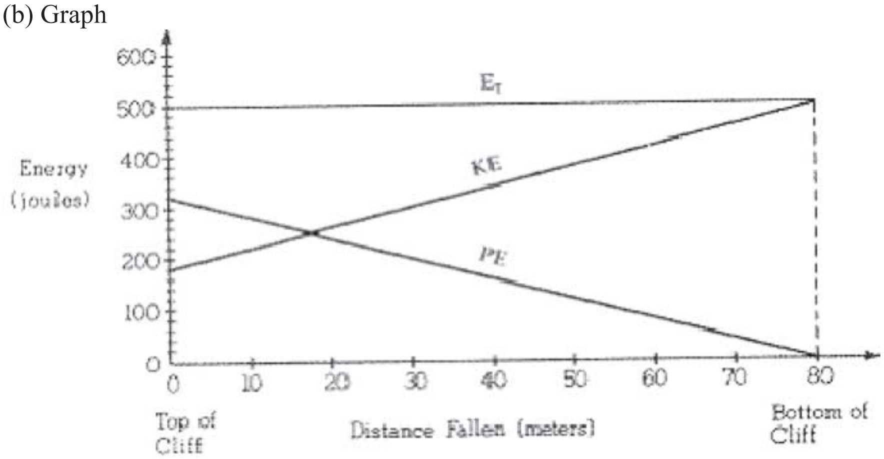
1979B1.

(a) $U = mgh = 320$ J

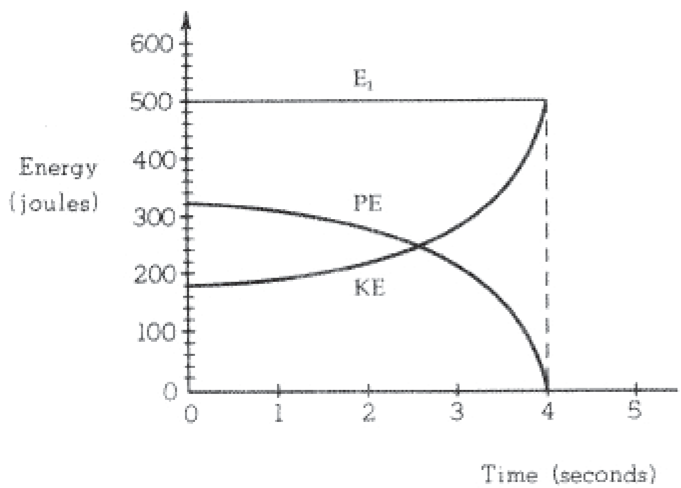
$K = ½ m v^2 = 180$ J

Total = U + K = 500 J

(b) Graph



(c) First determine the time at which the ball hits the ground, using $d_y = 0 + ½ g t^2$, to find it hits at 4 seconds.



1981B1.

(a) constant velocity means $F_{net} = 0$, $F - f_k = ma$ $F - \mu_k mg = 0$ $F - (0.2)(10)(10) = 0$
$F = 20$ N

(b) A change in K would require net work to be done. By the work–energy theorem:

$W_{net} = \Delta K$

$F_{net}\, d = 60$ J

$F_{net}\,(4\text{m}) = 60$ $F_{net} = 15$ N

$F' - f_k = 15$

$F' - 20 = 15$ $F = 35$ N

(c) $F_{net} = ma$

$(15) = (10)\, a$ $a = 1.5\ \text{m/s}^2$

1981B2.

The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring

$W = ½\, m v^2 = ½\,(3)\,(10)^2 = 150$ J

1982B3.

Same geometry as in problem 1975B7.

(a) Apply energy conservation top to bottom

$U_{top} = K_{bot}$

$mgh = ½\, m v^2$

$mg\,(R - R\cos\theta) = ½\, m v^2$

$$v = \sqrt{2g(R - R\cos\theta)}$$

(b) Use $F_{NET(C)} = mv^2 / r$

$F_t - mg = m\,(2g(R\text{-}R\cos\theta))\,/\,R$

$1.5\, mg - mg = 2mg(1 - 1\cos\theta)$

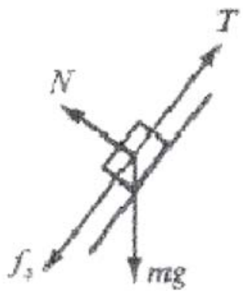
$.5 = 2(1 - \cos\theta)$

$2\cos\theta = 1.5$ → $\cos\theta = ¾$

1985B2.

(a) The tension in the string can be found easily by isolating the 10 kg mass. Only two forces act on this mass, the Tension upwards and the weight down (mg) …. Since the systems is at rest, T = mg = 100 N

(b) FBD



(c) Apply $F_{net} = 0$ along the plane. $T - f_s - mg \sin\theta = 0$ $(100\text{ N}) - f_s - (10)(10)(\sin 60)$
$f_s = 13$ N

(d) Loss of mechanical energy = Work done by friction while sliding
First find kinetic friction force   Perpendicular to plane $F_{net} = 0$ $F_n = mg\cos\theta$
$F_k = \mu_k F_n = \mu_k mg \cos\theta$

$W_{fk} = f_k d = \mu_k mg \cos\theta\,(d) = (0.15)(10)(10)(\cos(60)) = 15$J converted to thermal energy

(e) Using work-energy theorem … The U at the start – loss of energy from friction = K left over
$U - W_{fk} = K$
$mgh - W_{fk} = K$
$mg(d \sin 60) - 15 = K$
$(10)(10)(2) \sin 60 - 15 = K$ K = 158 J

1986B2.

(a) Use projectile methods to find the time. $d_y = v_{iy}t + ½\, a\, t^2$ $h = 0 + g\, t_2 / 2$

$$t = \sqrt{\frac{2h}{g}}$$

(b) v_x at ground is the same as v_x top $V_x = d_x / t$

$$v_x = \frac{D}{\sqrt{\frac{2h}{g}}}$$

multiply top and bottom by reciprocal to rationalize

$$v_x = D\sqrt{\frac{g}{2h}}$$

(c) The work done by the spring to move the block is equal to the amount of K gained by it = K_f

$W = K_f = ½\, m\, v^2 = (½\, M\, (D^2 / (2h/g)) = MD^2\, g / 4h$

(d) Apply energy conservation $U_{sp} = K$

$½\, k\Delta x^2 = ½\, mv^2$ (plug in V from part b) $$v_x = \frac{MD^2 g}{2hX^2}$$

If using $F=k\Delta x$ you have to plug use F_{avg} for the force

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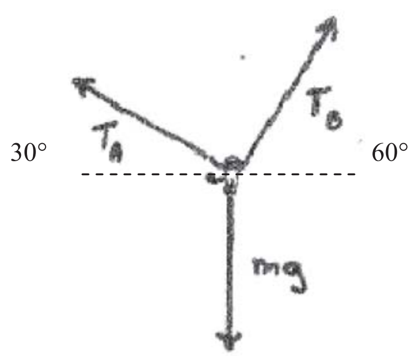
1991B1.

(a) FBD



(b) SIMULTANEOUS EQUATIONS

$F_{net(X)} = 0$ $F_{net(Y)} = 0$

$T_a \cos 30 = T_b \cos 60$ $T_a \sin 30 + T_b \sin 60 – mg = 0$

…. Solve above for T_b and plug into Fnet(y) eqn and solve

$T_a = 24$ N $T_b = 42$ N

(c) Using energy conservation with similar diagram as 1974B1 geometry

$U_{top} = U_p + K_p$

$mgh = ½\, m\, v^2$

$g(L – L\sin\theta) = ½\, v^2$

$(10)(10 – 10\sin60) = ½\, v^2$ $v = 5.2$ m/s

(d) $F_{net(C)} = mv^2/r$

$F_t – mg = mv^2 / r$ $F_t = m(g + v^2/r)$ $F_t = (5)(9.8+(5.2)^2/10) = 62$ N

1992B1.

(a) K + U   ½ m v^2 + mgh   ½ $(0.1)(6)^2 + (0.1)(9.8)(1.8) = 3.6$ J

(b) Apply energy conservation using ground as h=0

$E_{top} = E_p$

3.6 J = K + U

$3.6 = ½ m v^2 + mgh$

$3.6 = ½ (0.1)(v^2) + (0.1)(9.8)(.2)$ v = 8.2 m/s

(c) Apply net centripetal force with direction towards center as +

i) Top of circle = Ft points down and Fg points down

$F_{net(c)} = mv^2/r$

$Ft + mg = mv^2/r$ $F_t = mv^2/r – mg$ $(0.1)(6)^2/(0.8) – (.1)(9.8)$ $F_t = 3.5$ N

ii) Bottom of circle = Ft points up and Fg points down

$F_{net(c)} = mv^2/r$

$F_t – mg = mv^2/r$ $F_t = mv^2/r + mg$ $(0.1)(8.2)^2/(0.8) + (0.1)(9.8)$ $F_t = 9.5$ N

(d) Ball moves as a projectile.

First find time of fall in y direction

$d_y = v_{iy}t + ½ a t^2$

$(–0.2) = 0 + ½ (–9.8) t^2$

t = .2 sec

Then find range in x direction

$d_x = v_x t$

$d_x = (8.2)(0.2)$

$d_x = 1.6$ m

1996B2.

(a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k \Delta x$ and solve for k

(b) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan \theta$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} – \mu_s$ mg, Giving $mg = F_{spring} / \mu$.
Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid dynamics unit.

1997B1.

(a) The force is constant, so simple $F_{net} = ma$ is sufficient. (4) = (0.2) a a = 20 m/s^2

(b) Use $d = v_i t + ½ a t^2$ 12 = (0) + ½ (20) t^2 t = 1.1 sec

(c) W = Fd   W = (4 N) (12 m) = 48 J

(d) Using work energy theorem   W = ΔK   (K_i = 0)   W = Kf - Ki

W = ½ m v_f^2

Alternatively, use $v_f^2 = v_i^2 + 2 a d$ 48J = ½ (0.2) (v_f^2) v_f = 22 m/s

(e) The area under the triangle will give the extra work for the last 8 m

½ (8)(4) = 16J   + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem   W = ½ m v_f^2 64 J = ½ (0.2) v_f^2 v_f = 25.3 m/s

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

1999B1.

(a) Plug into $g = GM_{planet} / r_{planet}^2$ lookup earth mass and radius

g_{mars} = 3.822 m/s^2 to get it in terms of g_{earth} divide by 9.8 $g_{mars} = 0.39\ g_{earth}$

(b) Since on the surface, simply plug into F_g = mg = (11.5)(3.8) = 44 N

(c) On the incline, F_n = mg cos θ = (44) cos (20) = 41 N

(d) moving at constant velocity → F_{net} = 0

(e) W = P t   (5.4x10^5 J) = (10 W) t   t = 54000 sec

d = v t   (6.7x10^{-3})(54000 s)   d = 362 m

(f) P = Fv   (10) = F (6.7x10^{-3}) F_{push} =1492.54 N total pushing force used

* (.0001) use for drag

→ F_{drag} = 0.15 N

2002B2.

(a) From graph U = 0.05 J

(b) Since the total energy is 0.4 J, the farthest position would be when all of that energy was potential spring energy. From the graph, when all of the spring potential is 0.4 J, the displacement is 10 cm

(c) At –7 cm we read the potential energy off the graph as 0.18 J. Now we use energy conservation.

ME = U_{sp} + K   0.4J = 0.18 J + K   → K = 0.22 J

(d) At x=0 all of the energy is kinetic energy K= ½ m v^2 0.4 = ½ (3) v^2 v = 0.5 m/s

(e) The object moves as a horizontally launched projectile when it leaves.

First find time of fall in y direction	Then find range in x direction
$d_y = v_{iy}t + ½ a t^2$	$d_x = v_x t$
(– 0.5) = 0 + ½ (– 9.8) t^2	d_x = (0.5)(0.3)
t = 0.3 sec	d_x = 0.15 m

2004B1.

(a) i) fastest speed would be the lowest position which is the bottom of the first hill where you get all sick and puke your brains out.

ii) Applying energy conservation from the top of the hill where we assume the velocity is approximately zero we have

$U_{top} = K_{bottom}$

$mgh = ½\ m\ v^2$ $\qquad$ $(9.8)\ (90) = ½\ v^2$ $\qquad$ $v = 42$ m/s

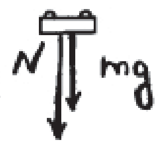
(b) Again applying energy conservation from the top to position B

$U_{top} = K_b + U_b$

$mgh = ½\ m\ v_B{}^2 + mgh_B$

$(9.8)(90) = ½\ v_B{}^2 + (9.8)(50)$ $\qquad$ $v_B = 28$ m/s

(c) i) FBD



ii) $mg = (700)(9.8) = 6860$ N

$F_{net(C)} = mv^2 /r$

$F_n + mg = mv^2 / r$

$F_n = mv^2/r - mg \quad = m\ (v^2/r - g) \ = (700)\ (28^2/20 - 9.8) = 20580$ N

(d) The friction will remove some of the energy so there will not be as much Kinetic energy at the top of the loop. In order to bring the KE back up to its original value to maintain the original speed, we would need less PE at that location. A lower height of the loop would reduce the PE and compensate to allow the same KE as before. To actually modify the track, you could flatten the inclines on either side of the loop to lower the height at B.

B2004B1.

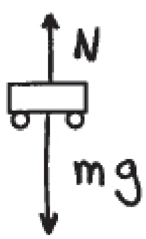
(a) set position A as the h=0 location so that the PE=0 there.

Applying energy conservation with have

$U_{top} + K_{top} = K_A$ $\qquad$ $mgh + ½\ m\ v^2 = ½\ m\ v_A{}^2$

$(9.8)(0.1) + ½\ (1.5)^2 = ½\ v_A{}^2$ $\qquad$ $v_A = 2.05$ m/s

(b) FBD



(c) $F_{net(C)} = mv^2 /r$

$mg - F_N = mv^2 / r$

$Fn = mg - mv^2/r \quad = m\ (g - v^2/r\ ) = (0.5)(9.8 - 2.05^2 / 0.95) \quad = 2.7$ N

(c) To stop the cart at point A, all of the kinetic energy that would have existed here needs to be removed by the work of friction which does negative work to remove the energy.

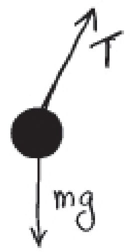
$W_{fk} = - K_A$

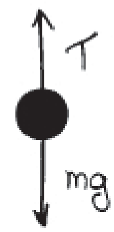
$W_{fk} = -\ ½\ m\ v_A{}^2 = -\ ½\ (0.5)(2.05^2) = -\ 1.1$ J

(d) The car is rolling over a hill at point A and when F_n becomes zero the car just barely loses contact with the track. Based on the equation from part (c) the larger the quantity (mv^2 / r) the more likely the car is to lose contact with the track (since more centripetal force would be required to keep it there) … to increase this quantity either the velocity could be increased or the radius could be decreased. To increase the velocity of the car, make the initial hill higher to increase the initial energy. To decrease the radius, simply shorten the hill length.

B2005B2.

FBD i) ii)





(b) Apply energy conservation?

$U_{top} = K_{bottom}$

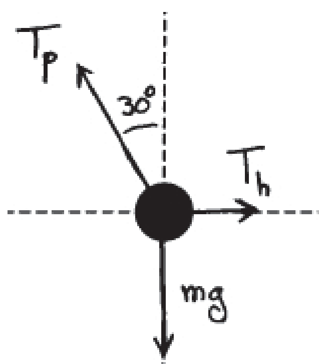
$mgh = ½\, m\, v^2$ $\qquad (9.8)(.08) – ½\, v^2$ $\qquad v = 1.3$ m/s

(c) $F_{net(c)} = mv^2/r$

$F_t – mg = mv^2/r$ $\qquad F_t = mv^2/r + mg$ $\qquad (0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $\qquad F_t = 0.93$ N

2005B2.

(a) FBD



(b) Apply

$F_{net(X)} = 0$ $\qquad F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $\qquad T_P \sin 30 = T_H$

$T_P = 20.37$ N $\qquad T_H = 10.18$ N

(c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$

$g(L – L \cos θ) = ½\, v^2$

$(10)\,(2.3 – 2.3 \cos 30) = ½\, v^2$ $\qquad v_{bottom} = 2.5$ m/s

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B2006B2.

(a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$ $\qquad Mgh = ½\,(M)\,(3.5v_o)^2$ $\qquad h = 6.125\, v_o^2 / g$

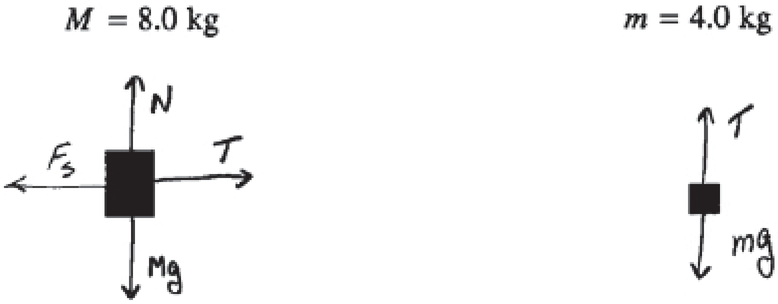
(b) $W_{NC} = ΔK\ \ (K_f\text{-}K_i)\quad K_f = 0$

$– f_k\, d = 0 – ½\,(1.5M)(2v_o)^2$

$μ_k\,(1.5\, M)\, g\,(d) = 3Mv_o^2$ $\qquad μ_k = 2v_o^2 / gD$

2006B1.

(a) FBD



(b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

(c) Tension in string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k\,(0.05)$ $k = 780$ N/m

(d) 4 kg mass is in free fall. $D = v_i t + ½\, g\, t^2$ $-0.7 = 0 + ½\,(-9.8)t^2$ $t = 0.38$ sec

(e) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $½\, k\, \Delta x^2 = ½\, mv^2$ $½\,(780)\,(0.05) = ½\,(8)\,v^2$ $v = 0.49$ m/s

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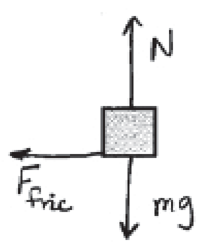
B2008B2.

(a) $d = v_i t + ½\, a\, t^2$ $(55) = (25)(3) + ½\, a\,(3)^2$ $a = -4.4$ m/s^2

(b) FBD



(c) using the diagram above and understanding that the static friction is actually responsible for decelerating the box to match the deceleration of the truck, we apply F_{net}
$F_{net} = ma$
$-f_s = -\mu_s F_n = ma$ $-\mu_s\, mg = ma$ $-\mu_s = a/g$ $-\mu_s = -4.4 / 9.8$ $\mu_s = 0.45$

Static friction applied to keep the box at rest relative to the truck bed.

(d) Use the given info to find the acceleration of the truck $a = \Delta v / t = 25/10 = 2.5$ m/s^2
To keep up with the trucks acceleration, the crate must be accelerated by the spring force, apply F_{net}
$F_{net} = ma$ $F_{sp} = ma$ $k\Delta x = ma$ $(9200)(\Delta x) = (900)(2.5)$ $\Delta x = 0.24$ m

(e) If the truck is moving at a constant speed the net force is zero. Since the only force acting directly on the crate is the spring force, the spring force must also become zero therefore the Δx would be zero and is **LESS** than before. Keep in mind the crate will stay on the frictionless truck bed because its inertia will keep it moving forward with the truck (remember you don't necessarily need forces to keep things moving)

2008B2.

(a) In a connected system, we must first find the acceleration of the system as a whole. The spring is internal when looking at the whole system and can be ignored.

$F_{net} = ma$ (4) = (10) a a = 0.4 m/s^2 → the acceleration of the whole system and also of each individual block when looked at separate

Now we look at just the 2 kg block, which has only the spring force acting on its FBD horizontal direction.

$F_{net} = ma$ $F_{sp} = (2)(.4)$ $F_{sp} = 0.8$ N

(b) Use $F_{sp} = k\Delta x$ $0.8 = (80)\,\Delta x$ $\Delta x = 0.01$ m

(c) Since the same force is acting on the same total mass and $F_{net} = ma$, the acceleration is the same

(d) The spring stretch will be MORE. This can be shown mathematically by looking at either block. Since the 8 kg block has only the spring force on its FBD we will look at that one.

$F_{sp} = ma$ $k\Delta x = ma$ $(80)(\Delta x) = (8)(0.4)$ $\Delta x = 0.04$ m

(e) When the block A hits the wall it instantly stops, then block B will begin to compress the spring and transfer its kinetic energy into spring potential energy. Looking at block B energy conservation:

$K_b = U_{sp}$ $\frac{1}{2}\, m\, v_b^2 = \frac{1}{2}\, k\, \Delta x^2$ $(8)(0.5)^2 = (80)\Delta x^2$ $\Delta x = 0.16$ m

2009B1.

(a) Apply energy conservation. All of the spring potential becomes gravitational potential

$U_{sp} = U$

$\frac{1}{2} k \Delta x^2 = mgh$ $\qquad$ $\frac{1}{2} k x^2 = mgh$ $\qquad$ $h = kx^2 / 2mg$

(b) You need to make a graph that is of the form y = m x, with the slope having "k" as part of it and the y and x values changing with each other. Other constants can also be included in the slope as well to make the y and x variables simpler. h is dependent on the different masses used so we will make h our y value and use m as part of our x value. Rearrange the given equation so that is it of the form y = mx with h being y and mass related to x.

We get $\qquad$ y = m x

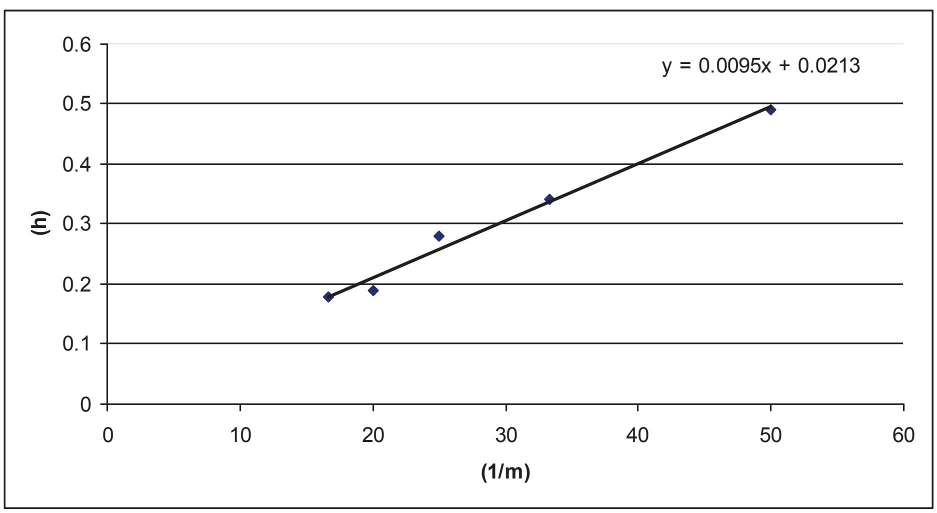
$$h = \left(\frac{kx^2}{2g}\right)\frac{1}{m}$$

so we use h as y and the value 1/m as x and graph it.

(note: we lumped all the things that do not change together as the constant slope term. Once we get a value for the slope, we can set it equal to this term and solve for k)

1/m	m (kg)	h (m)
50	0.020	0.49
33.33	0.030	0.34
25	0.040	0.28
20	0.050	0.19
16.67	0.060	0.18
X values		Y values

(c) Graph



(d) The slope of the best fit line is 0.01

We set this slope equal to the slope term in our equation, plug in the other known values and then solve it for k

$$0.01 = \left(\frac{kx^2}{2g}\right)$$

$$0.01 = \left(\frac{k(0.02)^2}{2(9.8)}\right)$$

Solving gives us k = 490 N/m

(e) - Use a stopwatch, or better, a precise laser time measurement system (such as a photogate), to determine the time it takes the toy to leave the ground and raise to the max height (same as time it takes to fall back down as well). Since its in free fall, use the down trip with v_i=0 and apply $d = \frac{1}{2} g t^2$ to find the height.
- Or, videotape it up against a metric scale using a high speed camera and slow motion to find the max h.

C1973M2

(a) Apply work-energy theorem

$W_{nc} = \Delta ME$

$W_{fk} = \Delta K \quad (K_f - K_i) \qquad K_f = 0$

$-f_k d = -\frac{1}{2} m v_i^2 \qquad -f_k (0.12) = -\frac{1}{2}(0.030)(500)^2 \qquad f_k = 31250$ N

(b) Find find acceleration $\quad -f_k = ma \qquad -(31250) = (0.03)\,a \qquad a = -1.04\text{x}10^6\ \text{m/s}^2$

Then use kinematics $\quad v_f = v_i + at \qquad 0 = 500 + (-1.04\text{x}10^6)\,t \qquad t = 4.8\text{x}10^{-4}$ sec

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C1982M1

(a) Apply energy conservation, set the top of the spring as h=0, therefore H at start = Lsin θ = 6 sin30 = 3 m

$U_{top} = K_{bot} \quad mgh = \frac{1}{2} mv^2 \quad (9.8)(3) = \frac{1}{2}(v^2) \qquad v = 7.67$ m/s

(b) Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.

$h = L \sin\theta + \Delta x \sin\theta$

$U_{top} = U_{sp}(bot)$

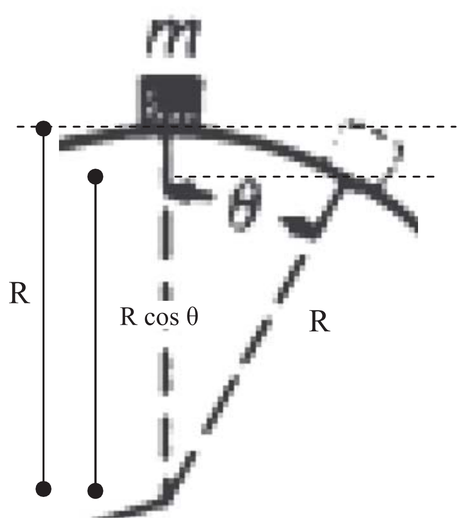
$mgh = \frac{1}{2} k \Delta x^2$

$mg(L \sin\theta + \Delta x \sin\theta) = \frac{1}{2} k \Delta x^2$

$(20)(9.8)(6 \sin 30 + 3 \sin 30) = \frac{1}{2} k (3)^2 \qquad k = 196$ N/m

(c) The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

C1983M3.



$h = R - R\cos\theta = R(1 - \cos\theta)$

i) $K_2 = U_{top}$

$K_2 = mg(R(1 - \cos\theta))$

ii) From, $K = \frac{1}{2} m v^2 = mgR(1 - \cos\theta)$ … $v^2 = 2gR(1 - \cos\theta)$

Then $a_c = v^2 / R = \quad 2g(1 - \cos\theta)$

C1985M1

(a) We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} - f_k = 0$ $mg \sin\theta = \mu_s(F_n)$
$mg \sin\theta = \mu_s\, mg \cos\theta$ $\mu_s = \tan\theta$

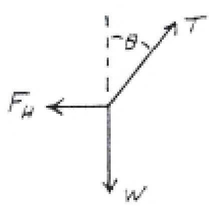
(b) Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so h= (d+x)sin θ
$U_f - U_i + U_{sp(F)} - U_{sp(i)}$
$0 - mgh + ½\, k\Delta x^2 - 0$ $½\, kx^2 - mg(d+x)\sin\theta$

(c) To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k
$W_{NC} = ½\, kx^2 - mg(d+x)\sin\theta$ $-f_k(d+x) = ½\, kx^2 - mg(d+x)\sin\theta$
$-\mu_k\, mg \cos\theta\, (d+x) = ½\, kx^2 - mg(d+x)\sin\theta$

$\mu_k = [mg(d+x)\sin\theta - ½\, kx^2] / [mg\,(d+x)\cos\theta]$

C1987M1

(a)



$F_{net(y)} = 0$
$T \cos\theta - W = 0$ $T = W / \cos\theta$

(b) Apply SIMULTANEOUS EQUATIONS
$F_{net(y)} = 0$ $F_{net(x)} = 0$
$T \cos\theta - W = 0$ $T \sin\theta - F_h = 0$
Sub T into X equation to get F_h $F_h = W \tan\theta$

(c) Using the same geometry diagram as solution 1975B7 solve for the velocity at the bottom using energy conservation

$U_{top} = K_{bot}$
$mgh = ½\, mv^2$

$$v = \sqrt{2g(L - L\cos\theta)}$$
$$v = \sqrt{2gL(1-\cos\theta)}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(T - W) = m(2gL(1-\cos\theta)) / L$

$T = W + 2mg - 2mg \cos\theta$
$T = W + 2W - 2W \cos\theta = W(3 - 2\cos\theta)$

C1988M2

(a) The graph is one of force vs Δx so the slope of this graph is the spring constant. Slope = 200 N/m

(b) Since there is no friction, energy is conserved and the decrease in kinetic energy will be equal to the gain in spring potential $|\Delta K| = U_{sp(f)} = ½\, k\, \Delta x^2 = ½\, (200)(0.1)^2 = 1$J.
Note: This is the same as the area under the line since the area would be the work done by the conservative spring force and the work done by a conservative force is equal to the amount of energy transferred.

(c) Using energy conservation. $K_i = U_{sp(f)}$ $½\, mv_o^2 = 1$ J $½\, (5)\, v_o^2 = 1$ $v_o = 0.63$ m/s

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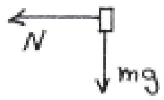
C1989M1

(a) Apply energy conservation from point A to point C setting point C as h=0 location
(note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

$U_A = K_C$ $mgh_a = ½ m v_c^2$ $(0.1)(9.8)(h_a) = ½ (0.1)(4)^2$ $h_a = 0.816m$
$h = h_a + 0.5 m = 1.32 m$

(b)



(c) Since the height at B and the height at C are the same, they would have to have the same velocities $v_b = 4$ m/s

(d) $F_{net(c)} = mv^2 / r$ $F_n = (0.1)(4)^2/(0.5) = 3.2$ N

(e) Using projectile methods … $V_{iy} = 4\sin30 = 2$ m/s Then $v_{fy}^2 = v_{iy}^2 + 2 a d_y$
$(0) = (2)^2 + 2(-9.8)(d_y)$ $d_y = 0.2$
$h_{max} = d_y$ + initial height $= 0.7$ m

Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

(f) Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $U_{new} - U_{old} = mgh_{net} - mgh_{old}$ $(0.1)(9.8)(2-1.32) = 0.67$ J lost.

C1989M3

(a) Apply energy conservation from start to top of spring using h=0 as top of spring.
$U = K$ $mgh = ½ m v^2$ $(9.8)(0.45) = ½ v^2$ $v = 3$ m/s

(b) At equilibrium the forces are balances $F_{net} = 0$ $F_{sp} = mg =(2)(9.8) = 19.6$ N

(c) Using the force from part b, $F_{sp} = k \Delta x$ $19.6 = 200 \Delta x$ $\Delta x = 0.098$ m

(d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the start position is now increased by the amount of Δx found in part c $h_{new} = h+\Delta x = 0.45 + 0.098 = 0.548$ m)
$U_{top} = U_{sp} + K$
$mgh = ½ k \Delta x^2 + ½ mv^2$ $(2)(9.8)(0.548) = ½ (200)(0.098)^2 + ½ (2)(v^2)$ $v = 3.13$ m/s

(e) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

C1990M2

(a) Energy conservation, $K_{bot} = U_{top}$ $½ m v^2 = mgh$ $½ (v_o^2) = gh$ $h = v_o^2 / 2g$

(b) Work-Energy theorem. $W_{nc} = \Delta K + \Delta U$ $(U_i = 0, K_f = 0)$
$-f_kd = (mgh - 0) + (0 - ½ m v_o^2)$ $-(\mu_k mg\cos\theta)\, h_2 / \sin\theta = mgh_2 - ½ mv_o^2$

$\mu mg\cos\theta\, h_2 / \sin\theta + mgh_2 = ½ mv_o^2$ $h_2 (\mu g\cos\theta / \sin\theta + g) = ½ v_o^2$
$h_2 = v_o^2 / (2g(\mu \cot\theta+1))$

C1991M1

(a) Apply energy conservation.

$K_{bottom} = U_p + K_p$ $\quad$ $\frac{1}{2} mv_{bot}^2 = mgh_p + K_p$

$\frac{1}{2}\, 3m\, (v_o/3)^2 = 3mg(r) + K_p$ $\quad$ $K_p = mv_o^2/6 - 3mgr$

(b) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set F_n=0 at the top of the loop so that only mg is acting down on the block. Then apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\quad$ $3mg = 3m\, v^2 /r$ $\quad$ $v = \sqrt{rg}$

(c) Energy conservation, top of loop to bottom of loop

$U_{top} + K_{top} = K_{bot}$

$mgh + \frac{1}{2} m\, v_{top}^2 = \frac{1}{2} m\, v_{bot}^2$ $\quad$ $g(2r) + \frac{1}{2} (\sqrt{rg})^2 = \frac{1}{2} (vo')^2$ $\quad$ $v_o' = \sqrt{5gr}$

C1993M1

- since there is friction on the surface the whole time, this is not an energy conservation problem, use work-energy.

(a) $U_{sp} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2} (400)(0.5)^2 = 50$ J

(b) Using work-energy $\quad$ $W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)} - U_{sp(i)}) + (K_f - K_i)$

$- f_k d = (\, 0 - 50J\,) + (\, \frac{1}{2} m\, v_f^2 - 0)$

$- \mu\, mg\, d = \frac{1}{2} mv_f^2 - 50$

$- (0.4)(4)(9.8)(0.5) = \frac{1}{2} (4)(v_c^2) - 50$ $\quad$ $v_c = 4.59$ m/s

(c) $W_{nc} = (K_f - K_i)$

$-f_k d = (0 - \frac{1}{2} m\, v_i^2\,)$ $\quad$ $- \mu\, mg\, d = - \frac{1}{2} m\, v_i^2$ $\quad$ $(0.4)(6)(9.8)\, d = \frac{1}{2} (6)(3)^2$ $\quad$ d = 1.15 m

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C2002M2

(a) Energy conservation, potential top = kinetic bottom $\quad$ $v = \sqrt{2gh}$

(b) Energy conservation, potential top = spring potential $\quad$ $U = U_{sp}$ $\quad$ $(2m)gh = \frac{1}{2} k\, x_m^2$

$$x_m = 2\sqrt{\frac{mgh}{k}}$$

C2004M1

(a) Energy conservation with position B set as h=0. $U_a = K_b$ $\quad$ $v_b = \sqrt{2gL}$

(b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = mv_b^2 / r$ $\quad$ $F_t - mg = m(2gL) / L$ $\quad$ $F_t = 3mg$

(c) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + \frac{1}{2} g\, t^2$ $\quad$ $L = 0 + gt^2 / 2$

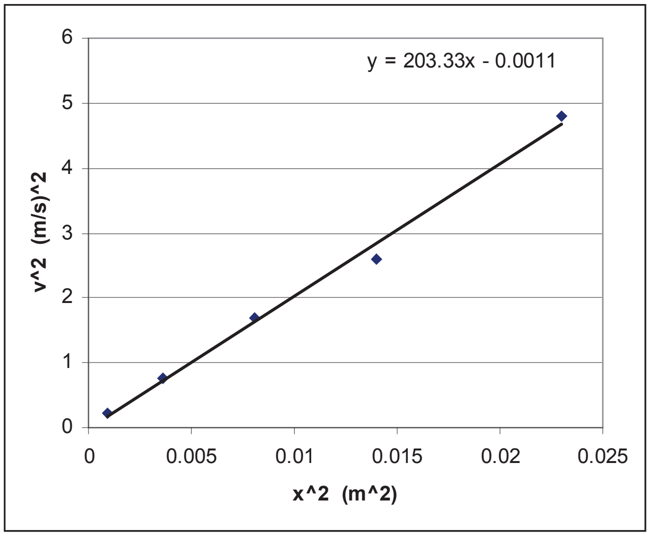
$t = \sqrt{\frac{2L}{g}}$ $\quad$ Then use $v_x = d_x / t$ $\quad$ $d_x = v'\sqrt{\frac{2L}{g}}$ $\quad$ total distance $\quad$ $x = v'\sqrt{\frac{2L}{g}} + L$

total distance includes the initial horizontal displacement L so it is added to the range

C2007M3

(a) Spring potential energy is converted into kinetic energy $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

(b) (c) i)



ii) using the equation above and rearrange to the form $y = m\,x$ with v^2 as y and x^2 as x

$y = m \quad x$

$v^2 = (k/m)\, x^2$

Slope = 200 = k /m

200 = (40) / m

m = 0.2 kg

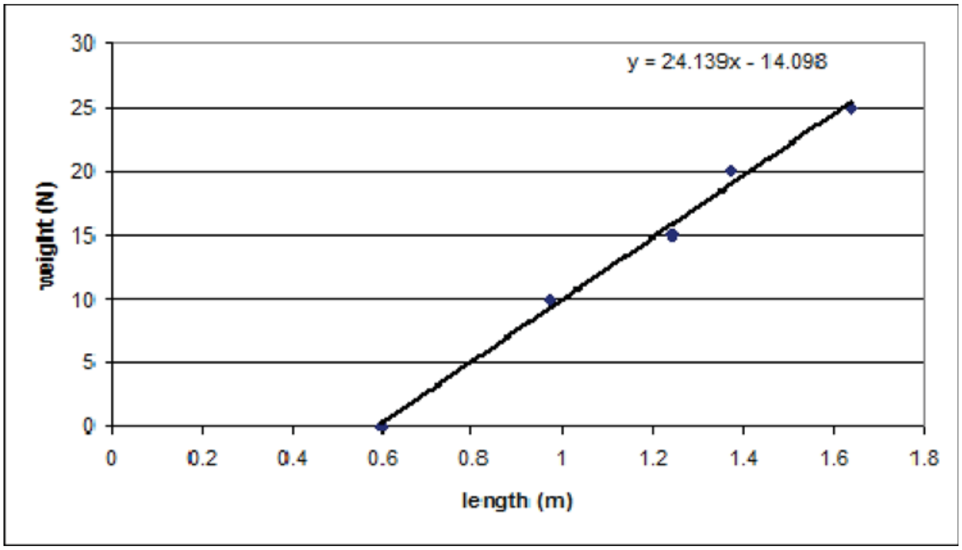
(d) Now you start with spring potential and gravitational potential and convert to kinetic. Note that at position A the height of the glider is given by h + the y component of the stretch distance x. $h_{initial} = h + x \sin\theta$

$U + U_{sp} = K$

$mgh + \frac{1}{2} k\, x^2 = \frac{1}{2} m\, v^2$ $\qquad mg(h + x \sin\theta) + \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

C2008M3

(a)



(b) The slope of the line is $F / \Delta x$ which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

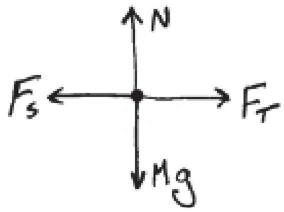
$mgh = \frac{1}{2} k \Delta x^2$ $\quad m(9.8)(1.5) = \frac{1}{2} (24)(0.9)^2$ $\quad m = 0.66$ kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $\quad F_{sp} = (0.66)(9.8)$ $\quad F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $\quad 6.5 = (24) \Delta x$ $\quad \Delta x = 0.27$ m

Supplemental

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} k\Delta x_2^2 + \frac{1}{2} mv^2$
$k (\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

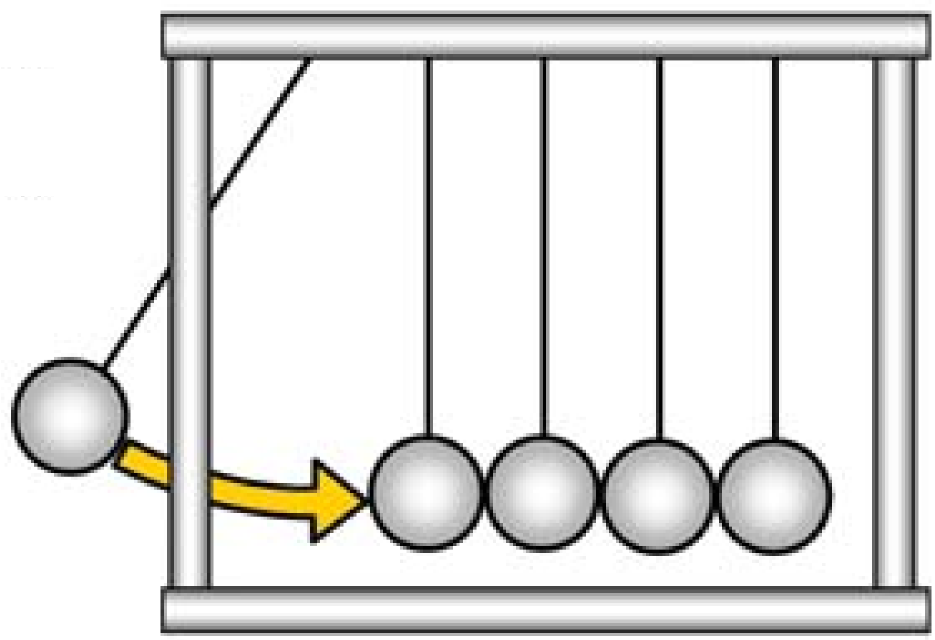
$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$

$$v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$$

(d) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get
$F_{net} = ma$ $F_{sp} - f_k = ma$
From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$

Chapter 5

Momentum and Impulse



AP Physics Multiple Choice Practice – Momentum and Impulse

1. A car of mass m, traveling at speed v, stops in time t when maximum braking force is applied. Assuming the braking force is independent of mass, what time would be required to stop a car of mass $2m$ traveling at speed v?
(A) $\frac{1}{2} t$ (B) t (C) $\sqrt{2}\, t$ (D) $2t$

2. A block of mass M is initially at rest on a frictionless floor. The block, attached to a massless spring with spring constant k, is initially at its equilibrium position. An arrow with mass m and velocity v is shot into the block. The arrow sticks in the block. What is the maximum compression of the spring?

$$(A)\ v\sqrt{\frac{m}{k}} \qquad (B)\ v\sqrt{\frac{m+M}{k}} \qquad (C)\ \frac{(m+M)v}{\sqrt{mk}} \qquad (D)\ \frac{mv}{\sqrt{(m+M)k}}$$

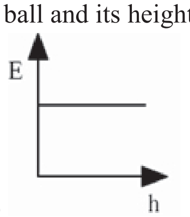
3. Two objects, P and Q, have the same momentum. Q can have more kinetic energy than P if it has:
(A) More mass than P (B) The same mass as P (C) More speed than P (D) The same speed at P

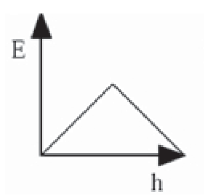
4. A spring is compressed between two objects with unequal masses, m and M, and held together. The objects are initially at rest on a horizontal frictionless surface. When released, which of the following is true?
(A) The total final kinetic energy is zero.
(B) The two objects have equal kinetic energy.
(C) The speed of one object is equal to the speed of the other.
(D) The total final momentum of the two objects is zero.

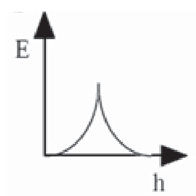
5. Two football players with mass 75 kg and 100 kg run directly toward each other with speeds of 6 m/s and 8 m/s respectively. If they grab each other as they collide, the combined speed of the two players just after the collision would be:
(A) 2 m/s (B) 3.4 m/s (C) 4.6 m/s (D) 7.1 m/s

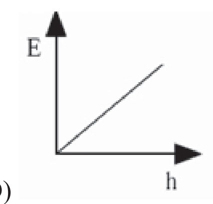
6. A 5000 kg freight car moving at 4 km/hr collides and couples with an 8000 kg freight car which is initially at rest. The approximate common final speed of these two cars is
(A) 1 km/h (B) 1.3 km/h (C) 1.5 km/h (D) 2.5 km/h

7. A rubber ball is held motionless a height h_o above a hard floor and released. Assuming that the collision with the floor is elastic, which one of the following graphs best shows the relationship between the total energy E of the ball and its height h above the surface.

(A) 

(B) 

(C) 

(D) 

8. Two carts are held together. Cart 1 is more massive than Cart 2. As they are forced apart by a compressed spring between them, which of the following will have the same magnitude for both carts.
(A) change of velocity (B) force (C) speed (D) velocity

9. A ball with a mass of 0.50 kg and a speed of 6 m/s collides perpendicularly with a wall and bounces off with a speed of 4 m/s in the opposite direction. What is the magnitude of the impulse acting on the ball?
(A) 1 Ns (B) 5 Ns (C) 2 m/s (D) 10 m/s

10. A cart with mass 2m has a velocity v before it strikes another cart of mass 3m at rest. The two carts couple and move off together with a velocity of
(A) $v/5$ (B) $2v/5$ (C) $2v/3$ (D) $(2/5)^{1/2}\, v$

11. **Multiple Correct:** Consider two laboratory carts of different masses but identical kinetic energie . Which of the following statements must be correct? Select two answers.
    (A) The one with the greatest mass has the greatest momentum
    (B) The same impulse was required to accelerate each cart from rest
    (C) Both can do the same amount of work as they come to a stop
    (D) The same amount of force was required to accelerate each cart from rest

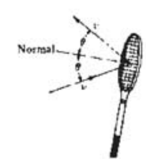
12. A mass m has speed v. It then collides with a stationary object of mass 2m. If both objects stick together in a perfectly inelastic collision, what is the final speed of the newly formed object?
    (A) v / 3 (B) v / 2 (C) 2v / 3 (D) 3v / 2

13. A 50 kg skater at rest on a frictionless rink throws a 2 kg ball, giving the ball a velocity of 10 m/s. Which statement describes the skater's subsequent motion?
    (A) 0.4 m/s in the same direction as the ball's motion.
    (B) 0.4 m/s in the opposite direction of the ball's motion.
    (C) 2 m/s in the same direction as the ball's motion.
    (D) 2 m/s in the opposite direction of the ball's motion.

14. A student initially at rest on a frictionless frozen pond throws a 1 kg hammer in one direction. After the throw, the hammer moves off in one direction while the student moves off in the other direction. Which of the following correctly describes the above situation?
    (A) The hammer will have the momentum with the greater magnitude
    (B) The student will have the momentum with the greater magnitude
    (C) The hammer will have the greater kinetic energy
    (D) The student will have the greater kinetic energy

15. Two toy cars with different masses originally at rest are pushed apart by a spring between them. Which TWO of the following statements would be true?
    (A) both toy cars will acquire equal but opposite momenta
    (B) both toy cars will acquire equal kinetic energies
    (C) the more massive toy car will acquire the least speed
    (D) the smaller toy car will experience an acceleration of the greatest magnitude

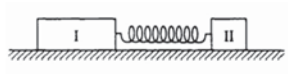
16. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially as shown. The magnitude of the momentum change of the ball is



    (A) 0 (B) 2mv (C) 2mv sinθ (D) 2mv cosθ

17. Two bodies of masses 5 and 7 kilograms are initially at rest on a horizontal frictionless surface. A light spring is compressed between the bodies, which are held together by a thin thread. After the spring is released by burning through the thread, the 5 kilogram body has a speed of 0.2 m/s. The speed of the 7 kilogram body is (in m/s)
    (A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{1}{5}$ (D) $\frac{1}{\sqrt{35}}$

18. **Multiple Correct:** A satellite of mass M moves in a circular orbit of radius R at a constant speed v around the Earth which has mass M_E. Which of the following statements must be true? Select two answers:
    (A) The net force on the satellite is equal to $Mv^2/2$ and is directed toward the center of the orbit.
    (B) The net work done on the satellite by gravity in one revolution is zero.
    (C) The angular momentum of the satellite is a constant.
    (D) The net force on the satellite is equal to GMM_E/R



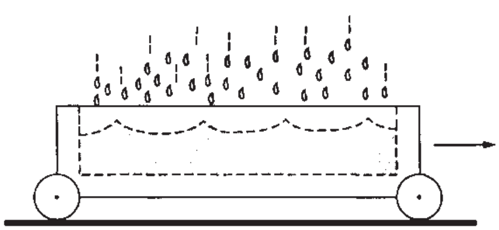
19. Two pucks are firmly attached by a stretched spring and are initially held at rest on a frictionless surface, as shown above. The pucks are then released simultaneously. If puck I has three times the mass of puck II, which of the following quantities is the same for both pucks as the spring pulls the two pucks toward each other?
(A) Speed (B) Magnitude of acceleration (C) Kinetic energy (D) Magnitude of momentum

20. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?
(A) The collision is elastic.
(B) The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.
(C) The speed of the objects that are stuck together will be less than the initial speed of the less massive object.
(D) The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

21. Two objects having the same mass travel toward each other on a flat surface each with a speed of 1.0 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 2.0 meters per second relative to the surface. Which of the following assessments of this report is most accurate?
(A) Momentum was not conserved therefore the report is false.
(B) If potential energy was released to the objects during the collision the report could be true.
(C) If the objects had different masses the report could be true.
(D) If the surface was inclined the report could be true.

22. A solid metal ball and a hollow plastic ball of the same external radius are released from rest in a large vacuum chamber. When each has fallen 1 meter, they both have the same
(A) inertia (B) speed (C) momentum (D) change in potential energy

23. A railroad car of mass *m* is moving at speed *v* when it collides with a second railroad car of mass *M* which is at rest. The two cars lock together instantaneously and move along the track. What is the kinetic energy of the cars immediately after the collision?
(A) $\frac{1}{2} mv^2$ (B) $\frac{1}{2} (M+m)(mv/M)^2$ (C) $\frac{1}{2} (M+m)(Mv/m)^2$ (D) $\frac{1}{2} (M+m)(mv/(m+M))^2$

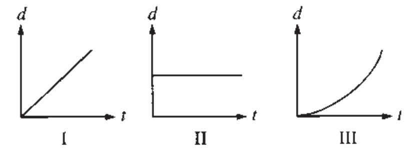


24. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of mechanical energy
(B) decrease because of conservation of momentum
(C) decrease because of conservation of mechanical energy
(D) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

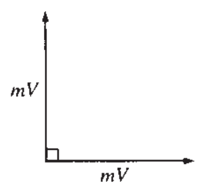
Questions 25-26

Three objects can only move along a straight, level path. The graphs below show the position *d* of each of the objects plotted as a function of time *t*.

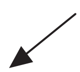
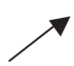


25. The magnitude of the momentum of the object is increasing in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only

26. The sum of the forces on the object is zero in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only



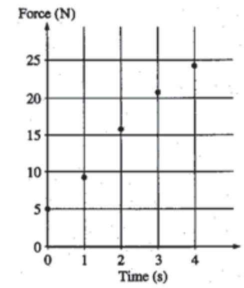
27. A stationary object explodes, breaking into three pieces of masses *m*, *m*, and *3m*. The two pieces of mass *m* move off at right angles to each other with the same magnitude of momentum mV, as shown in the diagram above. What are the magnitude and direction of the velocity of the piece having mass *3m* ?

Magnitude	Direction
(A) $\frac{V}{\sqrt{2}}$	
(B) $\frac{V}{\sqrt{2}}$	
(C) $\frac{\sqrt{2}V}{3}$	
(D) $\frac{\sqrt{2}V}{3}$	

28. An empty sled of mass M moves without friction across a frozen pond at speed v_o. Two objects are dropped vertically into the sled one at a time: first an object of mass m and then an object of mass 2m. Afterward the sled moves with speed v_f. What would be the final speed of the sled if the objects were dropped into it in reverse order?
(A) $v_f / 3$
(B) $v_f / 2$
(C) v_f
(D) $2v_f$

29. A student obtains data on the magnitude of force applied to an object as a function of time and displays the data on the graph shown. The increase in the momentum of the object between t=0 s and t=4 s is most nearly

(A) 40 N·s
(B) 50 N·s
(C) 60 N·s
(D) 80 N·s

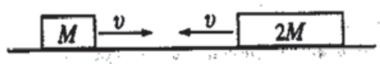


30. How does an air mattress protect a stunt person landing on the ground after a stunt?

(A) It reduces the kinetic energy loss of the stunt person.
(B) It reduces the momentum change of the stunt person.
(C) It shortens the stopping time of the stunt person and increases the force applied during the landing.
(D) It lengthens the stopping time of the stunt person and reduces the force applied during the landing.

31. Two objects, A and B, initially at rest, are "exploded" apart by the release of a coiled spring that was compressed between them. As they move apart, the velocity of object A is 5 m/s and the velocity of object B is – 2 m/s. The ratio of the mass of object A to the mass object B, m_a/m_b is

(A) 4/25 (B) 2/5 (C) 5/2 (D) 25/4

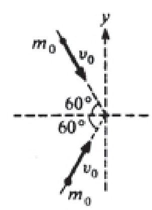


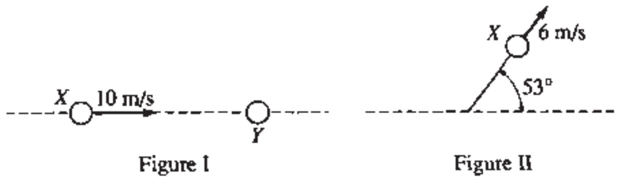
32. The two blocks of masses M and 2M shown above initially travel at the same speed v but in opposite directions. They collide and stick together. How much mechanical energy is lost to other forms of energy during the collision?

(A) $1/2\ M\ v^2$
(B) $3/4\ M\ v^2$
(C) $4/3\ M\ v^2$
(D) $3/2\ M\ v^2$

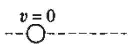
33. Two particles of equal mass m_o, moving with equal speeds v_o along paths inclined at 60° to the x-axis as shown, collide and stick together. Their velocity after the collision has magnitude

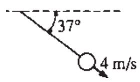
(A) $\frac{v_0}{4}$ (B) $\frac{v_0}{2}$ (C) $\frac{\sqrt{3}v_0}{2}$ (D) v_o

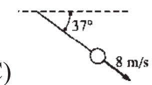


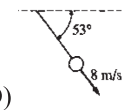


34. Two balls are on a frictionless horizontal tabletop. Ball X initially moves at 10 meters per second, as shown in Figure I above. It then collides elastically with identical ball Y which is initially at rest. After the collision, ball X moves at 6 meters per second along a path at 53° to its original direction, as shown in Figure II above. Which of the following diagrams best represents the motion of ball Y after the collision?

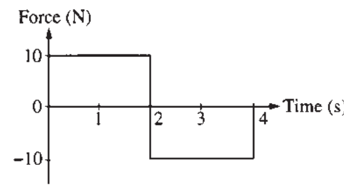
(A) 

(B) 

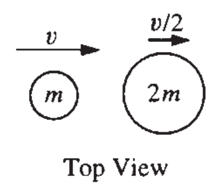
(C) 

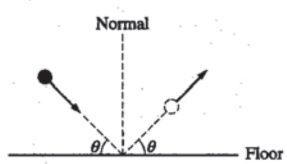
(D) 

35. The graph shows the force on an object of mass M as a function of time. For the time interval 0 to 4 s, the total change in the momentum of the object is
(A) 40 kg m/s (B) 20 kg m/s
(C) 0 kg m/s (D) – 20 kg m/s

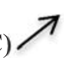


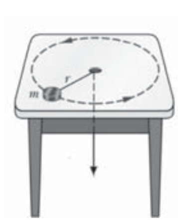
36. As shown in the top view, a disc of mass m is moving horizontally to the right with speed v on a table with negligible friction when it collides with a second disc of mass $2m$. The second disc is moving horizontally to the right with speed $v/2$ at the moment before impact. The two discs stick together upon impact. The kinetic energy of the composite body immediately after the collision is
(A) $(1/6)mv^2$ (B) $(1/2)mv^2$ (C) $2/3mv^2$ (D) $(9/8)mv^2$





37. A 2 kg ball collides with the floor at an angle θ and rebounds at the same angle and speed as shown above. Which of the following vectors represents the impulse exerted on the ball by the floor?

(A)  (B)  (C)  (D) 

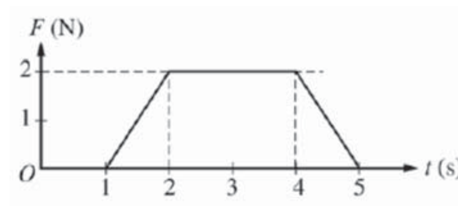


38. An object m, on the end of a string, moves in a circle (initially of radius r) on a horizontal frictionless table as shown. As the string is pulled very slowly through a small hole in the table, which of the following is correct for an observer measuring from the hole in the table?
(A) The angular momentum of m remains constant.
(B) The angular momentum of m decreases.
(C) The kinetic energy of m remains constant
(D) The kinetic energy of m decreases

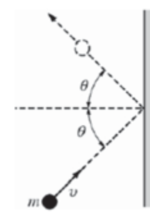
39. A boy of mass *m* and a girl of mass 2*m* are initially at rest at the center of a frozen pond. They push each other so that she slides to the left at speed *v* across the frictionless ice surface and he slides to the right. What is the total work done by the children?
(A) mv (B) mv^2 (C) $2mv^2$ (D) $3mv^2$

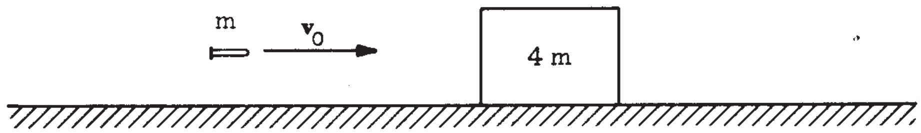
40. An object of mass *M* travels along a horizontal air track at a constant speed *v* and collides elastically with an object of identical mass that is initially at rest on the track. Which of the following statements is true for the two objects after the impact?
(A) The total momentum is *Mv* and the total kinetic energy is ½ Mv^2
(B) The total momentum is *Mv* and the total kinetic energy is less than ½ Mv^2
(C) The total momentum is less than *Mv* and the total kinetic energy is ½ Mv^2
(D) The momentum of each object is ½ *Mv*

41. A 2 kg object initially moving with a constant velocity is subjected to a force of magnitude F in the direction of motion. A graph of F as a function of time t is shown. What is the increase, if any, in the velocity of the object during the time the force is applied?
(A) 0 m/s (B) 3.0 m/s (C) 4.0 m/s (D) 6.0 m/s



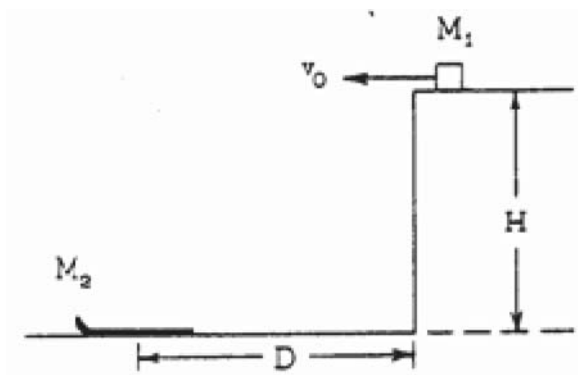
42. A ball of mass m with speed v strikes a wall at an angle θ with the normal, as shown. It then rebounds with the same speed and at the same angle. The impulse delivered by the ball to the wall is
(A) mv sin θ (B) mv cos θ (C) 2mv sin θ (D) 2mv cos θ





1976B2.
A bullet of mass m and velocity v_o is fired toward a block of mass 4m. The block is initially at rest on a frictionless horizontal surface. The bullet penetrates the block and emerges with a velocity of $\frac{v_o}{3}$

(a) Determine the final speed of the block.
(b) Determine the loss in kinetic energy of the bullet.
(c) Determine the gain in the kinetic energy of the block.



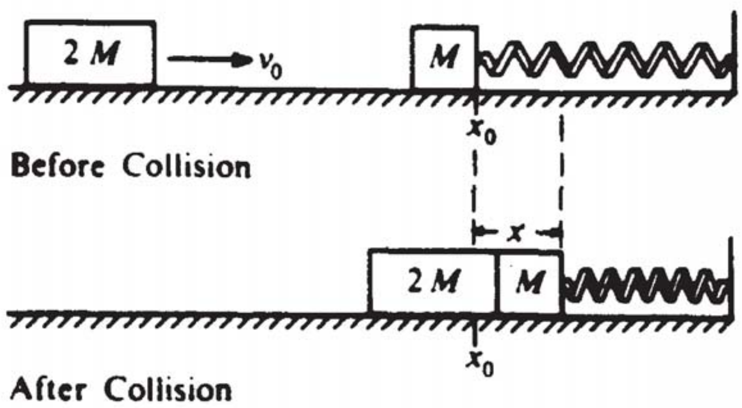
1978B2. A block of mass M_1 travels horizontally with a constant speed v_o on a plateau of height H until it comes to a cliff. A toboggan of mass M_2 is positioned on level ground below the cliff as shown above. The center of the toboggan is a distance D from the base of the cliff.

(a) Determine D in terms of v_o, H, and g so that the block lands in the center of the toboggan.
(b) The block sticks to the toboggan which is free to slide without friction. Determine the resulting velocity of the block and toboggan.

1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table.

In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second.
a. Determine the minimum work needed to compress the spring in this experiment.

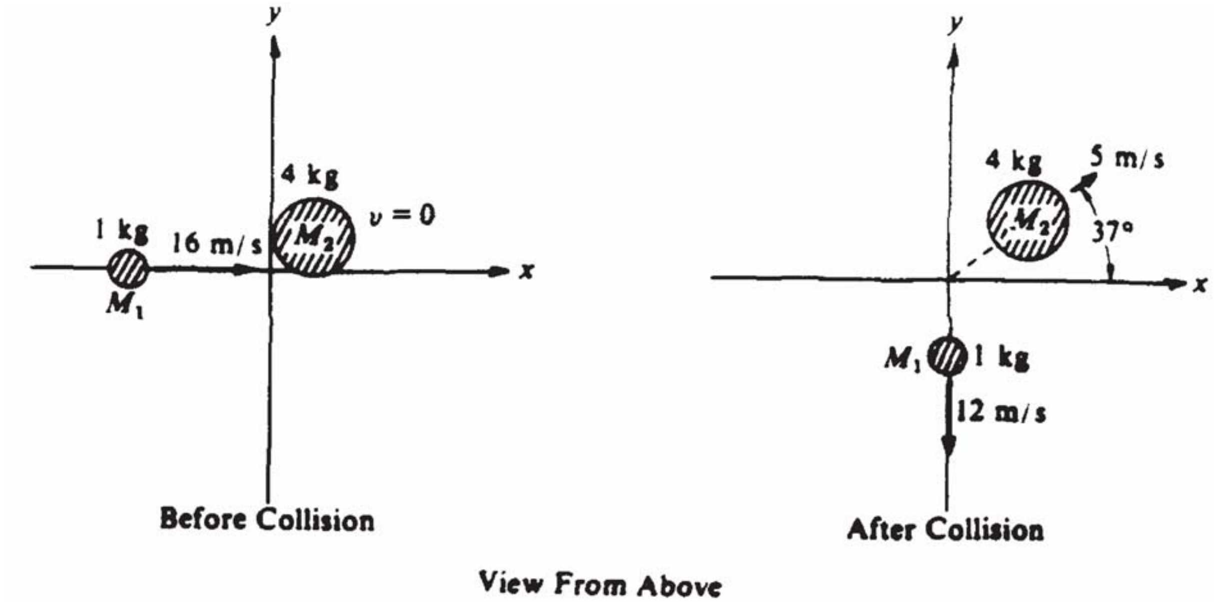
In a different experiment, the spring is compressed again exactly as above, but this time both masses are released simultaneously and each mass moves off separately at unknown speeds.
b. Determine the final velocity of each mass relative to the cable after the masses are released.



1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact

b. x, the maximum distance the spring is compressed



1984B2. Two objects of masses M_1 = 1 kilogram and M_2 = 4 kilograms are free to slide on a horizontal frictionless surface. The objects collide and the magnitudes and directions of the velocities of the two objects before and after the collision are shown on the diagram above. (sin 37° = 0.6, cos 37° = 0.8, tan 37º = 0.75)

a. Calculate the x and y components (p_x and p_y, respectively) of the momenta of the two objects before and after the collision, and write your results in the proper places in the following table.

	M_1 = 1 kg		M_2 = 4 kg	
	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$
Before Collision				
After Collision				

b. Show, using the data that you listed in the table, that linear momentum is conserved in this collision.

c. Calculate the kinetic energy of the two-object system before and after the collision.

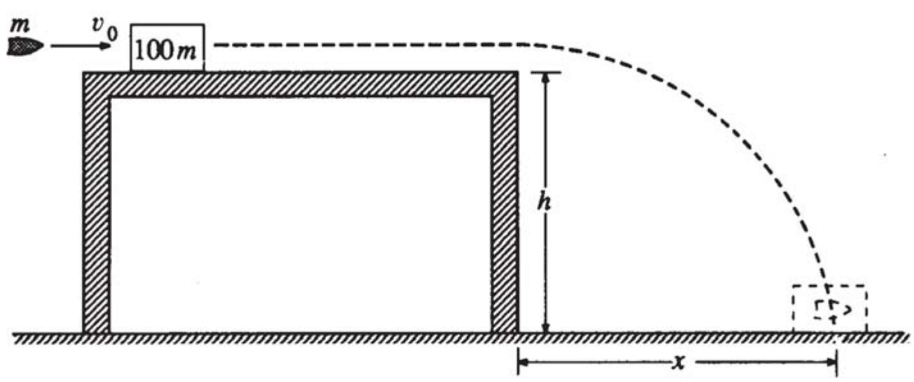
d. Is kinetic energy conserved in the collision?

1985B1. A 2-kilogram block initially hangs at rest at the end of two 1-meter strings of negligible mass as shown on the left diagram above. A 0.003-kilogram bullet, moving horizontally with a speed of 1000 meters per second, strikes the block and becomes embedded in it. After the collision, the bullet/ block combination swings upward, but does not rotate.

a. Calculate the speed v of the bullet/ block combination just after the collision.

b. Calculate the ratio of the initial kinetic energy of the bullet to the kinetic energy of the bullet/ block combination immediately after the collision.

c. Calculate the maximum vertical height above the initial rest position reached by the bullet/block combination.



1990B1. A bullet of mass m is moving horizontally with speed v_o when it hits a block of mass 100m that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact, the bullet and the block slide off the table and hit the floor a distance x from the edge of the table. Derive expressions for the following quantities in terms of m, h, v_o, and appropriate constants:

a. the speed of the block as it leaves the table

b. the change in kinetic energy of the bullet-block system during impact

c. the distance x

Suppose that the bullet passes through the block instead of remaining in it.

d. State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.

e. State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

1992B2. A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

a. Determine the speed of the child-sled system after the child jumps onto the sled.

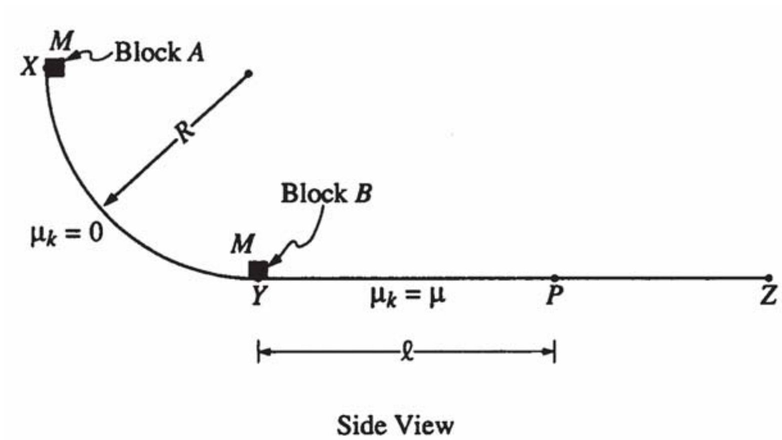
b. Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.

c. Determine the speed of the sled after the child jumps off it.

d. Determine the kinetic energy of the child-sled system when the child is at rest on the ice.

e. Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?



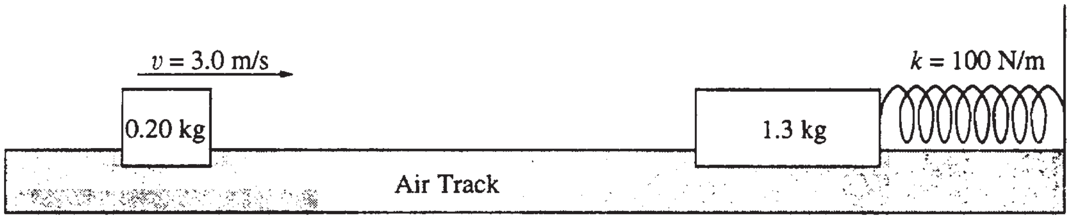
1994B2. A track consists of a frictionless arc XY, which is a quarter-circle of radius R, and a rough horizontal section YZ. Block A of mass M is released from rest at point X, slides down the curved section of the track, and collides instantaneously and inelastically with identical block B at point Y. The two blocks move together to the right, sliding past point P, which is a distance *L* from point Y. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ Express your answers in terms of M, *L*, μ, R, and g.

a. Determine the speed of block A just before it hits block B.

b. Determine the speed of the combined blocks immediately after the collision.

c. Assuming that no energy is transferred to the track or to the air surrounding the blocks. Determine the amount of internal energy transferred in the collision

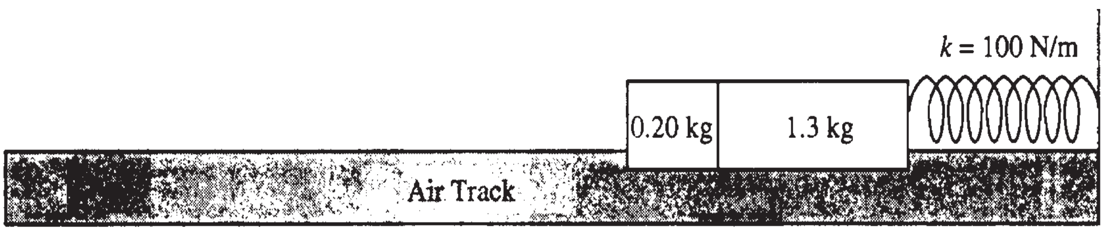
d. Determine the additional thermal energy that is generated as the blocks move from Y to P



1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

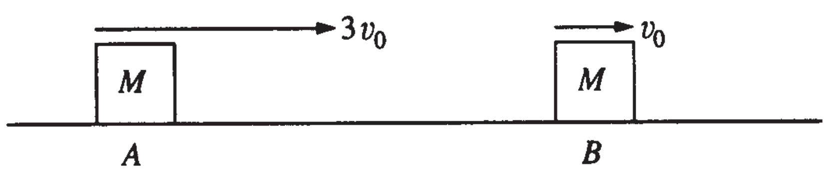
a. Determine the following for the <u>0.20-kilogram mass</u> immediately <u>before</u> the impact.
   i. Its linear momentum ii. Its kinetic energy

b. Determine the following for the <u>combined masses</u> immediately <u>after</u> the impact.
   i. The linear momentum ii. The kinetic energy



After the collision, the two masses compress the spring as shown.

c. Determine the maximum compression distance of the spring.



1996B1. Two identical objects A and B of mass M move on a one-dimensional, horizontal air track. Object B initially moves to the right with speed v_o. Object A initially moves to the right with speed $3v_o$, so that it collides with object B. Friction is negligible. Express your answers to the following in terms of M and v_o.

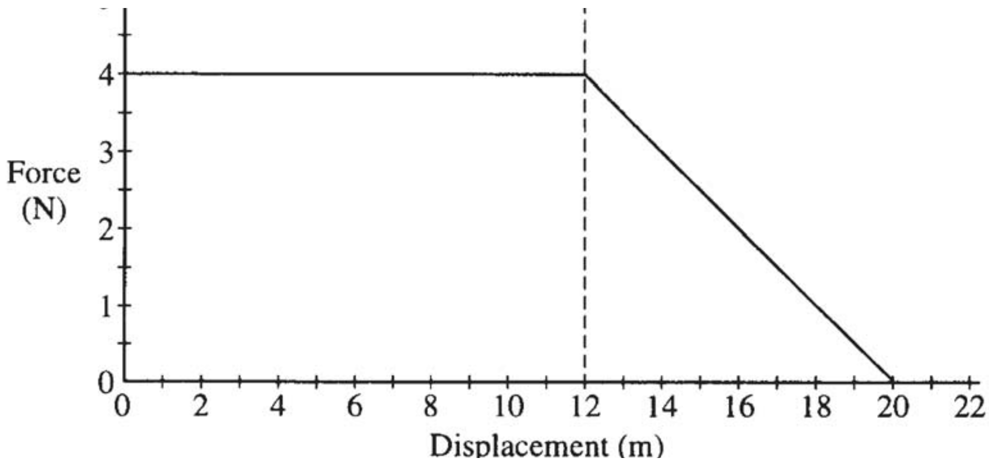
a. Determine the total momentum of the system of the two objects.

b. A student predicts that the collision will be totally inelastic (the objects stick together on collision). Assuming this is true, determine the following for the two objects immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)

When the experiment is performed, the student is surprised to observe that the objects separate after the collision and that object B subsequently moves to the right with a speed $2.5v_o$.

c. Determine the following for object A immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)

d. Determine the kinetic energy dissipated in the actual experiment.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

a. The acceleration of the particle when its displacement x is 6 m.

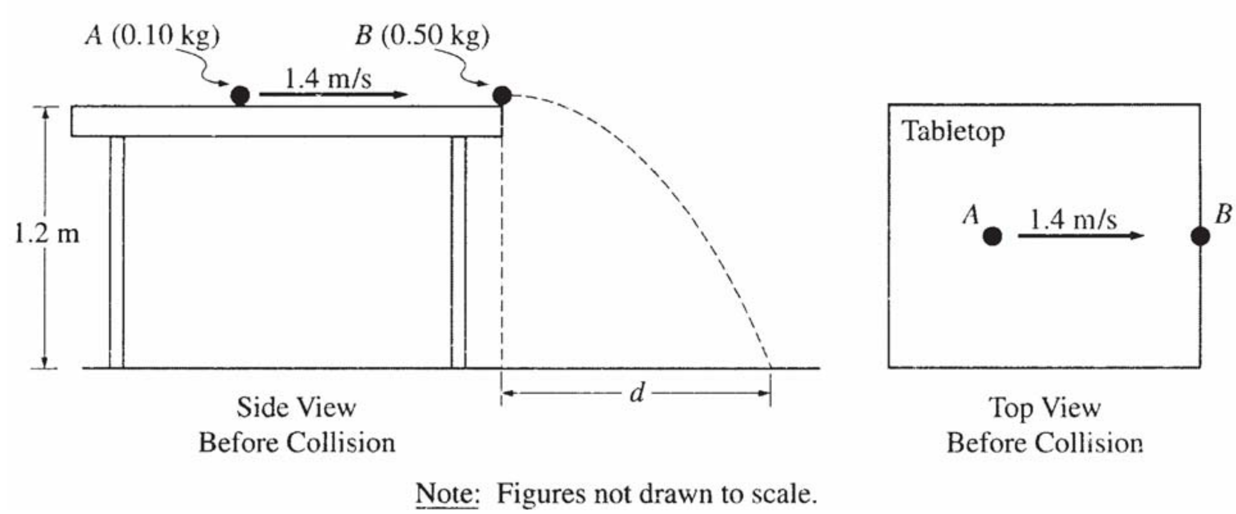
b. The time taken for the object to be displaced the first 12 m.

c. The amount of work done by the net force in displacing the object the first 12 m.

d. The speed of the object at displacement x = 12 m.

e. The final speed of the object at displacement x = 20 m.

f. The change in the momentum of the object as it is displaced from x = 12 m to x = 20 m



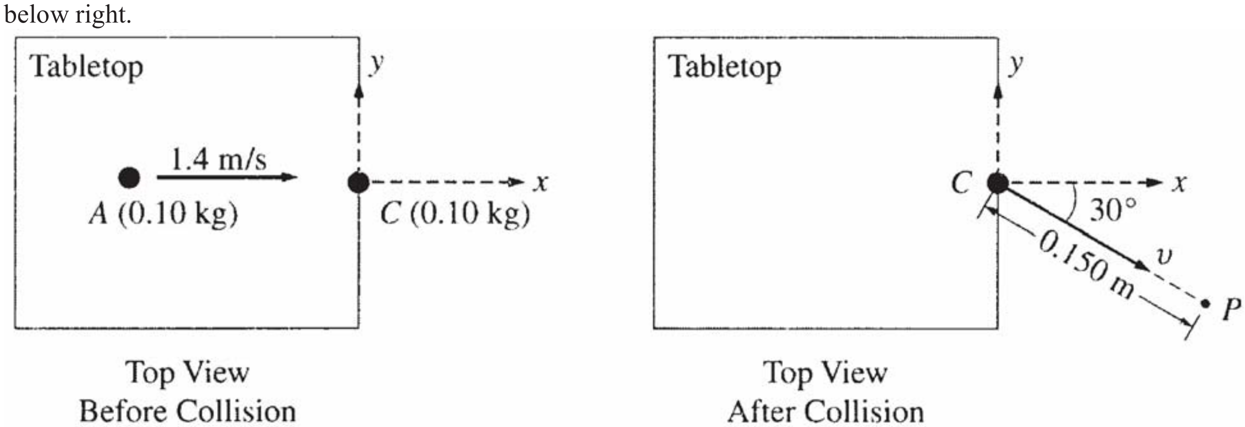
2001B2. An incident ball A of mass 0.10 kg is sliding at 1.4 m/s on the horizontal tabletop of negligible friction as shown above. It makes a head-on collision with a target ball B of mass 0.50 kg at rest at the edge of the table. As a result of the collision, the incident ball rebounds, sliding backwards at 0.70 m/s immediately after the collision.

a. Calculate the speed of the 0.50 kg target ball immediately after the collision.

The tabletop is 1.20 m above a level, horizontal floor. The target ball is projected horizontally and initially strikes the floor at a horizontal displacement *d* from the point of collision.

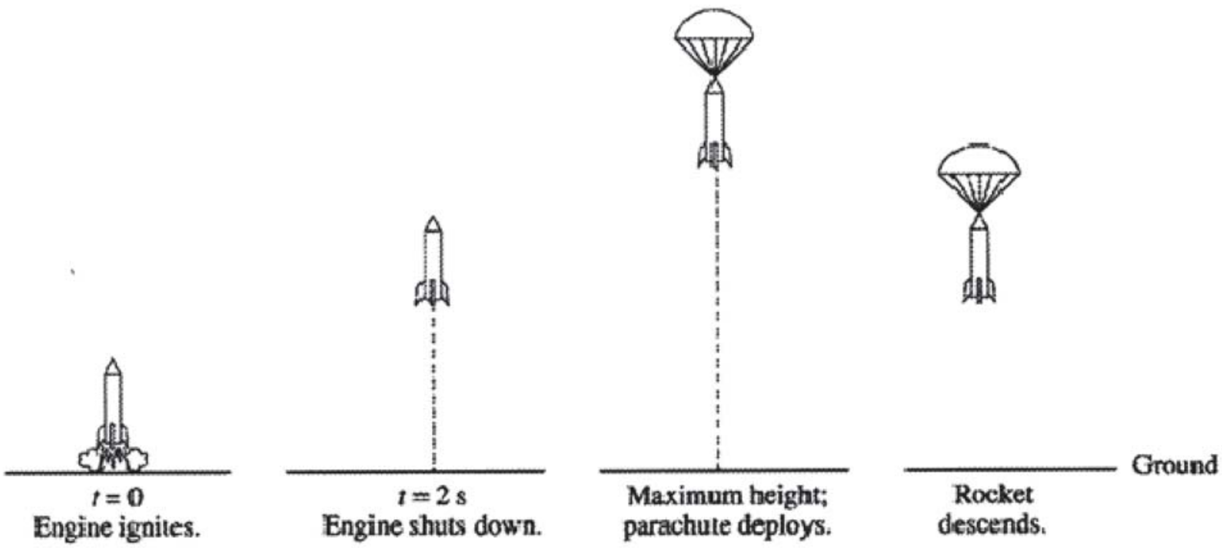
b. Calculate the horizontal displacement

In another experiment on the same table, the target ball B is replaced by target ball C of mass 0.10 kg. The incident ball A again slides at 1.4 m/s, as shown below left, but this time makes a glancing collision with the target ball C that is at rest at the edge of the table. The target ball C strikes the floor at point P, which is at a horizontal displacement of 0.15 m from the point of the collision, and at a horizontal angle of 30° from the +x-axis, as shown below right.



c. Calculate the speed v of the target ball C immediately after the collision.

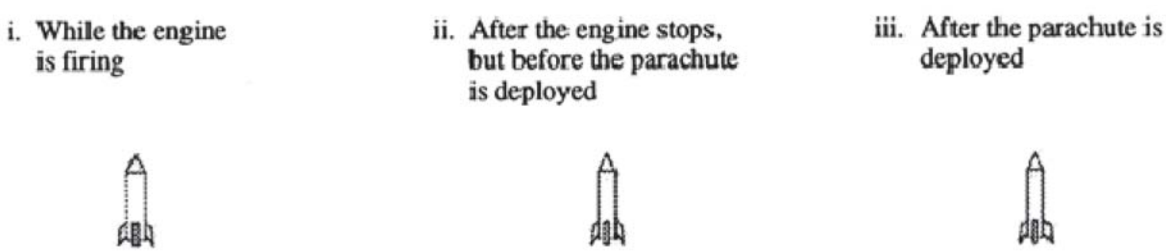
d. Calculate the y-component of incident ball A's momentum immediately after the collision.



Note: Figures not drawn to scale.

2002B1. A model rocket of mass 0.250 kg is launched vertically with an engine that is ignited at time t = 0, as shown above. The engine provides an impulse of 20.0 N•s by firing for 2.0 s. Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

(a) On the figures below, draw and label a free-body diagram for the rocket during each of the following intervals.

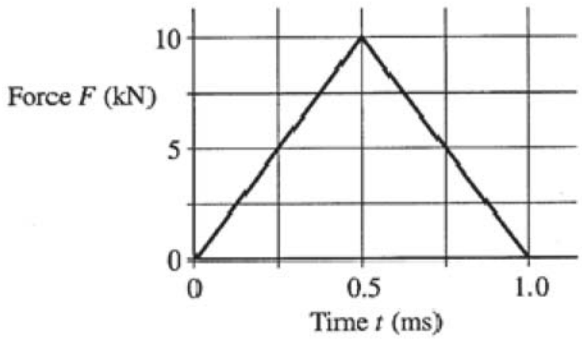


(b) Determine the magnitude of the average acceleration of the rocket during the 2 s firing of the engine.

(c) What maximum height will the rocket reach?

(d) At what time after t = 0 will the maximum height be reached?

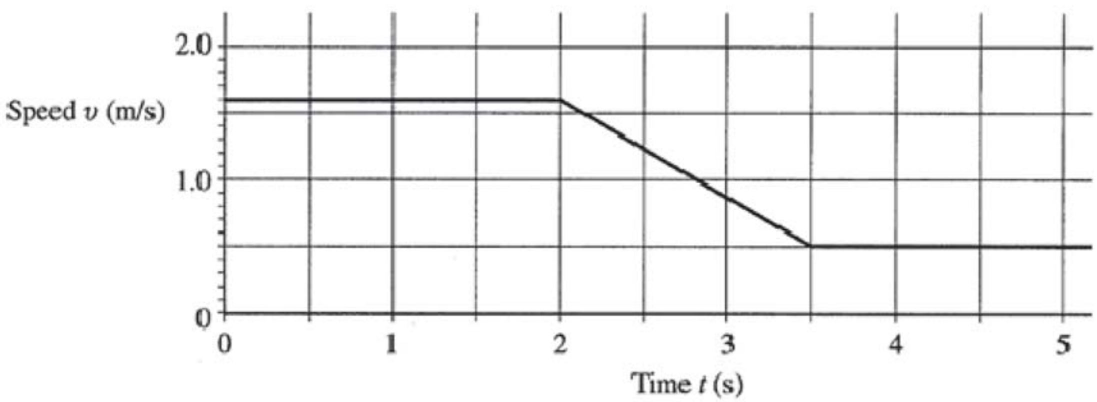
2002B1B. A 2.0 kg frictionless cart is moving at a constant speed of 3.0 m/s to the right on a horizontal surface, as shown above, when it collides with a second cart of undetermined mass m that is initially at rest. The force F of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 m/s to the right. Assume that friction is negligible before, during, and after the collision.



(a) Calculate the magnitude and direction of the velocity of the 2.0 kg cart after the collision.

(b) Calculate the mass m of the second cart.

After the collision, the second cart eventually experiences a ramp, which it traverses with no frictional losses. The graph below shows the speed v of the second cart as a function of time t for the next 5.0 s, where $t = 0$ is now the instant at which the carts separate.

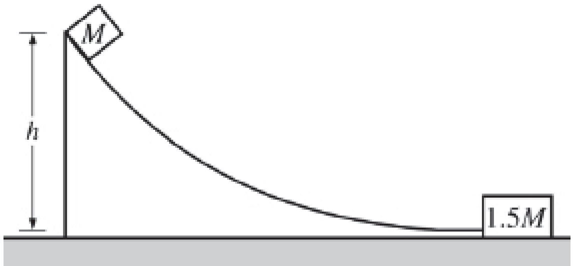


(c) Calculate the acceleration of the cart at $t = 3.0$ s.

(d) Calculate the distance traveled by the second cart during the 5.0 s interval after the collision (0 s < t < 5.0 s).

(e) State whether the ramp goes up or down **and** calculate the maximum elevation (above or below the initial height) reached by the second cart on the ramp during the 5.0 s interval after the collision (0 s < t < 5.0 s).

2006B2B



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2\ v_o$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

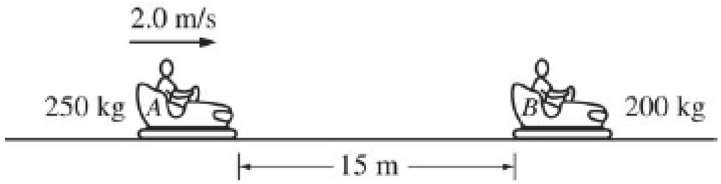
(a) Determine the height h of the ramp from which the small block was released.
(b) Determine the speed of the small block after the collision.
(c) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.
(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

2008B1B

A 70 kg woman and her 35 kg son are standing at rest on an ice rink, as shown above. They push against each other for a time of 0.60 s, causing them to glide apart. The speed of the woman immediately after they separate is 0.55 m/s. Assume that during the push, friction is negligible compared with the forces the people exert on each other.

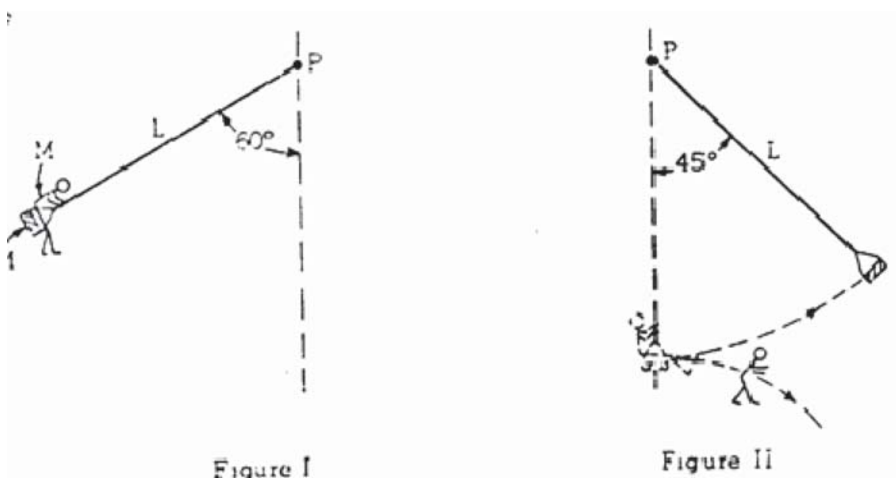
(a) Calculate the initial speed of the son after the push.
(b) Calculate the magnitude of the average force exerted on the son by the mother during the push.
(c) How do the magnitude and direction of the average force exerted on the mother by the son during the push compare with those of the average force exerted on the son by the mother? Justify your answer.
(d) After the initial push, the friction that the ice exerts cannot be considered negligible, and the mother comes to rest after moving a distance of 7.0 m across the ice. If their coefficients of friction are the same, how far does the son move after the push?

2008B1



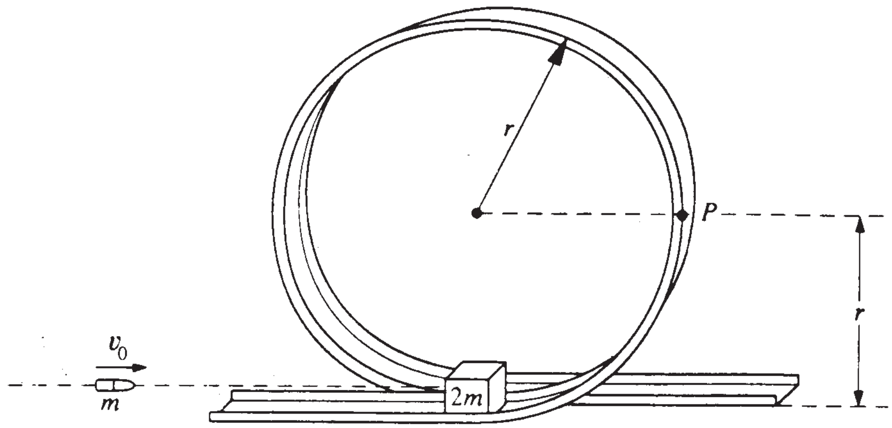
Several students are riding in bumper cars at an amusement park. The combined mass of car *A* and its occupants is 250 kg. The combined mass of car *B* and its occupants is 200 kg. Car *A* is 15 m away from car *B* and moving to the right at 2.0 m/s, as shown, when the driver decides to bump into car *B*, which is at rest.

(a) Car *A* accelerates at 1.5 m/s^2 to a speed of 5.0 m/s and then continues at constant velocity until it strikes car *B*. Calculate the total time for car *A* to travel the 15 m.
(b) After the collision, car *B* moves to the right at a speed of 4.8 m/s .
i. Calculate the speed of car *A* after the collision.
ii. Indicate the direction of motion of car *A* after the collision.
____ To the left ____ To the right ____ None; car *A* is at rest.
(c) Is this an elastic collision?
____ Yes ____ No
Justify your answer.



C1981M2. A swing seat of mass M is connected to a fixed point P by a massless cord of length L. A child also of mass M sits on the seat and begins to swing with zero velocity at a position at which the cord makes a 60° angle with the vertical is shown in Figure I. The swing continues down until the cord is exactly vertical at which time the child jumps off in a horizontal direction. The swing continues in the same direction until its cord makes a 45° angle with the vertical as shown in Figure II: at that point it begins to swing in the reverse direction.

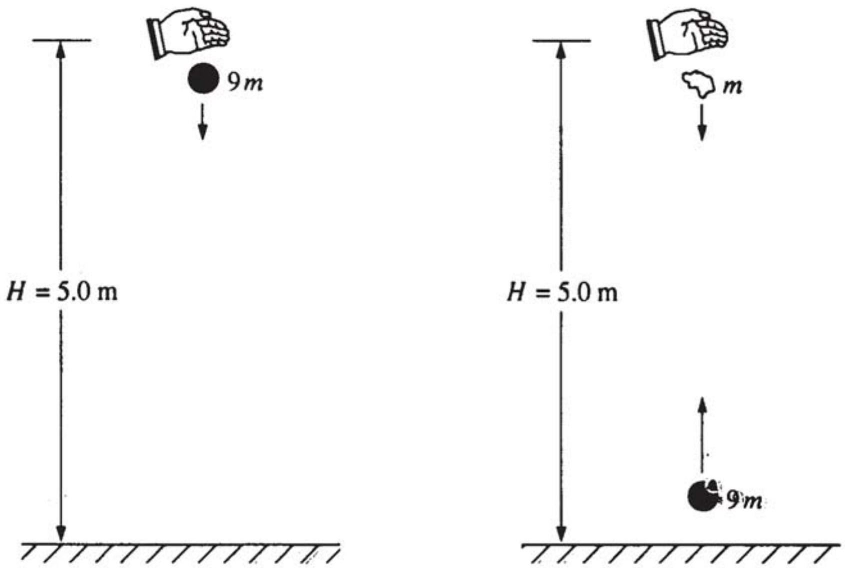
a) Determine the speed of the child and seat just at the lowest position prior to the child's dismount from the seat
b) Determine the speed if the seat immediately after the child dismounts
c) Determine the speed of the child immediately after he dismounts from the swing?



C1991M1. A small block of mass 2m initially rests on a track at the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_o and remains embedded in the block as the block and bullet circle the loop.

Determine each of the following in terms of m, v_o, r, and g.

a. The speed of the block and bullet immediately after impact
b. The kinetic energy of the block and bullet when they reach point P on the loop
c. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times
d The new required entry speed v_o' of the block and bullet at the bottom of the loop such that the conditions in part c apply.
e. The new initial speed of the bullet to produce the speed from part d above.



C1992M1. A ball of mass 9m is dropped from rest from a height H = 5.0 meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, collides with the ball 0.5 seconds later, which is ascending. Assume that g = 10 m/s^2, that air resistance is negligible, and that the collision process takes negligible time.

a. Determine the speed of the ball immediately before it hits the ground.
b. Determine the rebound speed of the ball immediately after it collides with the ground, justify your answer.
c. Determine the height above the ground at which the clay-ball collision takes place.
d. Determine the speeds of the ball and the clay blob immediately before the collision.
e. If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

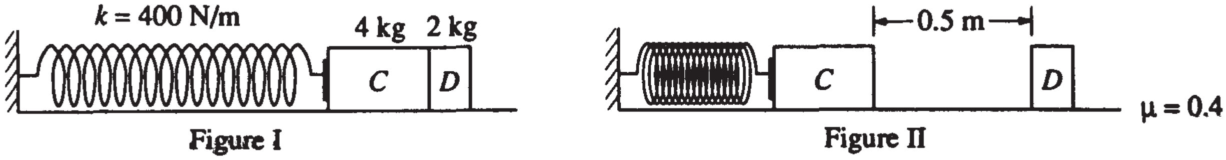


Figure I

Figure II

C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

c. The speed v_f of blocks C and *D* just after they collide

d. The horizontal distance the blocks move before coming to rest

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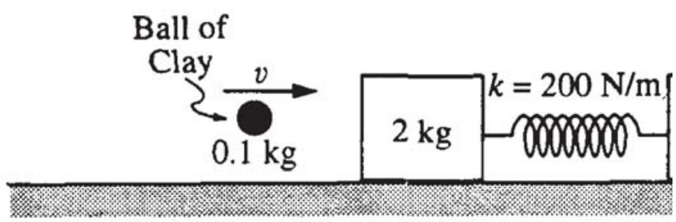


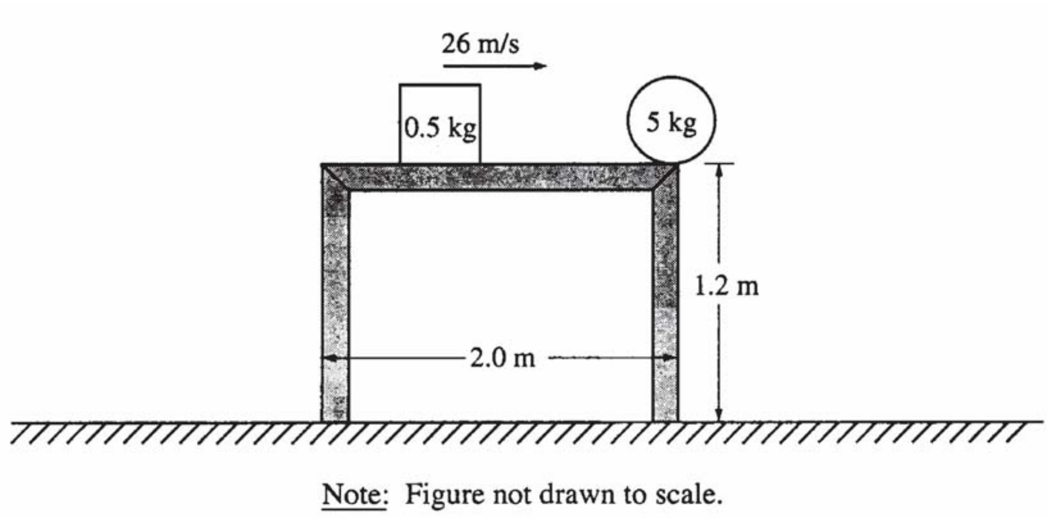
C1994M1. A 2-kilogram block is attached to an ideal spring (for which k = 200 N/m) and initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The spring is attached to a wall as shown. As a result, the spring compresses a maximum distance of 0.4 meters.

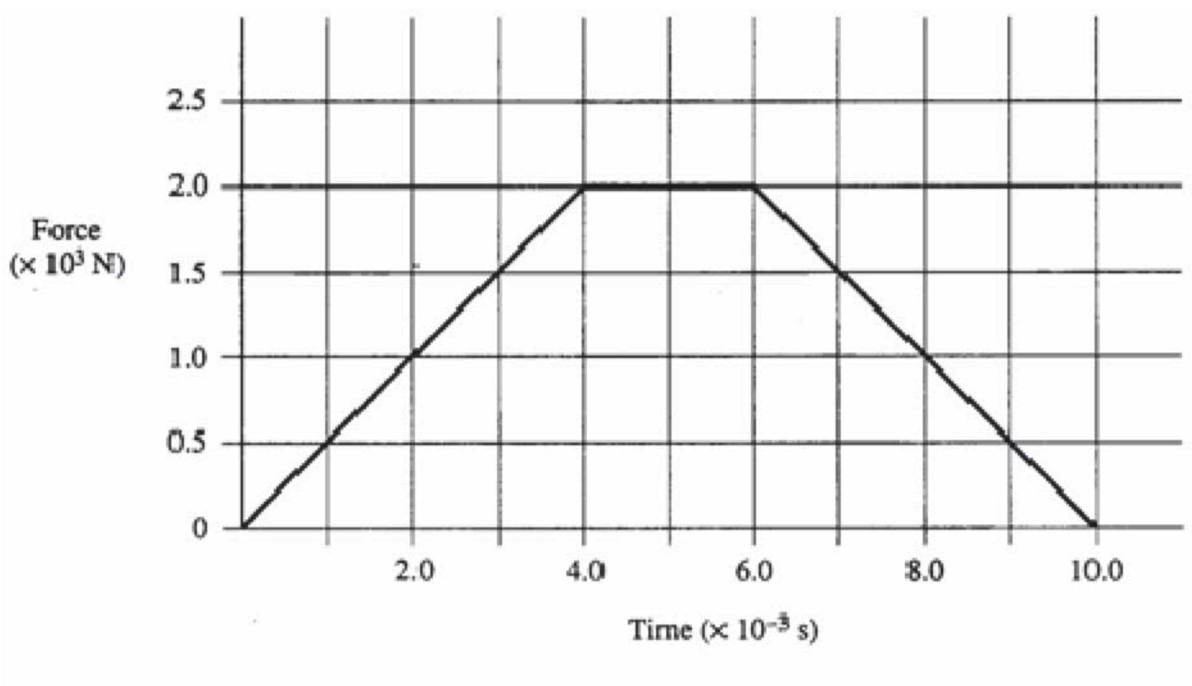
a. Calculate the energy stored in the spring at maximum compression.

b. Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

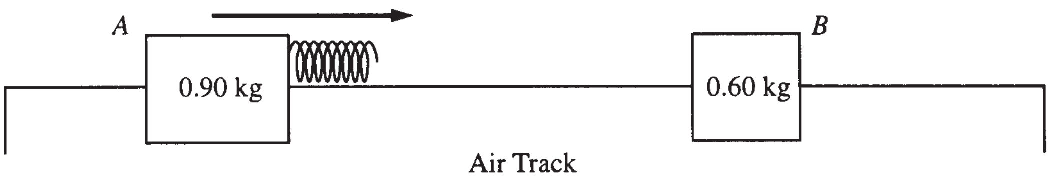
c. Calculate the initial speed v of the clay.



C1995M1. A 5-kilogram ball initially rests at the edge of a 2-meter-long, 1.2-meter-high frictionless table, as shown above. A hard plastic cube of mass 0.5 kilogram slides across the table at a speed of 26 meters per second and strikes the ball, causing the ball to leave the table in the direction in which the cube was moving. The figure below shows a graph of the force exerted **on the ball** by the cube as a function of time.

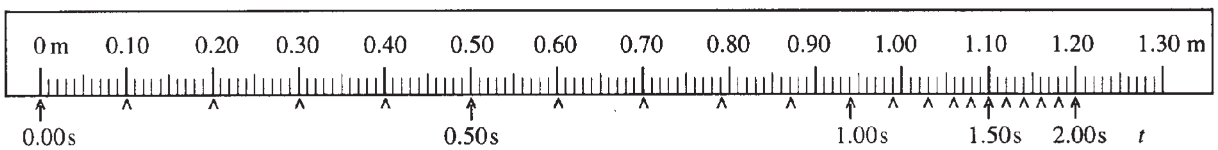


a. Determine the total impulse given to the ball.
b. Determine the horizontal velocity of the ball immediately after the collision.
c. Determine the following for the cube immediately after the collision.
   i. Its speed
   ii. Its direction of travel (right or left), if moving
d. Determine the kinetic energy dissipated in the collision.
e. Determine the distance between the two points of impact of the objects with the floor.



C1998M1. Two gliders move freely on an air track with negligible friction, as shown above. Glider A has a mass of 0.90 kg and glider B has a mass of 0.60 kg. Initially, glider A moves toward glider B, which is at rest. A spring of negligible mass is attached to the right side of glider A. Strobe photography is used to record successive positions of glider A at 0.10 s intervals over a total time of 2.00 s, during which time it collides with glider B.

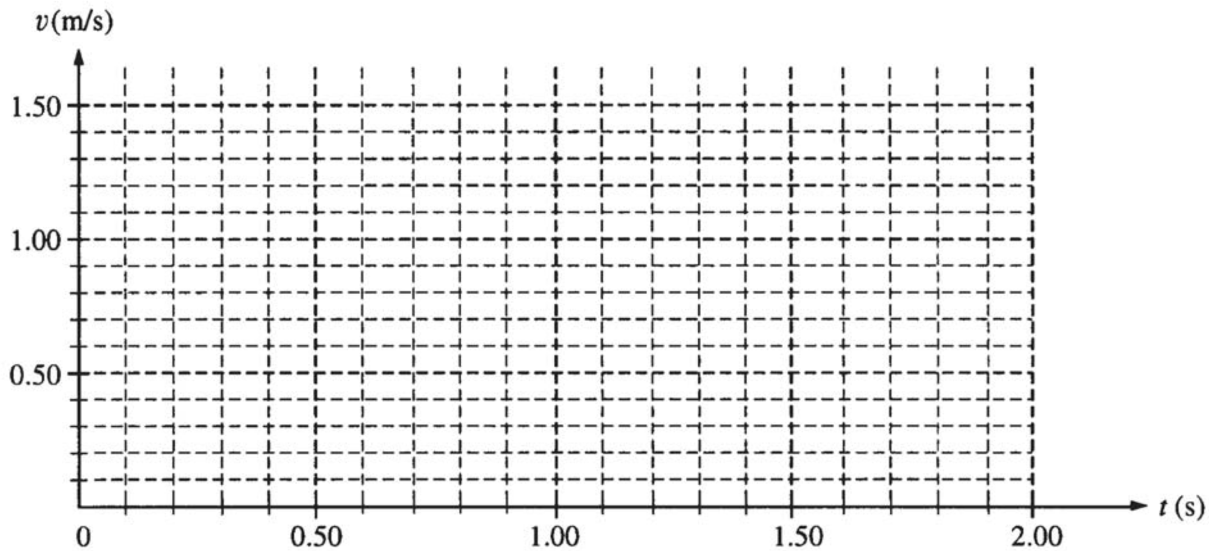
The following diagram represents the data for the motion of glider A. Positions of glider A at the end of each 0.10s interval are indicated by the symbol **Λ** against a metric ruler. The total elapsed time t after each 0.50 s is also indicated.



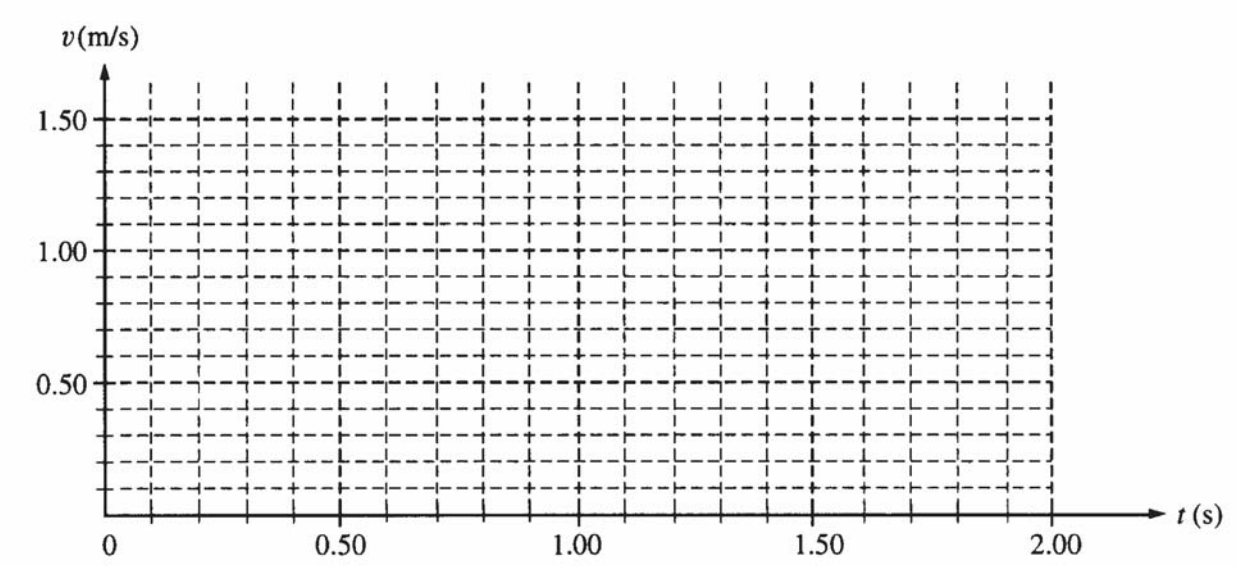
a. Determine the average speed of glider A for the following time intervals.

i. 0.0 s to 0.30 s ii. 0.90 s to 1.10 s iii. 1.70 s to 1.90 s

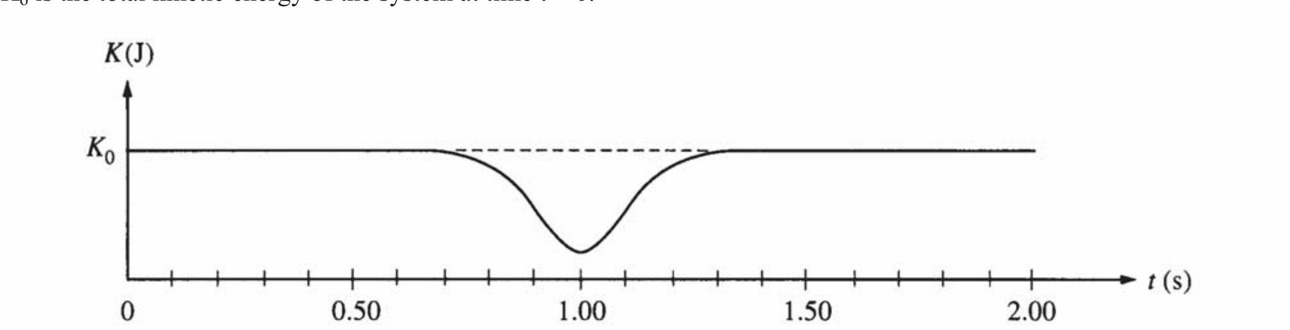
b. On the axes below, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.



c. i. Use the data to calculate the speed of glider B immediately after it separates from the spring.
ii. On the axes below, sketch a graph of the speed of glider B as a function of time t.



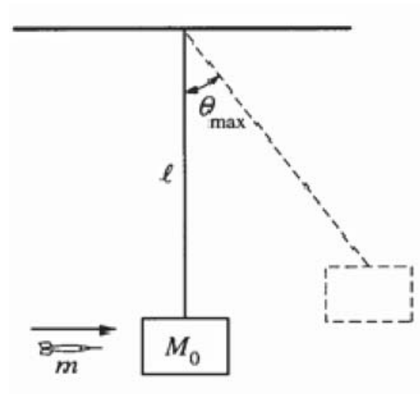
A graph of the total kinetic energy K for the two-glider system over the 2.00 s interval has the following shape. K_o is the total kinetic energy of the system at time t = 0.



d. i. Is the collision elastic? Justify your answer.

ii. Briefly explain why there is a minimum in the kinetic energy curve at t = 1.00 s.

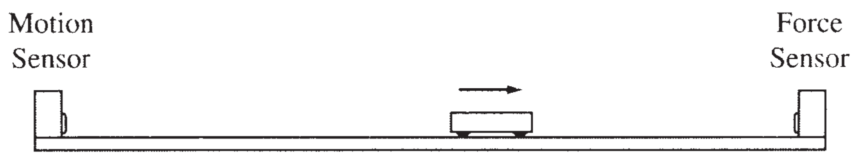
C1999M1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_o which hangs from a cord of length l and negligible mass, as shown. Assume the size of the block is negligible compared to l, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of
m, M_0, l, θ_{max}, and g.

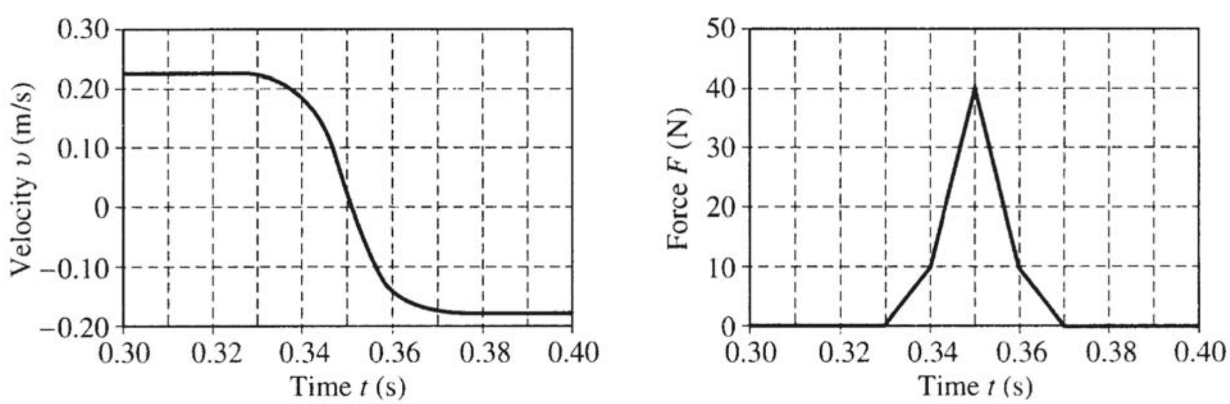


a. Determine the speed v_o of the dart immediately before it strikes the block.
b. The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.
c. At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses	Protractor
5 m of string	Five more blocks of mass M_0	Spring	

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.





2001M1. A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the following graphs.
a. Determine the cart's average acceleration between t = 0.33 s and t = 0.37 s.
b. Determine the magnitude of the change in the cart's momentum during the collision.
c. Determine the mass of the cart.
d. Determine the energy lost in the collision between the force sensor and the cart

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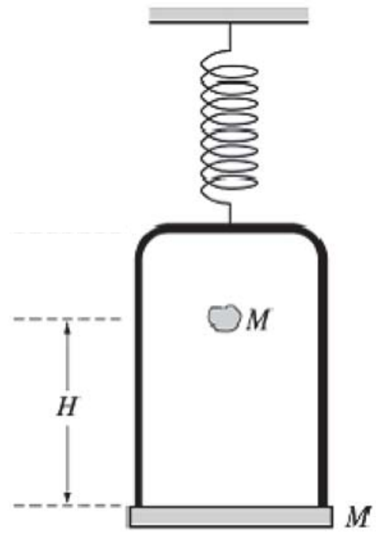
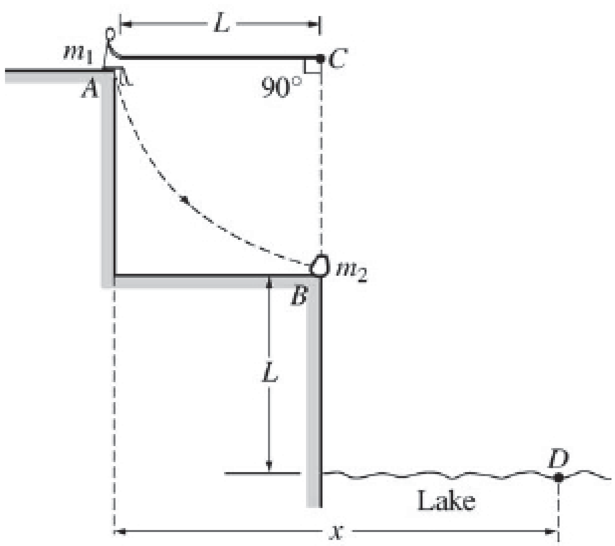
C2003M2.

An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.



(a) Determine the speed of the clay at the instant it hits the pan.
(b) Determine the speed of the pan just after the clay strikes it.
(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

C2004M1.



A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) The speed of the person and object just after the collision
(d) The total horizontal displacement x of the person from position A until the person and object land in the water at point D.

ANSWERS - AP Physics Multiple Choice Practice – Momentum and Impulse

	Solution	Answer
1.	Based on $Ft = m\Delta v$, doubling the mass would require twice the time for same momentum change	D
2.	Two step problem. I) find velocity after collision with arrow. $m_a v_{ai} = (m_a+m_b)\, v_f$ $v_f = mv / (m+M)$ II) now use energy conservation. $K_i = U_{sp(f)}$ $\frac{1}{2}(m+M)v_f^2 = \frac{1}{2} k \Delta x^2$, sub in v_f from I	D
3.	Since the momentum is the same, that means the quantity $m_1v_1 = m_2v_2$. This means that the mass and velocity change proportionally to each other so if you double m_1 you would have to double m_2 or v_2 on the other side as well to maintain the same momentum. Now we consider the energy formula $KE= \frac{1}{2} mv^2$ since the v is squared, it is the more important term to increase in order to make more energy. So if you double the mass of 1, then double the velocity of 2, you have the same momentum but the velocity of 2 when squared will make a greater energy, hence we want more velocity in object 2 to have more energy.	C
4.	Due to momentum conservation, the total before is zero therefore the total after must also be zero	D
5.	Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(75)(6) + (100)(–8) = (175)\, v_f$	A
6.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(5000)(4) = (13000)v_f$	C
7.	Energy is conserved during fall and since the collision is elastic, energy is also conserved during the collision and always has the same total value throughout.	A
8.	To conserve momentum, the change in momentum of each mass must be the same so each must receive the same impulse. Since the spring is in contact with each mass for the same expansion time, the applied force must be the same to produce the same impulse.	B
9.	Use $J=\Delta p$ $J = mv_f – mv_i$ $J = (0.5)(– 4) – (0.5)(6)$	B
10.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(2m)(v) = (5m)\, v_f$	B
11.	First of all, if the kinetic energies are the same, then when brought to rest, the non conservative work done on each would have to be the same based on work-energy principle. Also, since both have the same kinetic energies we have $\frac{1}{2} m_1v_1^2 = \frac{1}{2} m_2v_2^2$ … since the velocity is squared an increase in mass would need a proportionally smaller decrease in velocity to keep the terms the same and thus make the quantity mv be higher for the larger mass. This can be seen through example: If mass m_1 was double mass m_2 its velocity would be $v / \sqrt{2}$ times in comparison to mass m_2's velocity. So you get double the mass but less than half of the velocity which makes a larger mv term.	A,C
12.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (3m)\, v_f$	A
13.	Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (50)(v_{1f}) + (2)(10)$	B
14.	Explosion, momentum before is zero and after must also be zero. To have equal momentum the heavier student must have a much smaller velocity and since that smaller velocity is squared it has the effect of making the heavier object have less energy than the smaller one	C

15. Based on momentum conservation both carts have the same magnitude of momentum "mv" but based on $K = ½ m v^2$ the one with the larger mass would have a directly proportional smaller velocity that then gets squared. So by squaring the smaller velocity term it has the effect of making the bigger mass have less energy. This can be shown with an example of one object of mass m and speed v compared to a second object of mass 2m and speed v/2. The larger mass ends up with less energy even through the momenta are the same. B

16. A 2d collision must be looked at in both x-y directions always. Since the angle is the same and the v is the same, v_y is the same both before and after therefore there is no momentum change in the y direction. All of the momentum change comes from the x direction. D
$v_{ix} = v \cos\theta$ and $v_{fx} = -v \cos\theta$. $\Delta p = mv_{fx} - mv_{ix}$ … $- mv \cos\theta - mv \cos\theta$

17. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (7)(v_{1f}) + (5)(0.2)$ B

18. In a circle at constant speed, work is zero since the force is parallel to the incremental distance moved during revolution. Angular momentum is given by mvr and since none of those quantities are changing it is constant. However the net force is NOT = MR, its Mv^2/R B,C

19. Since the momentum before is zero, the momentum after must also be zero. Each mass must have equal and opposite momentum to maintain zero total momentum. D

20. In a perfect inelastic collision with one of the objects at rest, the speed after will always be less no matter what the masses. The 'increase' of mass in 'mv' is offset by a decrease in velocity C

21. Since the total momentum before and after is zero, momentum conservation is not violated, however the objects gain energy in the collision which is not possible unless there was some energy input which could come in the form of inputting stored potential energy in some way. B

22. The plastic ball is clearly lighter so anything involving mass is out, this leaves speed which makes sense based on free-fall B

23. Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (m+M)\, v_f$ D

24. As the cart moves forward it gains mass due to the rain but in the x direction the rain does not provide any impulse to speed up the car so its speed must decrease to conserve momentum B

25. Momentum increases if velocity increases. In a d-t graph, III shows increasing slope (velocity) B

26. The net force is zero if velocity (slope) does not change, this is graphs I and II C

27. Since the initial object was stationary and the total momentum was zero it must also have zero total momentum after. To cancel the momentum shown of the other two pieces, the 3m piece would need an x component of momentum $p_x = mV$ and a y component of momentum $p_y = mV$ giving it a total momentum of $\sqrt{2}\, mV$ using Pythagorean theorem. Then set this total momentum equal to the mass * velocity of the 3rd particle. D
$\sqrt{2}\, mV = (3m)\, V_{m3}$ and solve for V_{m3}

28. Its does not matter what order to masses are dropped in. Adding mass reduces momentum proportionally. All that matters is the total mass that was added. This can be provided by finding the velocity after the first drop, then continuing to find the velocity after the second drop. Then repeating the problem in reverse to find the final velocity which will come out the same C

29. Increase in momentum is momentum change which is the area under the curve C

30. Basic principle of impulse. Increased time lessens the force of impact. D

31. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = m_1(5) + m_2(-2)$ B

32. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $Mv + (-2Mv) = (3M)\,v_f$ gives $v_f = v/3$. C
Then to find the energy loss subtract the total energy before – the total energy after
$[\tfrac{1}{2} Mv^2 + \tfrac{1}{2} (2M)v^2] - \tfrac{1}{2} (3M) (v/3)^2 = 3/6\, Mv^2 + 6/6\, Mv^2 - 1/6\, Mv^2$

33. 2D collision. The y momentums are equal and opposite and will cancel out leaving only the x momentums which are also equal and will add together to give a total momentum equal to twice the x component momentum before hand. $p_{before} = p_{after}$ $2m_ov_o\cos 60 = (2m_o)\, v_f$ B

34. Since there is no y momentum before, there cannot be any net y momentum after. The balls have equal masses so you need the y velocities of each ball to be equal after to cancel out the momenta. By inspection, looking at the given velocities and angles and without doing any math, the only one that could possibly make equal y velocities is choice D C

35. The area of the Ft graph is the impulse which determines the momentum change. Since the net impulse is zero, there will be zero total momentum change. C

36. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(m)(v) + (2m)(v / 2) = (3m)v_f$ C

37. Since the angle and speed are the same, the x component velocity has been unchanged which means there could not have been any x direction momentum change. The y direction velocity was reversed so there must have been an upwards y impulse to change and reverse the velocity. D

38. Just as linear momentum must be conserved, angular momentum must similarly be conserved. Angular momentum is given by L = mvr, so to conserve angular momentum, these terms must all change proportionally. In this example, as the radius decreases the velocity increases to conserve momentum. A

39. Each child does work by pushing to produce the resulting energy. This kinetic energy is input through the stored energy in their muscles. To transfer this energy to each child, work is done. The amount of work done to transfer the energy must be equal to the amount of kinetic energy gained. Before hand, there was zero energy so if we find the total kinetic energy of the two students, that will give us the total work done. First, we need to find the speed of the boy using momentum conservation, explosion: D
$p_{before} = 0 = p_{after}$ $0 = m_bv_b + m_gv_g$ $0 = (m)(v_b) = (2m)(v_g)$ so $v_b = 2v$
Now we find the total energy $K_{tot} = K_b + K_g = \tfrac{1}{2} m(2v)^2 + \tfrac{1}{2} 2m(v)^2 = 2mv^2 + mv^2 = 3mv^2$

40. Since it is an elastic collision, the energy after must equal the energy before, and in all collisions momentum before equals momentum after. So if we simply find both the energy before and the momentum before, these have the same values after as well. $p = Mv$, $K = \tfrac{1}{2} Mv^2$ A

41. The area under the F-t graph will give the impulse which is equal to the momentum change. With the momentum change we can find the velocity change. B
$J = \text{area} = 6$ Then $J = \Delta p = m\Delta v$ $6 = (2)\Delta v$ $\Delta v = 3$ m/s

42. This is the same as question 16 except oriented vertically instead of horizontally. D

1976B2.

a) Apply momentum conservation. $p_{before} = p_{after}$ $mv_o = (m)(v_o/3) + (4m)(v_{f2})$ $v_{f2} = v_o / 6$

b) $KE_f – KE_i = ½ mv_o^2 – ½ m (v_o / 3)^2 = 4/9 mv_o^2$

c) $KE = ½ (4m)(v_o / 6)^2 = 1/18 mv_o^2$

1978B1.

a) Projectile methods. Find t in y direction. $d_y = v_{iy}t + ½ a t^2$ $t = \sqrt{\frac{2H}{g}}$

D is found with $v_x = d_x / t$ $D = v_ot$ $v_o\sqrt{\frac{2H}{g}}$

b) Apply momentum conservation in the x direction. $p_{before(x)} = p_{after(x)}$ $M_1v_o = (M_1+M_2)v_f$ $v_f = M_1v_0 / (M_1+M_2)$

1981B2.

a) The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring $W = ½ m v^2 = ½ (3) (10)^2 = 150$ J

b) Apply momentum conservation to the explosion
$p_{before} = 0 = p_{after}$ $0 = m_1v_{1f} + m_2v_{2f}$ $0 = (1)v_{1f} + (3)v_{2f}$ $v_{1f} = 3 v_{2f}$

Apply energy conservation … all of the spring energy is converted into the kinetic energy of the masses
$150 \text{ J} = K_1 + K_2$ $150 = ½ m v_{1f}^2 + ½ m v_{2f}^2$ sub in above for v_{2f}
$150 = ½ (1)(3v_{2f})^2 + ½ (3)(v_{2f})^2$
$v_{2f} = 5$ m/s $v_{1f} = 15$ m/s

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $½ (3M)(2/3\ v_o)^2 = ½ k \Delta x^2$ $\sqrt{\frac{4Mv_o^2}{3k}}$

1984B2.

a) Before the collision there is only an x direction momentum of mass M_1 … $p_x = m_1v_{1x} = 16$, all the rest are 0
After the collision, M_1 has y direction momentum = $m_1v_{1fy} = 12$ and M_2 has x and y direction momentums.
Using trig to find the x and y velocities of mass M_2 … $v_x = 5 \cos 37 = 3$, and $v_y = 5 \sin 37 = 3.75$.
Then plug into mv to get each x and y momentum after.

	$M_1 = 1$ kg		$M_2 = 4$ kg	
	p_x (kg m /s)	p_y (kg m /s)	p_x (kg m /s)	p_y (kg m /s)
Before	16	0	0	0
After	0	-12	16	12

b) SUM = 16 -12 16 12
When adding x's before they = x's after 16=16, when adding y's before they equal y's after |-12|=12

c) Kinetic Energy Before
$K = ½ m_1v_{1ix}^2$
$K = ½ (1)(16)^2 = 128$ J

Kinetic Energy After
$K = ½ m_1v_{1fy}^2 + ½ m_2v_2^2$
$K = ½ (1)(12)^2 + ½ (4)(5)^2 = 122$ J

d) From above, K is not conserved.

1985B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_{1i} = (m+M)v_f$ $v_f = 1.5$ m/s

b) KE_i / KE_f $½ m v_{1i}^2 / ½ (m+M)vf^2 = 667$

c) Apply conservation of energy of combined masses $K = U$ $½ (m+M)v^2 = (m+M)gh$ $h = 0.11$ m

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1990B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_o = (101m)v_f$ $v_f = v_o / 101$

b) $\Delta K = K_f - K_i = ½ (101m)v_f^2 - ½ mv_o^2 = ½ (101m)(v_o/101)^2 - ½ mv_o^2 = -(50/101) mv_o^2$

c) Using projectile methods. Find t in y direction. $d_y = v_{iy}t + ½ a t^2$ $t = \sqrt{\frac{2h}{g}}$

D is found with $v_x = d_x / t$ $D = v_xt$ $\frac{v_0}{101}\sqrt{\frac{2h}{g}}$

d) The velocity of the block would be different but the change in the x velocity has no impact on the time in the y direction due to independence of motion. v_{iy} is still zero so t is unchanged.

e) In the initial problem, all of the bullets momentum was transferred to the block. In the new scenario, there is less momentum transferred to the block so the block will be going slower. Based on $D = v_xt$ with the same time as before but smaller velocity the distance x will be smaller.

1992 B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$
$m_1v_{1i} = (m_1+m_2)v_f$ (30)(4) = (80)v_f v_f = 1.5 m/s

b) K= ½ $(m_1+m_2)\ v_f^2$ = ½ $(80)(1.5)^2$ = 90 J

c) Apply momentum conservation explosion.
$p_{before} = p_{after}$ $(m_1+m_2)v = m_1v_{1f} + m_2v_{2f}$ (80)(1.5) = 0 + (50)v_{2f} v_{2f} = 2.4 m/s

d) K = ½ $m_2v_{2f}^2$ = ½ $(50)(2.4)^2$ = 144 J

e) By inspection the energy in d is greater. The energy increased due to an energy input from the work of the child's muscles in pushing on the sled.

1994 B2.

a) Apply energy conservation top to bottom. U = K mgh = ½ mv^2 (gR) = ½ v^2 $v = \sqrt{2gR}$

b) Apply momentum conservation

$p_{before} = p_{after}$ $m_av_{ai} = (m_a+m_b)v_f$ $M(\sqrt{2gR}) = 2Mv_f$ $v_f = \dfrac{\sqrt{2gR}}{2}$

c) The loss of the kinetic energy is equal to the amount of internal energy transferred

$\Delta K = K_f - K_i = \frac{1}{2}\,2M\left(\dfrac{\sqrt{2gR}}{2}\right)^2 - \frac{1}{2}M\left(\sqrt{2gR}\right)^2$ = – MgR / 2 lost ➔ MgR / 2 internal energy gain.

d) Find the remaining kinetic energy loss using work-energy theorem which will be equal the internal energy gain.
$W_{nc} = \Delta K$ $-f_kd = -\mu F_nd = -\mu(2m)gL = -2\mu MgL$, kinetic loss = internal E gain ➔ 2μMgL

1995 B1.

a) i) p = mv = (0.2)(3) = 0.6 kg m/s
ii) K = ½ mv^2 = ½ $(0.2)(3)^2$ = 0.9 J

b) i.) Apply momentum conservation $p_{before} = p_{after}$ = 0.6 kg m/s
ii) First find the velocity after using the momentum above
0.6 = (1.3+0.2) v_f v_f= 0.4 m/s K = ½ $(m_1+m_2)\ v_f^2$ = ½ $(1.3+0.2)(0.4)^2$ = 0.12 J

c) Apply energy conservation K = U_{sp} 0.12 J = ½ $k\Delta x^2$ = ½ (100) Δx^2 Δx = 0.05 m

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1996 B1.

a) $p_{tot} = M(3v_o) + (M)(v_o) = 4mv_o$

b) i) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$
$4Mv_o = (m_1+m_2)v_f$ $4Mv_o = (2M)\ v_f$ $v_f = 2v_o$
ii) Since they are both moving right they would have to be moving right after

c) i) Apply momentum conservation $p_{before} = p_{after}$
$4Mv_o = m_1v_{1f} + m_2v_{2i}$ $4Mv_o = Mv_{af} + M(2.5v_o)$ $v_{af} = 1.5\ v_o$
ii) As before, the would have to be moving right.

d) $\Delta K = K_f - K_i = (\frac{1}{2} m_av_{af}^2 + \frac{1}{2} m_bv_{bf}^2) - (\frac{1}{2} m_av_{ai}^2 + \frac{1}{2} m_bv_{bi}^2) = 4.25\ Mv_o^2 - 5\ Mv_o^2 = -0.75\ Mv_o^2$

1997 B1.

a) The force is constant, so simple $F_{net} = ma$ is sufficient. $(4) = (0.2)\ a$ $a = 20\ m/s^2$

b) Use $d = v_it + \frac{1}{2} a\ t^2$ $12 = (0) + \frac{1}{2}(20)\ t^2$ $t = 1.1$ sec

c) $W = Fd$ $W = (4\ N)(12\ m) = 48\ J$

d) Using work energy theorem $W = \Delta K$ $(K_i = 0)$ $W = Kf - Ki$
$W = \frac{1}{2} m\ v_f^2$
Alternatively, use $v_f^2 = v_i^2 + 2\ a\ d$ $48J = \frac{1}{2}(0.2)(v_f^2)$ $v_f = 21.9\ m/s$

e) The area under the triangle will give the extra work for the last 8 m
$\frac{1}{2}(8)(4) = 16J$ + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem $W = \frac{1}{2} m\ v_f^2$ $64\ J = \frac{1}{2}(0.2)\ v_f^2$ $v_f = 25.3\ m/s$

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

f) The momentum change can simply be found with $\Delta p = m\Delta v = m(v_f - v_i) = 0.2\ (25.3 - 21.9) = 0.68$ kg m/s

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2001B2.

a) Apply momentum conservation $p_{before} = p_{after}$
$m_av_{ai} = m_av_{af} + m_bv_{bf}$ $(0.1)(1.4) = (0.1)(-0.7) + (0.5)v_{bf}$ $v_{bf} = 0.42\ m/s$

b) Using projectile methods. Find t in y direction. $d_y = v_{iy}t + \frac{1}{2} a\ t^2$ $-1.2m = 0 + \frac{1}{2}(-9.8)\ t^2$ $t = 0.49$
D is found with $v_x = d_x / t$ $D = v_xt$ $(0.42)(0.49)$ $D = 0.2$ m

c) The time of fall is the same as before since it's the same vertical distance. t = 0.49 s
The velocity of ball C leaving the table can be found using projectile methods. $v_x = d / t = 0.15/0.49 = 0.31$ m/s

d) Looking that the y direction. $p_{y(before)} = p_{y(after)}$
$0 = p_{ay} - p_{cy}$ $0 = p_{ay} - m_cv_{cy}$ $0 = p_{ay} - (0.1)(0.31)\sin 30$ $p_{ay} = 0.015$ kg m/s

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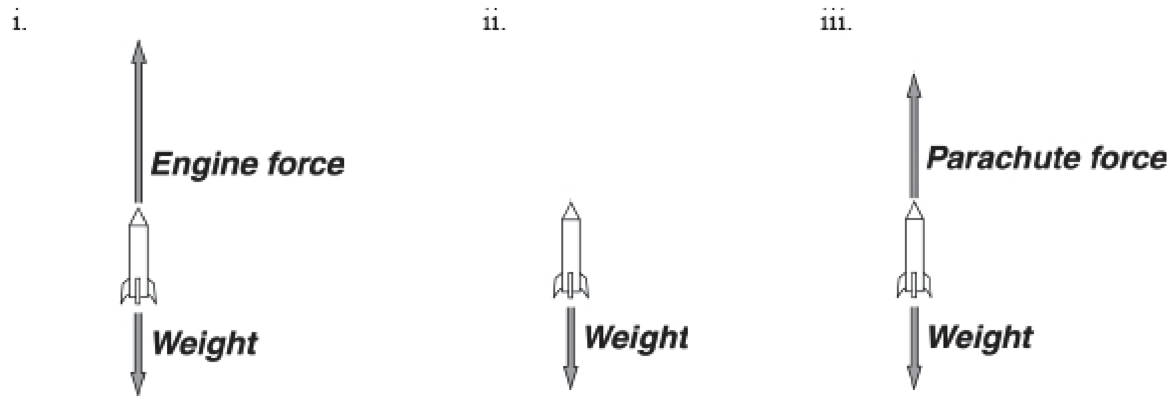
2002B1.

a)



b) $J_{engine} = F_{eng}\, t$ (20) = F_{eng} (2) F_{eng} = 10 N

F_{net} = ma (F_{eng}–mg)=ma (10 – 0.25(9.8))=(0.25)a a = 30 m/s^2

c) Find distance traveled in part (i) $d_1 = v_i t + ½ a t^2 = 0 + ½ (30)(2)^2 = 60$ m
Find distance in part (ii) free fall.
first find velocity at end of part (i) = v_i for part ii then find distance traveled in part ii
$v_{1f} = v_{1i} + a_1 t_1 = (0) + (30)(2) = 60$ m/s $v_{2f}^2 = v_{2i}^2 + 2gd_2 = (60)^2 + 2(–9.8)(d_2)$ d_2 = 184 m

d_{total} = 244 m

d) Find time in part ii. $v_{2f} = v_{2i} + gt$ 0 = 60 + –9.8 t t = 6.1 s
then add it to the part I time (2 s) total time ➔ 8.1 sec

2002B1B

a) The graph of force vs time uses area to represent the Impulse and the impulse equals change in momentum.
Area = 2 x ½ bh = (0.5 ms)(10kN). Milli and kilo cancel each other out. Area = 5 Ns = J

VERY IMPORTANT – Based on the problem, the force given and therefore impulse is actually negative because the graph is for the 2 kg cart and clearly the force would act opposite the motion of the cart.

$J = \Delta p = mv_f – mv_i$ (–5) = (2)(v_f) – 2(3) v_f = 0.5 m/s (for the 2 kg cart)

b) Apply momentum conservation $p_{before} = p_{after}$
$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ (2)(3) = (2)(0.5) + (m_b)(1.6) m_b = 3.125 kg

c) slope = acceleration = Δy / Δx = (0.5 – 1.6) / (3.5 – 3) = – 0.73 m/s^2

d) distance = area under line, using four shapes.
0-2 rectangle, 2-3.5 triangle top + rectangle bottom, 3.5-5, rectangle ➔ 5.5 m

e) Since the acceleration is negative the cart is slowing so it must be going up the ramp. Use energy conservation to find the max height. $K_{bot} = U_{top}$ ½ mv^2 = mgh ½ (1.6)^2 = (9.8) h h = 0.13 m

2006B2B.

a) Apply energy conservation
$U_{top} = K_{bottom}$
$mgh = ½ m v^2$ $Mgh = ½ (M) (3.5v_o)^2$ $h = 6.125 v_o^2 / g$

b) Apply momentum conservation $p_{before} = p_{after}$
$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(M)(3.5v_o) = (M)v_{af} + (1.5M)(2v_o)$ $v_{af} = ½ v_o$

c) $W_{NC} = \Delta K$ (K_f-K_i) $K_f = 0$
$– f_k d = 0 – ½ (1.5M)(2v_o)^2$
$\mu_k (1.5 M) g (d) = 3Mv_o^2$ $\mu_k = 2v_o^2 / gD$

d) Compare the kinetic energies before and after
Before After
$K = ½ M (3.5v_o)^2$ $½ M(½ v_o)^2 + ½ (1.5M)(2v_o)^2$ there are not equal so its inelastic

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2008B1B.

a) Apply momentum conservation to the explosion $p_{before} = 0 = p_{after}$
$0 = m_a v_{af} + m_b v_{bf}$ $0 = (70)(– 0.55) + (35)(v_{bf})$ $v_{bf} = 1.1$ m/s

b) $J_{son} = \Delta p_{son}$ $F_{on\text{-}son}\, t = m(v_f – v_i)$ $F (0.6) = (35)(0 – 1.1) =$ $F = – 64$ N

c) Based on newtons third law action/reaction, the force on the son must be the same but in the opposite direction as the force on the mother.

d) On the son $W_{fk} = \Delta K$ $– f_k d = ½ mv_f^2 – ½ mv_i^2$ $– \mu mg\, d = ½ m (0–v_i^2)$ $d = v_i^2 / (2\mu)$

This would be the same formula for the mother's motion with a different initial velocity. Since the mass cancels out we see the distance traveled is proportional to the velocity squared. The boy moves at twice the speed of the mother, so based on this relationship should travel 4 x the distance. The mother traveled 7 m so the son would have a sliding distance of 28 m.
(Alternatively, you could plug in the numbers for the mother to solve for μ and then plug in again using the same value of μ and the sons velocity to find the distance. μ is the same for both people.)

2008B1.

a) First determine the time to travel while the car accelerates. $v_{1f} = v_{1i} + a_1 t_1$ $(5) = (2) + (1.5)\, t_1$ $t_1 = 2$ sec
Also determine the distance traveled while accelerating $d_1 = v_{1i} t_1 + ½ a_1 t_1^2$ $d_1 = (2)(2) + ½ (1.5)(2)^2 = 7$ m

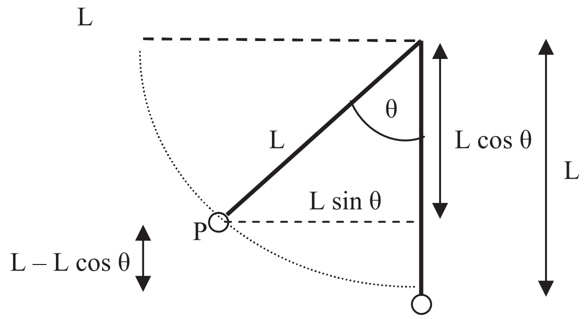
This leaves 8 m left for the constant speed portion of the trip.
The velocity at the end of the 7m is the average constant velocity for the second part of the trip
$v_2 = d_2 / t_2$ $5 = 8 / t_2$ $t_2 = 1.6$ sec ➔ total time = $t_1 + t_2 =$ 3.6 seconds

b) i) Apply momentum conservation $p_{before} = p_{after}$
$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(250)(5) = (250)v_{af} + (200)(4.8)$ $v_{af} = 1.2$ m/s
ii) Since the velocity is + the car is moving right

c) Check kinetic energy before vs after
$K_i = ½ (250)(5)^2 = 3125$ J $K_f = ½ (250) (1.2)^2 + ½ (200)(4.8)^2 = 2484$ J

Since the energies are not the same, it is inelastic

C1981M2.



a) $U_{top} = K_{bot}$
$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$
$$v = \sqrt{2g(L - \frac{L}{2})}$$
$$v = \sqrt{gL}$$

b) Use the max rise height on the opposite site to find the seats speed
$K_{bot} = U_{top}$ $\quad \frac{1}{2} mv^2 = mgh$

$$v = \sqrt{2g(L - L\cos 45)}$$
$$v = \sqrt{2g(L - \frac{\sqrt{2}L}{2})}$$
$$v = \sqrt{2gL\,(1 - \frac{\sqrt{2}}{2})} \qquad v = \sqrt{gL\,(2 - \sqrt{2})}$$

c) Apply momentum conservation $\quad p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $\quad (2m)\sqrt{gL} = m\, v_{af} + m(\sqrt{gL\,(2-\sqrt{2})}) \quad = \sqrt{gL}\left(2 - \sqrt{(2-\sqrt{2})}\right)$

C1991M1.

(a) Apply momentum conservation perfect inelastic $\quad p_{before} = p_{after}$
$mv_o = (m+2m)\, v_f$ $\quad v_f = v_o / 3$

(b) Apply energy conservation.
$K_{bottom} = U_p + K_p$ $\quad \frac{1}{2} mv_{bot}^2 = mgh_p + K_p$
$\frac{1}{2}\, 3m\,(v_o/3)^2 = 3mg(r) + K_p$ $\quad K_p = mv_o^2/6 - 3mgr$

(c) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set F_n=0 at the top of the loop so that only mg is acting down on the block. The apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\quad 3mg = 3m\, v^2 /r$ $\quad v = \sqrt{rg}$

(d) Energy conservation top of loop to bottom of loop
$U_{top} + K_{top} = K_{bot}$
$mgh + \frac{1}{2} m\, v_{top}^2 = \frac{1}{2} m\, v_{bot}^2$ $\quad g(2r) + \frac{1}{2}(\sqrt{rg})^2 = \frac{1}{2}(vo')^2$ $\quad v_o' = \sqrt{5gr}$

(e) Apply momentum conservation, perfect inelastic with v_f as the speed found above and v_i unknown

$p_{before} = p_{after}$ $\quad mv_b' = (m+2m)\, v_f$ $\quad v_b' = 3v_f =$ $\quad v_o' = 3\sqrt{5gr}$

C1992M1.

a) $U_{top} = K_{bot}$ $mgh = \frac{1}{2} mv^2$ $(10)(5) = \frac{1}{2} v^2$ $v = 10$ m/s

b) Since the ball hits the ground elastically, it would rebound with a speed equal to that it hit with 10 m/s

c) Free fall of clay $d = v_i t + \frac{1}{2} gt^2 = 0 + \frac{1}{2}(-10)(0.5)^2$
$d = -1.25$ m displaced down, so height from ground would be 3.75 m

d) Clay free fall (down) $v_f = v_i + gt = 0 + (-10)(0.5) = -5$ m/s speed = 5 m/s
Ball free fall (up) $v_f = v_i + gt = 10 + (-10)(0.5) = 5$ m/s speed = 5 m/s

e) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai} + m_b v_{bi} = (m_a + m_b) v_f$ $(9m)(5) + (m)(-5) = (10m) v_f$ $v_f = 4$ m/s, up (since +)

C1993M1. - since there is friction on the surface the whole time, energy conservation cannot be used

a) $U_{sp} = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(400)(0.5)^2 = 50$ J

b) Using work-energy $W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)} - U_{sp(i)}) + (K_f - K_i)$
$-f_k d = (0 - 50\text{J}) + (\frac{1}{2} m v_f^2 - 0)$
$-\mu mg d = \frac{1}{2} mv_f^2 - 50$
$-(0.4)(4)(9.8)(0.5) = \frac{1}{2}(4)(v_c^2) - 50$ $v_c = 4.59$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ci} = (m_c + m_d) v_f$ $(4)(4.59) = (4+2) v_f$ $v_f = 3.06$ m/s

d) $W_{nc} = (K_f - K_i)$
$-f_k d = (0 - \frac{1}{2} m v_i^2)$ $-\mu mg d = -\frac{1}{2} m v_i^2$ $(0.4)(6)(9.8) d = \frac{1}{2}(6)(3.06)^2$ $d = 1.19$ m

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C1994M1.

a) $U_{sp} = \frac{1}{2} k \Delta x^2 = \frac{1}{2}(200)(0.4)^2 = 16$ J

b) Apply energy conservation $K_{before\ compression} = U_{sp\text{-}after\ compression}$
$\frac{1}{2}(m_a + m_b) v^2 = U_{sp}$ $\frac{1}{2}(0.1+2) v^2 = 16$ $v = 3.9$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai} = (m_a + m_b) v_f$ $(0.1) v_{ai} = (0.1+2)(3.9)$ $v_{ai} = 81.9$ m/s

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C1995M1.

a) In the F vs t curve the impulse is the area under the curve. Area of triangle + rectangle + triangle = 12 Ns

b) $J_{on\text{-}ball} = \Delta p_{ball}$ $J = m(v_{bf} - v_{bi})$ $12 = 5(v_{bf} - 0)$ $v_{bf} = 2.4$ m/s

c) i) Due to action reaction, the force on the cube is the same as that on the ball but in the opposite direction so the impulse applied to it is –12 Ns. $J_{on\text{-}cube} = \Delta p_{cube}$ $J = m(v_{cf} - v_{ci})$ $-12 = 0.5(v_{cf} - 26)$ $v_{cf} = 2$ m/s

ii) since +, moving right

d) $\frac{1}{2} mv_{cf}^2 + \frac{1}{2} mv_{bf}^2 - \frac{1}{2} mv_{ci}^2 =$ 154 J

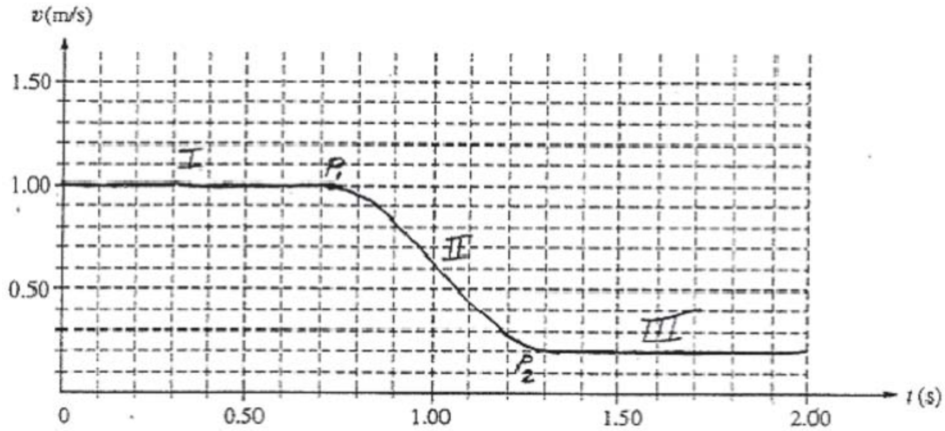
e) Using projectiles … both take same time to fall since $v_{iy} = 0$ for both and distance of fall same for both
$d_y = v_{iy}t + \frac{1}{2} g\, t_2$ $-1.2 = 0 + \frac{1}{2}(-9.8)\, t^2$ $t = 0.5$ sec

Each d_x is found using $d_x = v_x t$ for each respetive speed of cube and ball.
Gives d_x(cube) = 1m d_x(ball) = 1.2 m so they are spaced by 0.2 m when they hit.

C1998M1.

a) use v = d / t for each interval i) 1 m/s ii) 0.6 m/s iii) 0.2 m/s

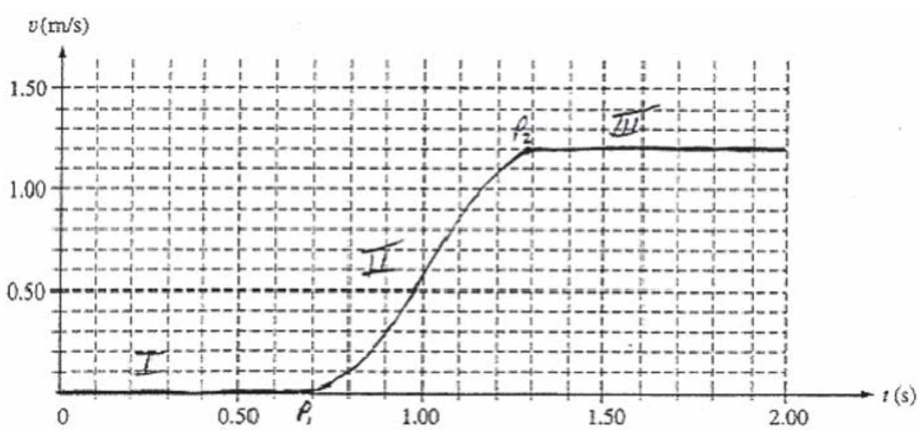
b) Based on the pattern of the **Λ** shapes of the ruler we can see the glider moves at a constant speed up until 0.70 s where the spacings start to change and it decelerates up until around the 1.3 second time where the speed becomes constant again. So the first constant speed is the initial velocity of the glider (1 m/s) and the second constant speed is the final velocity of the glider after the collision (0.2 m/s)



c)) i) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(0.9)(1) = (0.9)(0.2) + (0.6)(v_{bf})$ $v_{bf} = 1.2$ m/s

ii) Glider B is at rest up until 0.7 seconds where the collision accelerates to a final constant speed of 1.2 m/s



d) i) The collision is elastic because the kinetic energy before and after is the same

ii) The kinetic energy becomes a minimum because the energy is momentarily transferred to the spring

C1991M1. - The geometry of this problem is similar to C1981M2 in this document.

a) First determine the speed of the combined dart and block using energy conservation.

$K_{bot} = U_{top}$

$\frac{1}{2} mv^2 = mgh$ Then apply momentum conservation bullet to block collision

$$v = \sqrt{2g(L - L\cos\theta)}$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

perfect inelastic …$p_{before} = p_{after}$

$mv_o = (m+M_o)\,v$ $$v_0 = \frac{(m+M_o)}{m}\sqrt{2gL(1-\cos\theta)}$$

b) Apply $F_{net(c)} = mv^2 / r$, at the lowest point (tension acts upwards weight acts down)

$F_t - mg = mv^2/r$ $F_t = m(g+v^2/r)$ substitute v from above

$F_t = (m+M_o)(g + 2gL(1-\cos\theta)/L) = (m+M_o)(g + 2g - 2g\cos\theta) = (m+M_o)\,g\,(3 - 2\cos\theta)$

c) One way would be to hang the spring vertically, attach the five known masses, measure the spring stretch, and use these results to find the spring constant based on $F=k\Delta x$. Then attach the block to the spring and measure the spring stretch again. Fire the dart vertically at the block and measure the maximum distance traveled. Similar to the problem above, use energy conservation to find the initial speed of the block+dart then use momentum conservation in the collision to find the darts initial speed.

C2001M1.

a) Pick velocity from the graph and use $a = (v_f - v_i)/t$ $a = -10\ m/s^2$

b) The area of the force time graph gives the impulse which equals the momentum change. You can break the graph into three triangles and 1 rectangle and find the area = 0.6 Ns = 0.6 kg m/s of momentum change

c) Using the value above. $\Delta p = m(v_f - v_i)$ $-0.6 = m(-0.22 - 0.18)$ $m = 1.5$ kg.

The force sensor applies a – momentum since it would push in the negative direction as the cart collides with it.

d) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2}(1.5)(0.18^2 - 0.22^2) = -0.012$ J

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ $mgh = \frac{1}{2} mv^2$ $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\,v_f$ $M(\sqrt{2gH}) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Even though the position shown has an unknown initial stretch and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

Apply energy conservation $K + U = U_{sp\,(gained)}$ $\frac{1}{2} mv^2 + mgh = \frac{1}{2} k\,\Delta x^2$

Plug in mass (2m), h = H/2 and Δx = H/2 ➔ $\frac{1}{2}(2m)v^2 + (2m)g(H/2) = \frac{1}{2} k(H/2)^2$

plug in v_f from part b $m(2gH/4) + mgH = kH^2/8$ ….

Both sides * (1/H) ➔ $mg/2 + mg = kH/8$ ➔ $3/2\ mg = kH/8$ $k = 12mg / H$

C2004M1.

a) Energy conservation with position B set as h=0. $U_a = K_b$ $v_b = \sqrt{2gL}$

b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = m_1 v_b^2 / r$ $F_t - m_1 g = m_1(2gL) / L$ $F_t = 3m_1 g$

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$m_1 v_{1i} = (m_1+m_2)\, v_f$ $v_f = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}$

d) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + ½\, g\, t^2$ $L = 0 + gt^2 / 2$

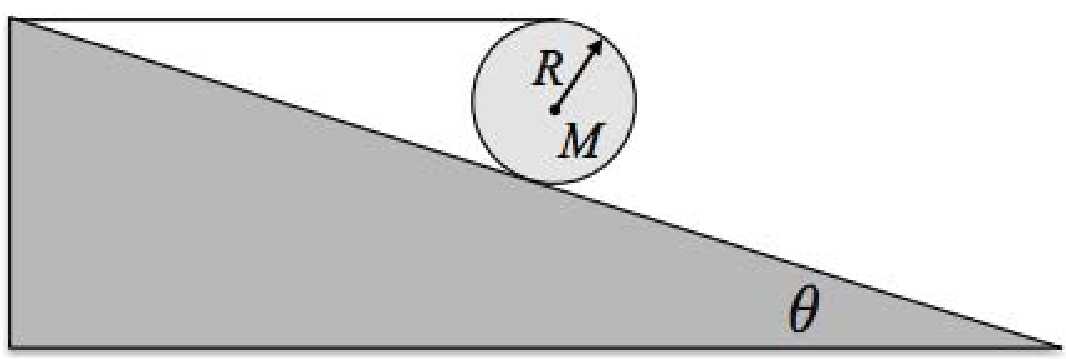
$t = \sqrt{\dfrac{2L}{g}}$ Then use $v_x = d_x / t$ $d_x = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}\sqrt{\dfrac{2L}{g}} = \dfrac{m_1}{(m_1+m_2)}2L$

The dx found is measured from the edge of the second lower cliff so the total horizontal distance would have to include the initial x displacement (L) starting from the first cliff.

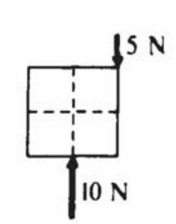
➔ $\dfrac{m_1}{(m_1+m_2)}2L + L = L\left[\dfrac{2m_1}{(m_1+m_2)} + 1\right]$

Chapter 6

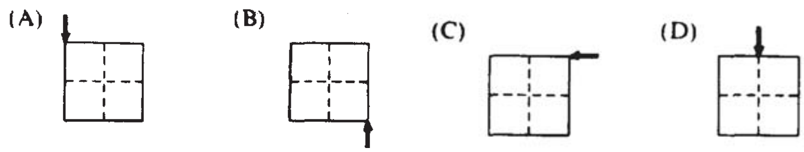
Rotation

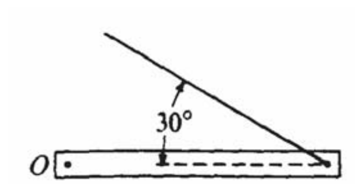


SECTION A – Torque and Statics

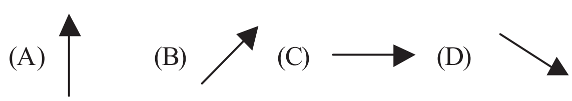


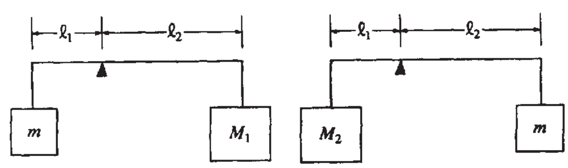
1. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown above. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?





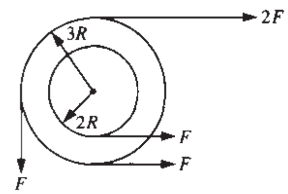
2. A uniform rigid bar of weight W is supported in a horizontal orientation as shown above by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?



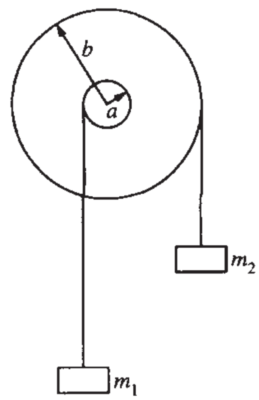


3. A rod of negligible mass is pivoted at a point that is off-center, so that length l_1 is different from length l_2. The figures above show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

(A) $M_1 + M_2$ (B) ½($M_1 + M_2$) (C) $M_1 M_2$ (D) $\sqrt{M_1 M_2}$



4. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown above. The magnitude of the net torque on the system about the axis is
(A) FR (B) 2FR (C) 5FR (D) 14FR

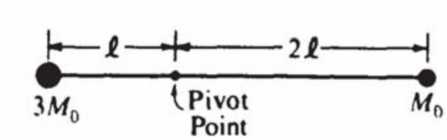


5. For the wheel-and-axle system shown above, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$ (B) $am_1 = bm_2$ (C) $am_2 = bm_1$ (D) $a^2m_1 = b^2m^2$

SECTION B – Rotational Kinematics and Dynamics

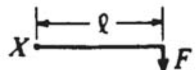
1. A uniform stick has length L. The moment of inertia about the center of the stick is I_o. A particle of mass M is attached to one end of the stick. The moment of inertia of the combined system about the center of the stick is

(A) $I_0 + \frac{1}{4}ML^2$ (B) $I_0 + \frac{1}{2}ML^2$ (C) $I_0 + \frac{3}{4}ML^2$ (D) $I_0 + ML^2$

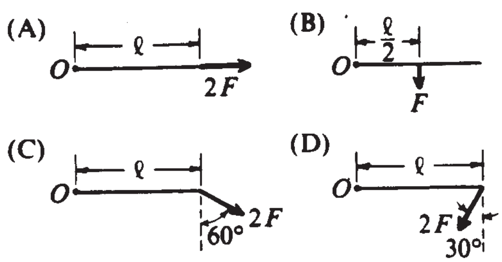


2. A light rigid rod with masses attached to its ends is pivoted about a horizontal axis as shown above. When released from rest in a horizontal orientation, the rod begins to rotate with an angular acceleration of magnitude

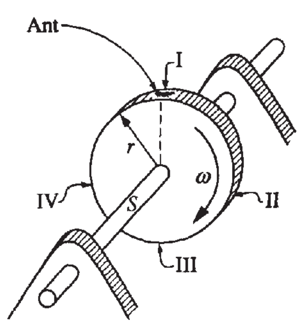
(A) $\frac{g}{7l}$ (B) $\frac{g}{5l}$ (C) $\frac{g}{4l}$ (D) $\frac{5g}{7l}$



3. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram above? (All forces lie in the plane of the paper.)



Questions 4-5



An ant of mass m clings to the rim of a flywheel of radius r, as shown above. The flywheel rotates clockwise on a horizontal shaft S with constant angular velocity ω. As the wheel rotates, the ant revolves past the stationary points I, II, III, and IV. The ant can adhere to the wheel with a force much greater than its own weight.

4. It will be most difficult for the ant to adhere to the wheel as it revolves past which of the four points?
(A) I (B) II (C) III (D) IV

5. What is the magnitude of the minimum adhesion force necessary for the ant to stay on the flywheel at point III?
(A) mg (B) $m\omega^2 r$ (C) $m\omega^2 r - mg$ (D) $m\omega^2 r + mg$

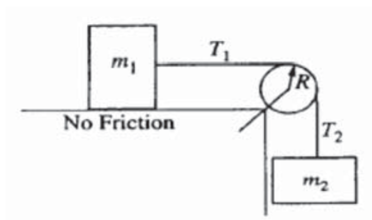
6. A turntable that is initially at rest is set in motion with a constant angular acceleration α. What is the angular velocity of the turntable after it has made one complete revolution?
(A) $\sqrt{2\alpha}$ (B) $\sqrt{2\pi\alpha}$ (C) $\sqrt{4\pi\alpha}$ (D) $4\pi\alpha$

Questions 7-8

A wheel with rotational inertia *I* is mounted on a fixed, frictionless axle. The angular speed ω of the wheel is increased from zero to ω_f in a time interval T.

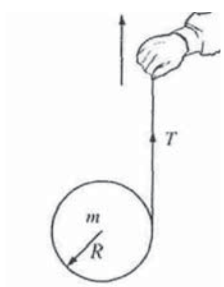
11. What is the average net torque on the wheel during this time interval?
(A) $\frac{\omega_f}{T}$ (B) $\frac{I\omega_f^2}{T}$ (C) $\frac{I\omega_f}{T^2}$ (D) $\frac{I\omega_f}{T}$

12. What is the average power input to the wheel during this time interval?
(A) $\frac{I\omega_f}{2T}$ (B) $\frac{I\omega_f^2}{2T}$ (C) $\frac{I\omega_f^2}{2T^2}$ (D) $\frac{I^2\omega_f}{2T^2}$



9. Two blocks are joined by a light string that passes over the pulley shown above, which has radius *R* and moment of inertia *I* about its center. T_1 and T_2 are the tensions in the string on either side of the pulley and *α* is the angular acceleration of the pulley. Which of the following equations best describes the pulley's rotational motion during the time the blocks accelerate?
(A) $m_2gR = I\alpha$ (B) $T_2R = I\alpha$ (C) $(T_2 - T_1)R = I\alpha$ (D) $(m_2 - m_1)gR = I\alpha$

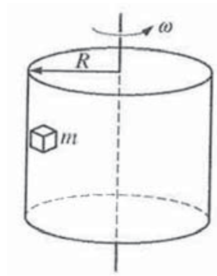
Questions 10-11



A solid cylinder of mass *m* and radius *R* has a string wound around it. A person holding the string pulls it vertically upward, as shown above, such that the cylinder is suspended in midair for a brief time interval *Δt* and its center of mass does not move. The tension in the string is *T,* and the rotational inertia of the cylinder about its axis is $\frac{1}{2}MR^2$

10. the net force on the cylinder during the time interval Δt is
(A) mg (B) $T - mgR$ (C) $mgR - T$ (D) zero

11. The linear acceleration of the person's hand during the time interval Δt is
(A) $\frac{T \square mg}{m}$ (B) $2g$ (C) $\frac{g}{2}$ (D) $\frac{T}{m}$



12. A block of mass *m* is placed against the inner wall of a hollow cylinder of radius *R* that rotates about a vertical axis with a constant angular velocity *ω,* as shown above. In order for friction to prevent the mass from sliding down the wall, the coefficient of static friction *μ* between the mass and the wall must satisfy which of the following inequalities?

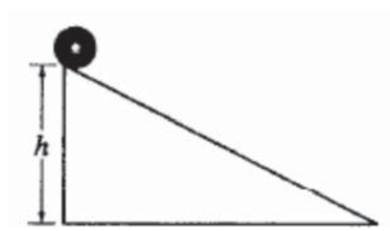
(A) $\mu \square \frac{g}{\square^2 R}$ (B) $\mu \square \frac{\square^2 R}{g}$ (C) $\mu \square \frac{g}{\square^2 R}$ (D) $\mu \square \frac{\square^2 R}{g}$

<u>SECTION C – Rolling</u>

1. A bowling ball of mass M and radius R. whose moment of inertia about its center is (2/5)MR^2, rolls without slipping along a level surface at speed *v*. The maximum vertical height to which it can roll if it ascends an incline is

(A) $\frac{v^2}{5g}$ (B) $\frac{2v^2}{5g}$ (C) $\frac{v^2}{2g}$ (D) $\frac{7v^2}{10g}$

<u>Questions 2-3</u>



A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown above.

2. If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?

(A) $\sqrt{2gh}$ (B) $\frac{2Mghr^2}{I}$ (C) $\sqrt{\frac{2Mghr^2}{I}}$ (D) $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$

3. If the plane has friction so that the sphere rolls without slipping, what is the speed v_{cm} of the center of mass at the bottom of the incline?

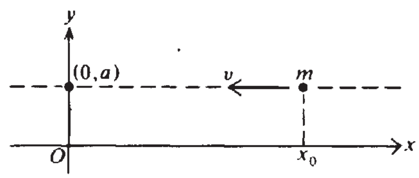
(A) $\sqrt{2gh}$ (B) $\frac{2Mghr^2}{I}$ (C) $\sqrt{\frac{2Mghr^2}{I}}$ (D) $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$

4. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
(A) vertically upward (B) horizontally forward (C) horizontally backward
(D) upward and forward, at approximately 45° to the horizontal

SECTION D – Angular Momentum

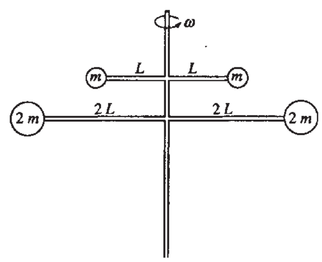
1. An ice skater is spinning about a vertical axis with arms fully extended. If the arms are pulled in closer to the body, in which of the following ways are the angular momentum and kinetic energy of the skater affected?

	Angular Momentum	Kinetic Energy
(A)	Increases	Increases
(B)	Increases	Remains Constant
(C)	Remains Constant	Increases
(D)	Remains Constant	Remains Constant



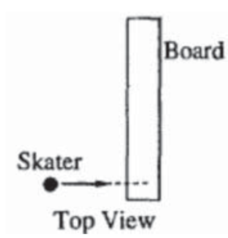
2. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is

   (A) zero (B) mva (C) mvx_o (D) $mv\sqrt{x_0^2 + a^2}$



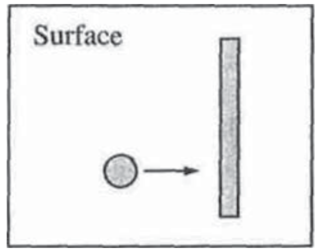
3. The rigid body shown in the diagram above consists of a vertical support post and two horizontal crossbars with spheres attached. The masses of the spheres and the lengths of the crossbars are indicated in the diagram. The body rotates about a vertical axis along the support post with constant angular speed ω. If the masses of the support post and the crossbars are negligible, what is the ratio of the angular momentum of the two upper spheres to that of the two lower spheres?

   (A) 2/1 (B) 1/2 (C) 1/4 (D) 1/8



4. A long board is free to slide on a sheet of frictionless ice. As shown in the top view above, a skater skates to the board and hops onto one end, causing the board to slide and rotate. In this situation, which of the following occurs?

   (A) Linear momentum is converted to angular momentum.
   (B) Rotational kinetic energy is conserved.
   (C) Translational kinetic energy is conserved.
   (D) Linear momentum and angular momentum are both conserved.



Top View

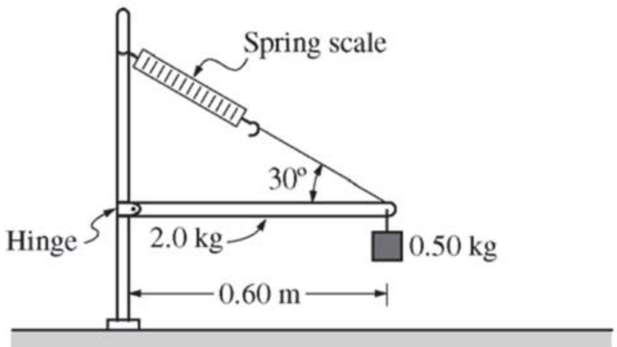
5. **Multiple Correct.** A disk sliding on a horizontal surface that has negligible friction collides with a rod that is free to move and rotate on the surface, as shown in the top view above. Which of the following quantities must be the same for the disk-rod system before and after the collision? Select two answers.

   I. Linear momentum
   II. Angular momentum
   III. Kinetic energy

   (A) Linear Momentum
   (B) Angular Momentum
   (C) Kinetic Energy
   (D) Mechanical Energy

WARNING: These are AP Physics C Free Response Practice – Use with caution!

SECTION A – Torque and Statics



2008M2. The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

a. On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

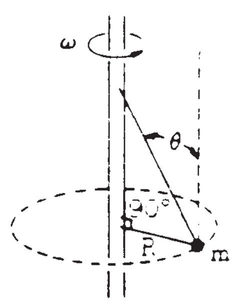
b. Calculate the reading on the spring scale.

The rotational inertia of a rod about its center is $\frac{1}{12}ML^2$, where M is the mass of the rod and L is its length.

c. Calculate the rotational inertia of the rod-block system about the hinge.

d. If the cord that supports the rod is cut near the end of the rod, calculate the initial angular acceleration of the rod-block system about the hinge.

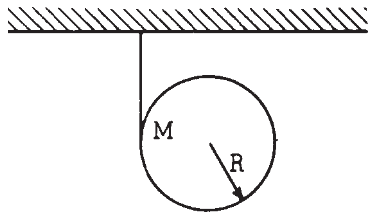
SECTION B – Rotational Kinematics and Dynamics



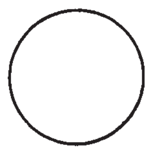
1973M3. A ball of mass m is attached by two strings to a vertical rod. as shown above. The entire system rotates at constant angular velocity ω about the axis of the rod.

a. Assuming ω is large enough to keep both strings taut, find the force each string exerts on the ball in terms of ω, m, g, R, and θ.

b. Find the minimum angular velocity, ω_{min} for which the lower string barely remains taut.



1976M2. A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is $\frac{1}{2}MR^2$.



a. On the circle above draw vectors showing all the forces acting on the cylinder after it is released. Label each force clearly.
b. In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.
c. While descending, does the center of the cylinder move toward the left, toward the right, or straight down? Explain.

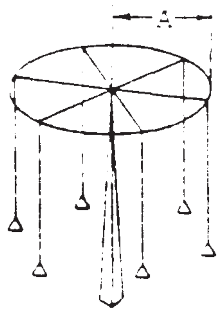


Figure I

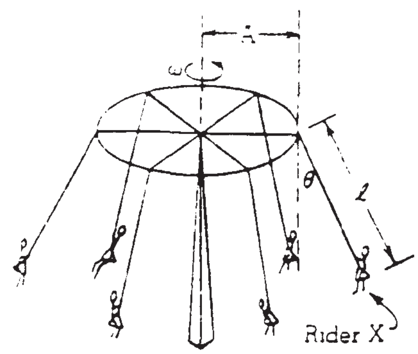


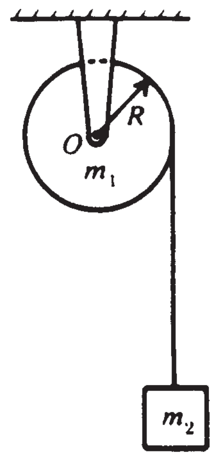
Figure II

1978M1. An amusement park ride consists of a ring of radius A from which hang ropes of length ℓ with seats for the riders as shown in Figure I. When the ring is rotating at a constant angular velocity ω each rope forms a constant angle θ with the vertical as shown in Figure II. Let the mass of each rider be m and neglect friction, air resistance, and the mass of the ring, ropes, and seats.

a. In the space below, draw and label all the forces acting on rider X (represented by the point below) under the constant rotating condition of Figure II. Clearly define any symbols you introduce.

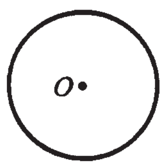
•

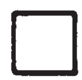
b. Derive an expression for ω in terms of A, ℓ, θ and the acceleration of gravity g.
c. Determine the minimum work that the motor that powers the ride would have to perform to bring the system from rest to the constant rotating condition of Figure II. Express your answer in terms of m, g, ℓ, θ, and the speed v of each rider.



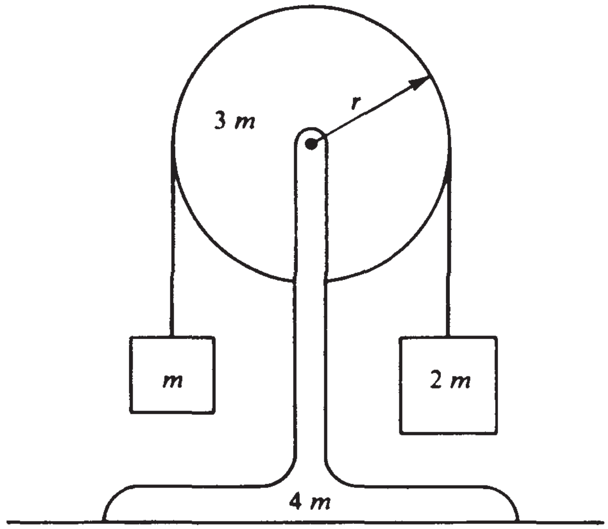
1983M2. A uniform solid cylinder of mass m_1 and radius R is mounted on frictionless bearings about a fixed axis through O. The moment of inertia of the cylinder about the axis is $I = \frac{1}{2}m_1R^2$. A block of mass m_2, suspended by a cord wrapped around the cylinder as shown above, is released at time t = 0.

a. On the diagram below draw and identify all of the forces acting on the cylinder and on the block.





b. In terms of m_1, m_2, R. and g, determine each of the following.
   i. The acceleration of the block
   ii. The tension in the cord



1985M3. A pulley of mass 3m and radius r is mounted on frictionless bearings and supported by a stand of mass 4m at rest on a table as shown above. The moment of inertia of this pulley about its axis is $1.5mr^2$.
Passing over the pulley is a massless cord supporting a block of mass m on the left and a block of mass 2m on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.

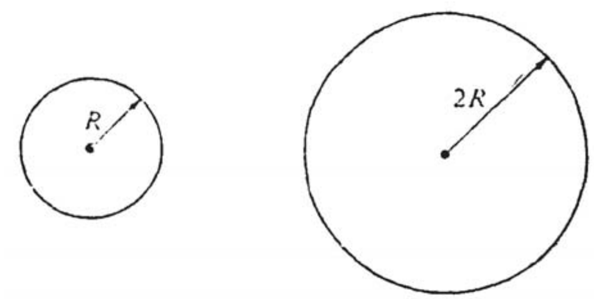
a. On the diagrams below, draw and label all the forces acting on each block.

b. Use the symbols identified in part a. to write each of the following.
   i. The equations of translational motion (Newton's second law) for each of the two blocks
   ii. The analogous equation for the rotational motion of the pulley

c. Solve the equations in part b. for the acceleration of the two blocks.

d. Determine the tension in the segment of the cord attached to the block of mass m.

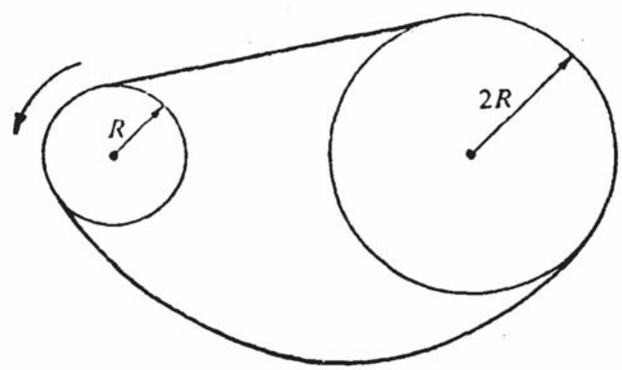
e. Determine the normal force exerted on the apparatus by the table while the blocks are in motion.



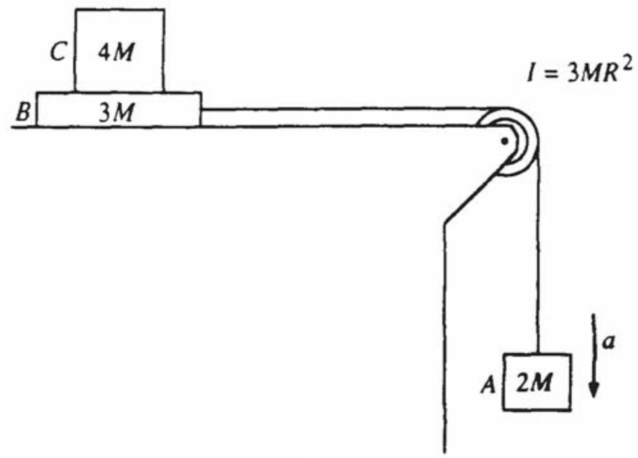
1988M3. The two uniform disks shown above have equal mass, and each can rotate on frictionless bearings about a fixed axis through its center. The smaller disk has a radius R and moment of inertia I about its axis. The larger disk has a radius 2R

a. Determine the moment of inertia of the larger disk about its axis in terms of I.

The two disks are then linked as shown below by a light chain that cannot slip. They are at rest when, at time t = 0, a student applies a torque to the smaller disk, and it rotates counterclockwise with constant angular acceleration α. Assume that the mass of the chain and the tension in the lower part of the chain, are negligible. In terms of I, R, α, and t, determine each of the following:



b. The angular acceleration of the larger disk
c. The tension in the upper part of the chain
d. The torque that the student applied to the smaller disk
e. The rotational kinetic energy of the smaller disk as a function of time



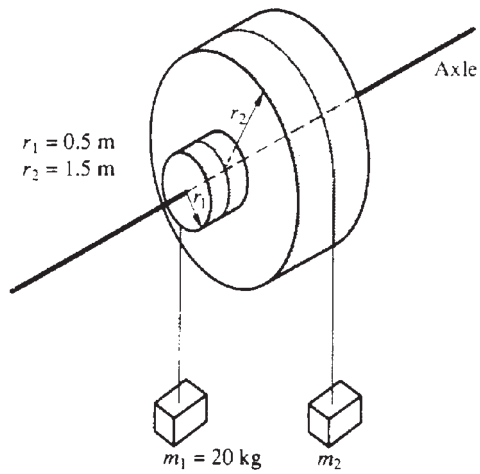
1989M2. Block A of mass 2M hangs from a cord that passes over a pulley and is connected to block B of mass 3M that is free to move on a frictionless horizontal surface, as shown above. The pulley is a disk with frictionless bearings, having a radius R and moment of inertia $3MR^2$. Block C of mass 4M is on top of block B. The surface between blocks B and C is NOT frictionless. Shortly after the system is released from rest, block A moves with a downward acceleration a, and the two blocks on the table move relative to each other.

In terms of M, g, and a, determine the

a. tension T_v in the vertical section of the cord
b. tension T_h in the horizontal section of the cord

If a = 2 meters per second squared, determine the

c. coefficient of kinetic friction between blocks B and C
d. acceleration of block C

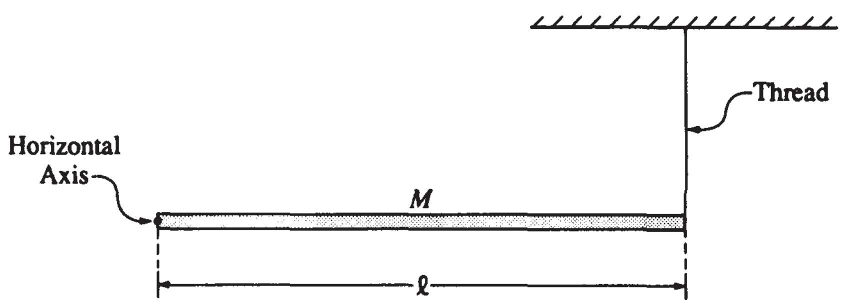


1991M2. Two masses. m_1 and m_2 are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively. as shown in the diagram above. The cylinders are rigidly connected to each other but are free to rotate without friction on a common axle. The moment of inertia of the pair of cylinders is I = 45 kg•m^2 Also r_1 = 0.5 meter, r_2 = 1.5 meters, and m_1 = 20 kilograms.

a. Determine m_2 such that the system will remain in equilibrium.

The mass m_2 is removed and the system is released from rest.

b. Determine the angular acceleration of the cylinders.
c. Determine the tension in the cable supporting m_1
d. Determine the linear speed of m_1 at the time it has descended 1.0 meter.



1993M3. A long, uniform rod of mass *M* and length *ℓ* is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. The moment of inertia of the rod about the axis at the end of the rod is $Ml^2/3$. Express the answers to all parts of this question in terms of *M*, *ℓ*, and *g*.

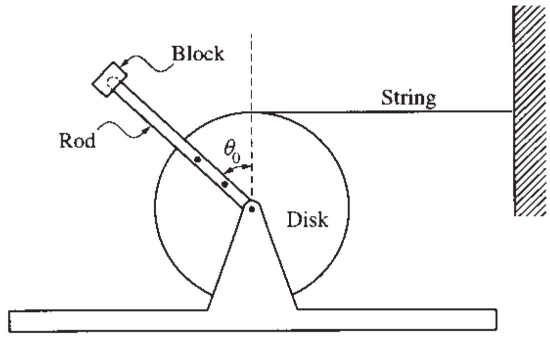
a. Determine the magnitude and direction of the force exerted on the rod by the axis.

The thread is then burned by a match. For the time immediately after the thread breaks, determine each of the following:

b. The angular acceleration of the rod about the axis
c. The translational acceleration of the center of mass of the rod
d. The force exerted on the end of the rod by the axis

The rod rotates about the axis and swings down from the horizontal position.

e. Determine the angular velocity of the rod as a function of θ, the arbitrary angle through which the rod has swung.



1999M3 As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk so that it will rotate with the disk. A block is attached to the end of the rod. Properties of the disk, rod, and block are as follows.

Disk: mass = 3m, radius = R, moment of inertia about center $I_D = 1.5mR^2$

Rod: mass = m, length = 2R, moment of inertia about one end $I_R = 4/3(mR^2)$

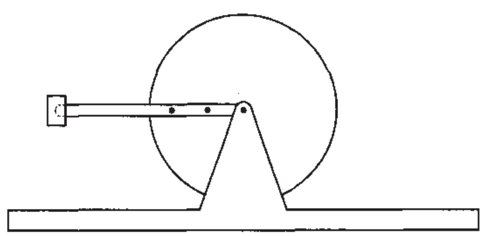
Block: mass = 2m

The system is held in equilibrium with the rod at an angle θ_0 to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Express your answers to the following in terms of m, R, θ_0, and g.

a. Determine the tension in the string.

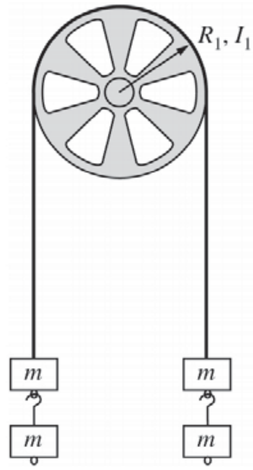
The string is now cut, and the disk-rod-block system is free to rotate.

b. Determine the following for the instant immediately after the string is cut.
i. The magnitude of the angular acceleration of the disk
ii. The magnitude of the linear acceleration of the mass at the end of the rod



As the disk rotates, the rod passes the horizontal position shown above.

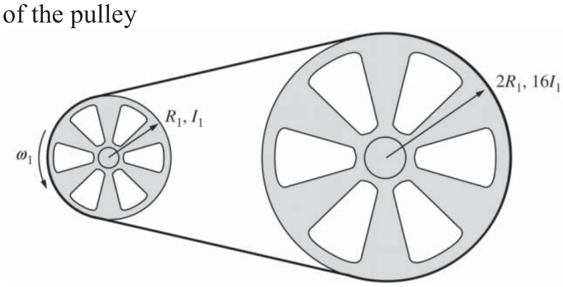
c. Determine the linear speed of the mass at the end of the rod for the instant the rod is in the horizontal position.



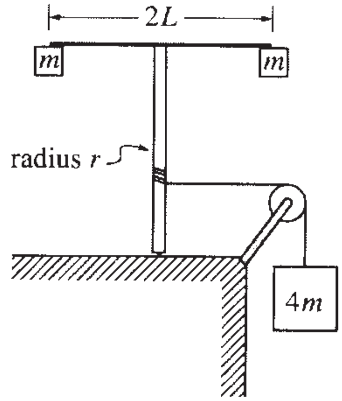
2000M3. A pulley of radius R_1 and rotational inertia I_1 is mounted on an axle with negligible friction. A light cord passing over the pulley has two blocks of mass m attached to either end, as shown above. Assume that the cord does not slip on the pulley. Determine the answers to parts a. and b. in terms of m, R_1, I_1, and fundamental constants.

a. Determine the tension T in the cord.

b. One block is now removed from the right and hung on the left. When the system is released from rest, the three blocks on the left accelerate downward with an acceleration g/3 . Determine the following.
   i. The tension T_3 in the section of cord supporting the three blocks on the left
   ii. The tension T_1 in the section of cord supporting the single block on the right
   iii. The rotational inertia I_1 of the pulley



c. The blocks are now removed and the cord is tied into a loop, which is passed around the original pulley and a second pulley of radius $2R_1$ and rotational inertia $16I_1$. The axis of the original pulley is attached to a motor that rotates it at angular speed ω_1, which in turn causes the larger pulley to rotate. The loop does not slip on the pulleys. Determine the following in terms of I_1, R_I, and ω_I.
   i. The angular speed ω_2 of the larger pulley
   ii. The angular momentum L_2 of the larger pulley
   iii. The total kinetic energy of the system

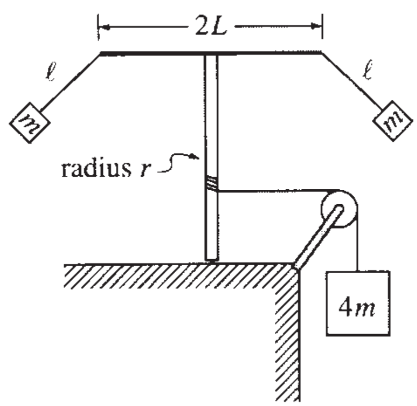


Experiment A

2001M3. A light string that is attached to a large block of mass *4m* passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length 2L, with a small block of mass *m* attached at each end. The rotational inertia of the pole and the rod are negligible.

a. Determine the rotational inertia of the rod-and-block apparatus attached to the top of the pole.
b. Determine the downward acceleration of the large block.
c. When the large block has descended a distance *D,* how does the instantaneous total kinetic energy of the three blocks compare with the value *4mgD*? Check the appropriate space below and justify your answer.

Greater than 4mgD ______ Equal to 4mgD ____ Less than 4mgD _____



Experiment B

The system is now reset. The string is rewound around the pole to bring the large block back to its original location. The small blocks are detached from the rod and then suspended from each end of the rod, using strings of length *l*. The system is again released from rest so that as the large block descends and the apparatus rotates, the small blocks swing outward, as shown in Experiment B above. This time the downward acceleration of the block decreases with time after the system is released.

d. When the large block has descended a distance D, how does the instantaneous total kinetic energy of the three blocks compare to that in part c.? Check the appropriate space below and justify your answer.

Greater before ___ Equal to before ___ Less than before ___

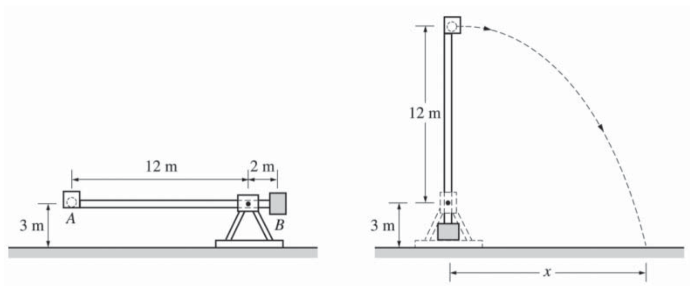


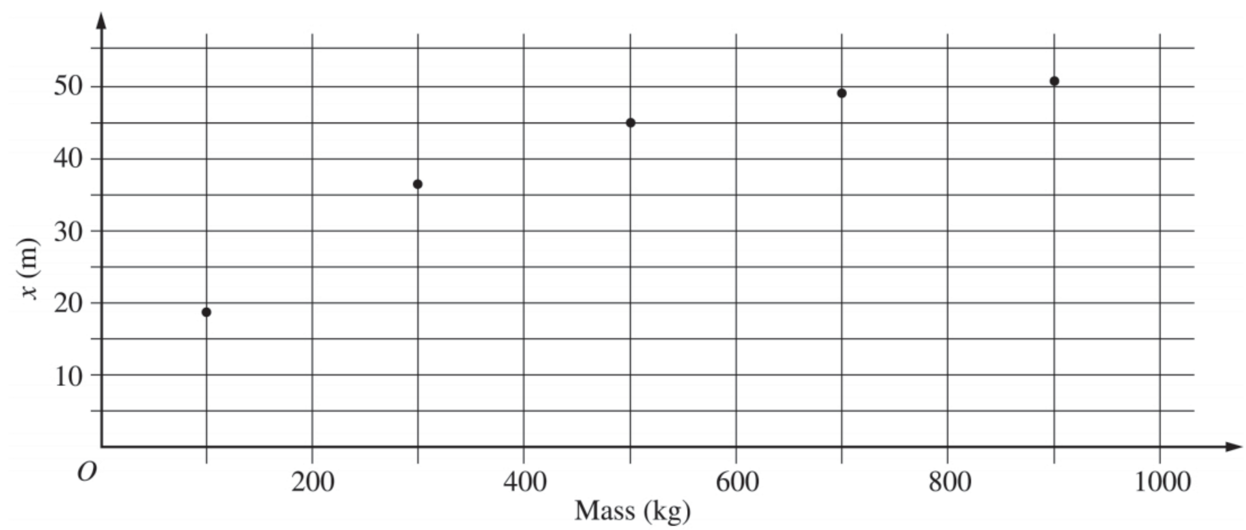
Figure 1 Figure 2

2003M3. Some physics students build a catapult, as shown above. The supporting platform is fixed firmly to the ground. The projectile, of mass 10 kg, is placed in cup *A* at one end of the rotating arm. A counterweight bucket *B* that is to be loaded with various masses greater than 10 kg is located at the other end of the arm. The arm is released from the horizontal position, shown in Figure 1, and begins rotating. There is a mechanism (not shown) that stops the arm in the vertical position, allowing the projectile to be launched with a horizontal velocity as shown in Figure 2.

a. The students load five different masses in the counterweight bucket, release the catapult, and measure the resulting distance x traveled by the 10 kg projectile, recording the following data.

Mass (kg)	100	300	500	700	900
x (m)	18	37	45	48	51

i. The data are plotted on the axes below. Sketch a best-fit curve for these data points.



ii. Using your best-fit curve, determine the distance x traveled by the projectile if 250 kg is placed in the counterweight bucket.

b. The students assume that the mass of the rotating arm, the cup, and the counterweight bucket can be neglected. With this assumption, they develop a theoretical model for x as a function of the counterweight mass using the relationship $x = v_x t$, where v, is the horizontal velocity of the projectile as it leaves the cup and t is the time after launch.

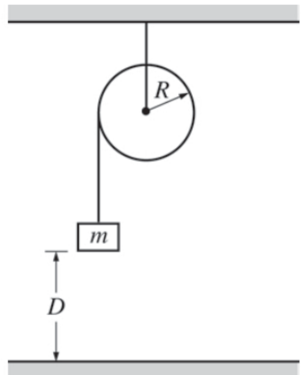
i. How many seconds after leaving the cup will the projectile strike the ground?

ii. Derive the equation that describes the gravitational potential energy of the system relative to the ground when in the position shown in Figure 1, assuming the mass in the counterweight bucket is M.

iii. Derive the equation for the velocity of the projectile as it leaves the cup, as shown in Figure 2.

c. i. Complete the theoretical model by writing the relationship for x as a function of the counterweight mass using the results from b. i and b. iii.

ii. Compare the experimental and theoretical values of x for a counterweight bucket mass of 300 kg. Offer a reason for any difference.



2004M2. A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor.

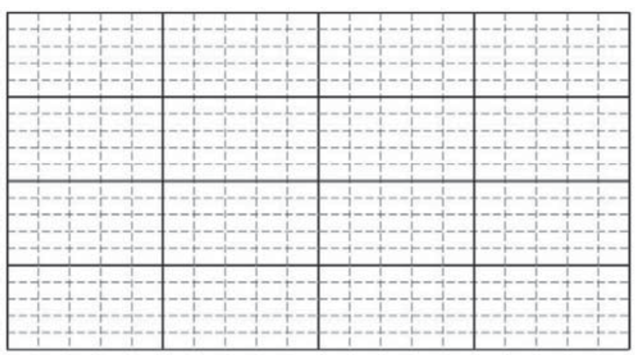
a. Calculate the linear acceleration a of the falling block in terms of the given quantities.

b. The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the acceleration of the block? Explain your reasoning.

ii. On the grid below, plot the quantities determined in b. i., label the axes, and draw the best-fit line to the data.

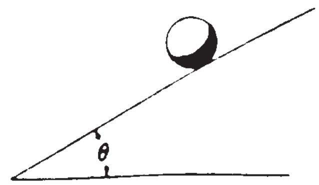


iii. Use your graph to calculate the magnitude of the acceleration.

c. Calculate the rotational inertia of the pulley in terms of m, R, a, and fundamental constants.

d. The value of acceleration found in b.iii, along with numerical values for the given quantities and your answer to c., can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be greater than this value. Give one explanation for this discrepancy.

SECTION C – Rolling



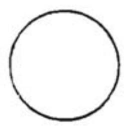
1974M2. The moment of inertia of a uniform solid sphere (mass M, radius R) about a diameter is $2MR^2/5$. The sphere is placed on an inclined plane (angle θ) as shown above and released from rest.

a. Determine the minimum coefficient of friction μ between the sphere and plane with which the sphere will roll down the incline without slipping

b. If μ were zero, would the speed of the sphere at the bottom be greater, smaller, or the same as in part a.? Explain your answer.

1977M2. A uniform cylinder of mass M, and radius R is initially at rest on a rough horizontal surface. The moment of inertia of a cylinder about its axis is $\frac{1}{2}MR^2$. A string, which is wrapped around the cylinder, is pulled upwards with a force T whose magnitude is 0.6Mg and whose direction is maintained vertically upward at all times. In consequence, the cylinder both accelerates horizontally and slips. The coefficient of kinetic friction is 0.5.

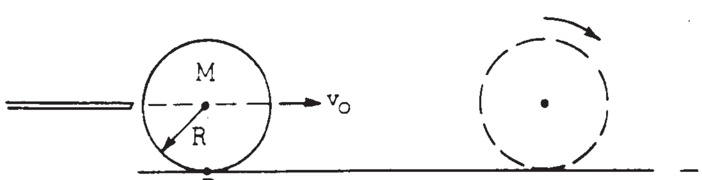
a. On the diagram below, draw vectors that represent each of the forces acting on the cylinder identify and clearly label each force.



b. Determine the linear acceleration a of the center of the cylinder.

c. Calculate the angular acceleration α of the cylinder.

d. Your results should show that a and αR are not equal. Explain.



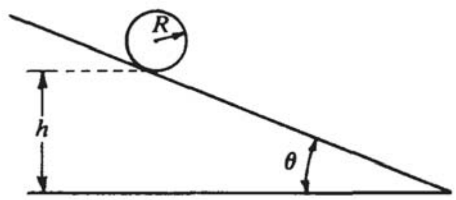
1980M3. A billiard ball has mass M, radius R, and moment of inertia about the center of mass $I_c = 2MR^2/5$
The ball is struck by a cue stick along a horizontal line through the ball's center of mass so that the ball initially slides with a velocity v_o as shown above. As the ball moves across the rough billiard table (coefficient of sliding friction μ_k), its motion gradually changes from pure translation through rolling with slipping to rolling without slipping.

a. Develop an expression for the linear velocity v of the center of the ball as a function of time while it is rolling with slipping.

b. Develop an expression for the angular velocity ω of the ball as a function of time while it is rolling with slipping.

c. Determine the time at which the ball begins to roll without slipping.

d. When the ball is struck it acquires an angular momentum about the fixed point P on the surface of the table. During the subsequent motion the angular momentum about point P remains constant despite the frictional force. Explain why this is so.

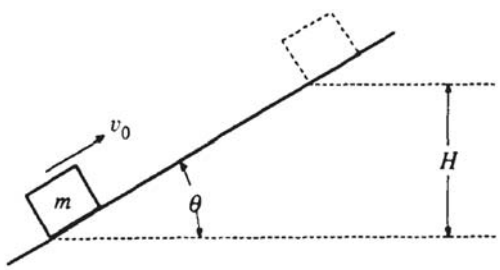


1986M2. An inclined plane makes an angle of θ with the horizontal, as shown above. A solid sphere of radius R and mass M is initially at rest in the position shown, such that the lowest point of the sphere is a vertical height *h* above the base of the plane. The sphere is released and rolls down the plane without slipping. The moment of inertia of the sphere about an axis through its center is $2MR^2/5$. Express your answers in terms of M, R. *h,* g, and θ.

a. Determine the following for the sphere when it is at the bottom of the plane:
   i. Its translational kinetic energy
   ii. Its rotational kinetic energy
b. Determine the following for the sphere when it is on the plane.
   i. Its linear acceleration
   ii. The magnitude of the frictional force acting on it

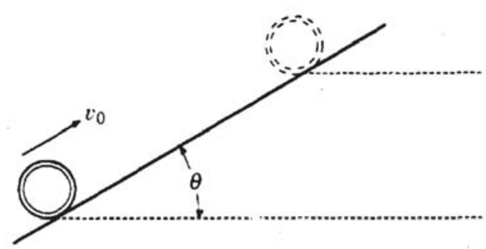
The solid sphere is replaced by a hollow sphere of identical radius R and mass M. The hollow sphere, which is released from the same location as the solid sphere, rolls down the incline without slipping.

c. What is the total kinetic energy of the hollow sphere at the bottom of the plane?
d. State whether the rotational kinetic energy of the hollow sphere is greater than, less than, or equal to that of the solid sphere at the bottom of the plane. Justify your answer.



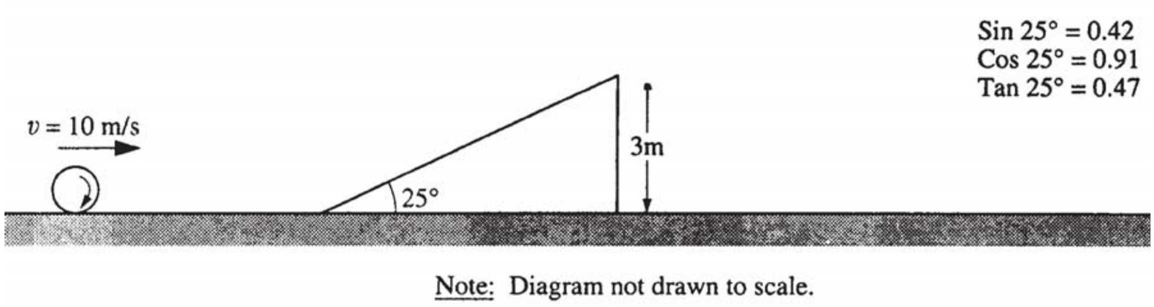
1990M2. A block of mass m slides up the incline shown above with an initial speed v_O in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, determine the maximum height to which the block will rise in terms of H and the given quantities.



A thin hoop of mass m and radius R moves up the incline shown above with an initial speed v_O in the position shown.

c. If the incline is rough and the hoop rolls up the incline without slipping, determine the maximum height to which the hoop will rise in terms of H and the given quantities.
d. If the incline is frictionless, determine the maximum height to which the hoop will rise in terms of H and the given quantities.



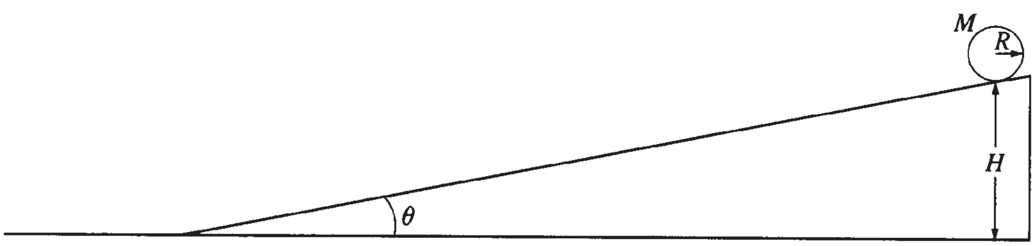
1994M2. A large sphere rolls without slipping across a horizontal surface. The sphere has a constant translational speed of 10 meters per second, a mass m of 25 kilograms, and a radius r of 0.2 meter. The moment of inertia of the sphere about its center of mass is $I = 2mr^2/5$. The sphere approaches a 25° incline of height 3 meters as shown above and rolls up the incline without slipping.

a. Calculate the total kinetic energy of the sphere as it rolls along the horizontal surface.

b. i. Calculate the magnitude of the sphere's velocity just as it leaves the top of the incline.
ii. Specify the direction of the sphere's velocity just as it leaves the top of the incline.

c. Neglecting air resistance, calculate the horizontal distance from the point where the sphere leaves the incline to the point where the sphere strikes the level surface.

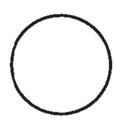
d. Suppose, instead, that the sphere were to roll toward the incline as stated above, but the incline were frictionless. State whether the speed of the sphere just as it leaves the top of the incline would be less than, equal to, or greater than the speed calculated in b. Explain briefly.



1997M3. A solid cylinder with mass M, radius R, and rotational inertia $\frac{1}{2}MR^2$ rolls without slipping down the inclined plane shown above. The cylinder starts from rest at a height H. The inclined plane makes an angle θ with the horizontal. Express all solutions in terms of M, R, H, θ, and g.

a. Determine the translational speed of the cylinder when it reaches the bottom of the inclined plane.

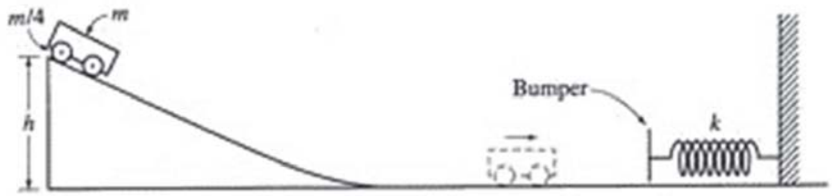
b. On the figure below, draw and label the forces acting on the cylinder as it rolls down the inclined plane Your arrow should begin at the **point of application** of each force.



c. Show that the acceleration of the center of mass of the cylinder while it is rolling down the inclined plane is $(2/3)g \sin\theta$.

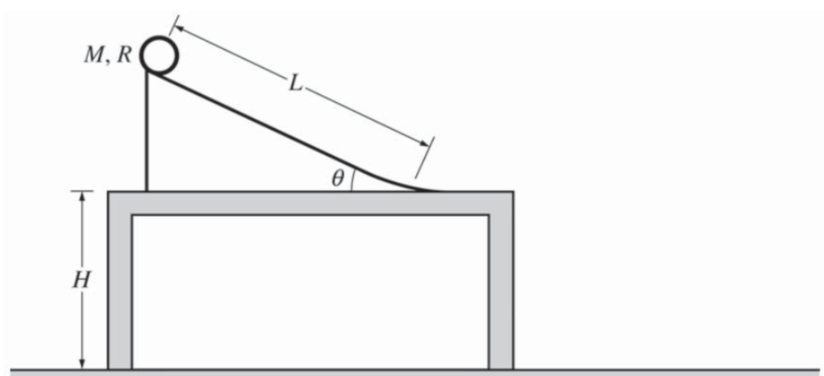
d. Determine the minimum coefficient of friction between the cylinder and the inclined plane that is required for the cylinder to roll without slipping.

e. The coefficient of friction μ is now made less than the value determined in part d., so that the cylinder both rotates and slips.
i. Indicate whether the translational speed of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the translational speed calculated in part a. Justify your answer.
ii. Indicate whether the total kinetic energy of the cylinder at the bottom of the inclined plane is greater than, less than, or equal to the total kinetic energy for the previous case of rolling without slipping. Justify your answer.



2002M2. The cart shown above is made of a block of mass m and four solid rubber tires each of mass m/4 and radius r. Each tire may be considered to be a disk. (A disk has rotational inertia ½ ML^2, where M is the mass and L is the radius of the disk.) The cart is released from rest and rolls without slipping from the top of an inclined plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the total rotational inertia of all four tires.
b. Determine the speed of the cart when it reaches the bottom of the incline.
c. After rolling down the incline and across the horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.
d. Now assume that the bumper has a non-negligible mass. After the collision with the bumper, the spring is compressed to a maximum distance of about 90% of the value of x_m in part c.. Give a reasonable explanation for this decrease.

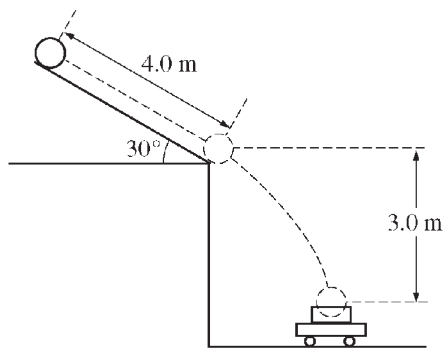


2006M3. A thin hoop of mass *M*, radius *R*, and rotational inertia MR^2 is released from rest from the top of the ramp of length *L* above. The ramp makes an angle θ with respect to a horizontal tabletop to which the ramp is fixed. The table is a height *H* above the floor. Assume that the hoop rolls without slipping down the ramp and across the table. Express all algebraic answers in terms of given quantities and fundamental constants.

a. Derive an expression for the acceleration of the center of mass of the hoop as it rolls down the ramp.
b. Derive an expression for the speed of the center of mass of the hoop when it reaches the bottom of the ramp.
c. Derive an expression for the horizontal distance from the edge of the table to where the hoop lands on the floor.
d. Suppose that the hoop is now replaced by a disk having the same mass *M* and radius *R*. How will the distance from the edge of the table to where the disk lands on the floor compare with the distance determined in part c. for the hoop?

Less than____ The same as____ Greater than____

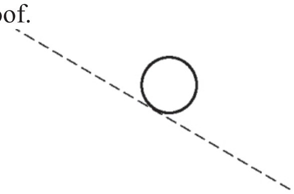
Briefly justify your response.



Note: Figure not drawn to scale.

2010M2. A bowling ball of mass 6.0 kg is released from rest from the top of a slanted roof that is 4.0 m long and angled at 30°, as shown above. The ball rolls along the roof without slipping. The rotational inertia of a sphere of mass M and radius R about its center of mass is $2MR^2/5$.

a. On the figure below, draw and label the forces (not components) acting on the ball at their points of application as it rolls along the roof.

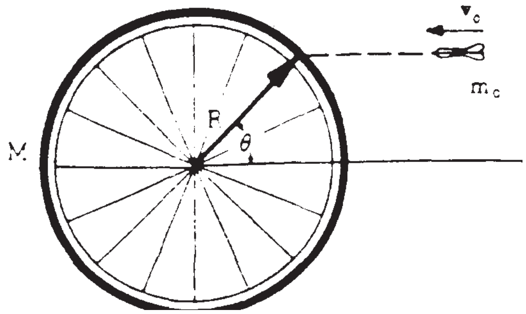


b. Calculate the force due to friction acting on the ball as it rolls along the roof. If you need to draw anything other than what you have shown in part a. to assist in your solution, use the space below. Do NOT add anything to the figure in part a.

c. Calculate the linear speed of the center of mass of the ball when it reaches the bottom edge of the roof.

d. A wagon containing a box is at rest on the ground below the roof so that the ball falls a vertical distance of 3.0 m and lands and sticks in the center of the box. The total mass of the wagon and the box is 12 kg. Calculate the horizontal speed of the wagon immediately after the ball lands in it.

SECTION D – Angular Momentum



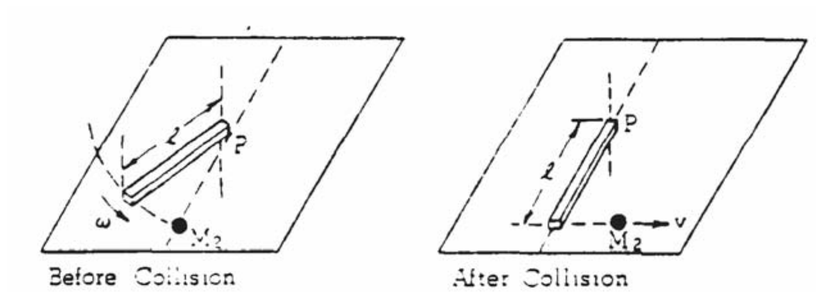
View From Above

1975M2. A bicycle wheel of mass M (assumed to be concentrated at its rim) and radius R is mounted horizontally so it may turn without friction on a vertical axle. A dart of mass m_o is thrown with velocity v_o as shown above and sticks in the tire.

a. If the wheel is initially at rest, find its angular velocity ω after the dart strikes.

b. In terms of the given quantities, determine the ratio:

$$\frac{\text{final kinetic energy of the system}}{\text{initial kinetic energy of the system}}$$



1978M2. A system consists of a mass M_2 and a uniform rod of mass M_1 and length l. The rod is initially rotating with an angular speed ω on a horizontal frictionless table about a vertical axis fixed at one end through point P. The moment of inertia of the rod about P is $Ml^2/3$. The rod strikes the stationary mass M_2. As a result of this collision, the rod is stopped and the mass M_2 moves away with speed v.

a. Using angular momentum conservation determine the speed v in terms of M_1, M_2, l, and ω.

b. Determine the linear momentum of this system just before the collision in terms of M_1, l, and ω.

c. Determine the linear momentum of this system just after the collision in terms of M_1 l, and ω.

d. What is responsible for the change in the linear momentum of this system during the collision?

e. Why is the angular momentum of this system about point P conserved during the collision?

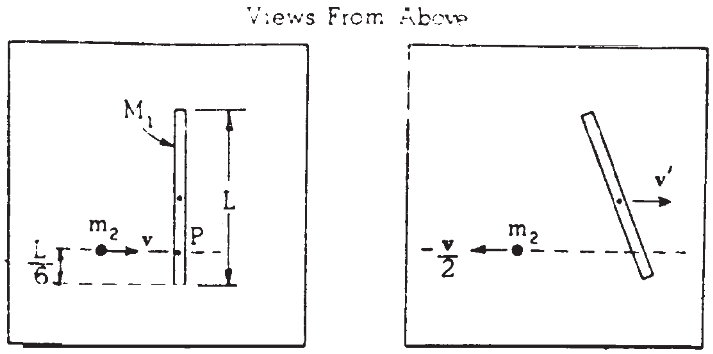


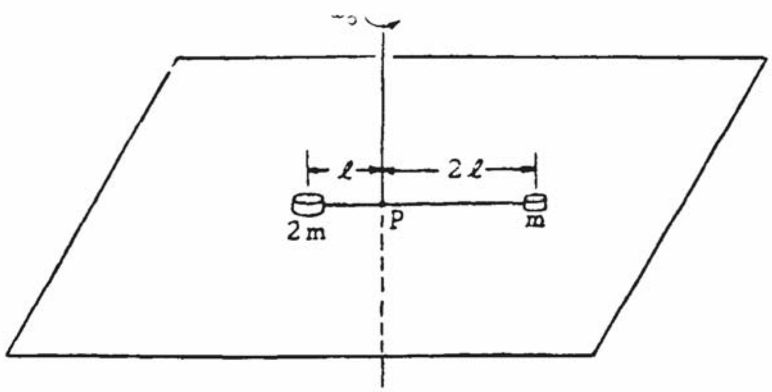
Figure I: Before Figure II: After

1981M3. A thin, uniform rod of mass M_1 and length L , is initially at rest on a frictionless horizontal surface. The moment of inertia of the rod about its center of mass is $M_1L^2/12$. As shown in Figure I, the rod is struck at point P by a mass m_2 whose initial velocity v is perpendicular to the rod. After the collision, mass m_2 has velocity $-\frac{1}{2}\mathbf{v}$ as shown in Figure II. Answer the following in terms of the symbols given.

a. Using the principle of conservation of linear momentum, determine the velocity **v'** of the center of mass of this rod after the collision.

b. Using the principle of conservation of angular momentum, determine the angular velocity ω of the rod about its center of mass after the collision.

c. Determine the change in kinetic energy of the system resulting from the collision.

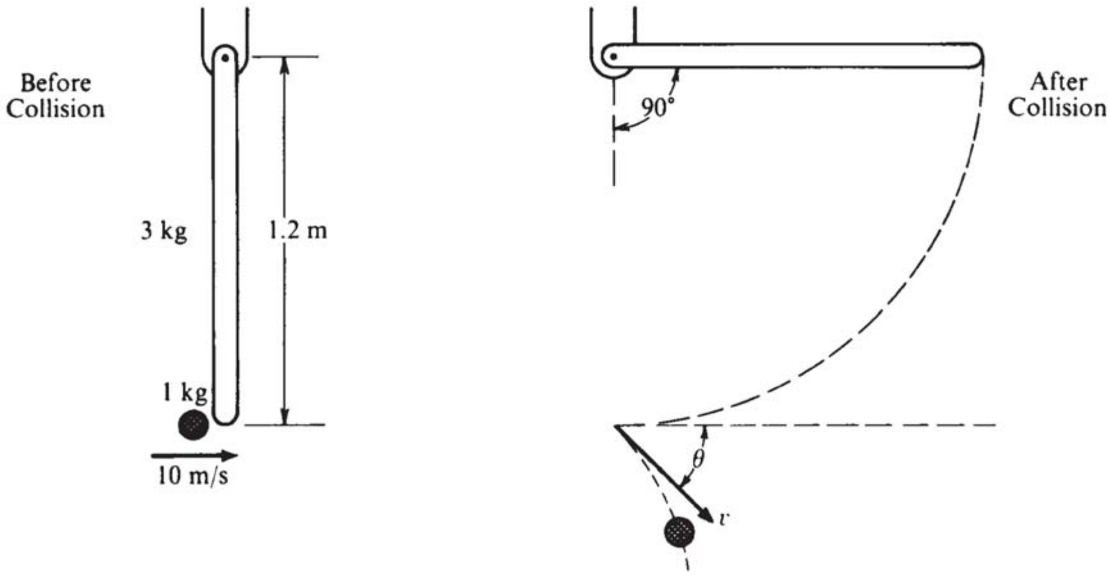


1982M3. A system consists of two small disks, of masses m and 2m, attached to a rod of negligible mass of length $3l$ as shown above. The rod is free to turn about a vertical axis through point P. The two disks rest on a rough horizontal surface; the coefficient of friction between the disks and the surface is μ. At time t = 0, the rod has an initial counterclockwise angular velocity ω_o about P. The system is gradually brought to rest by friction. Develop expressions for the following quantities in terms of μ m, l, g, and ω_o

a. The initial angular momentum of the system about the axis through P

b. The frictional torque acting on the system about the axis through P

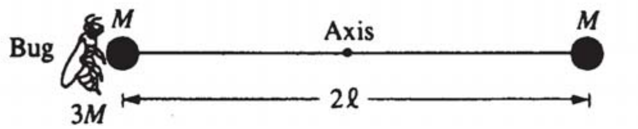
c. The time T at which the system will come to rest.



Note: You may use $g = 10\ m/s^2$.

1987M3. A l.0-kilogram object is moving horizontally with a velocity of 10 meters per second, as shown above, when it makes a glancing collision with the lower end of a bar that was hanging vertically at rest before the collision. For the system consisting of the object and bar, linear momentum is not conserved in this collision, but kinetic energy is conserved. The bar, which has a length l of 1.2 meters and a mass m of 3.0 kilograms, is pivoted about the upper end. Immediately after the collision the object moves with speed v at an angle θ relative to its original direction. The bar swings freely, and after the collision reaches a maximum angle of 90° with respect to the vertical. The moment of inertia of the bar about the pivot is $I_{bar} = ml^2/3$ Ignore all friction.

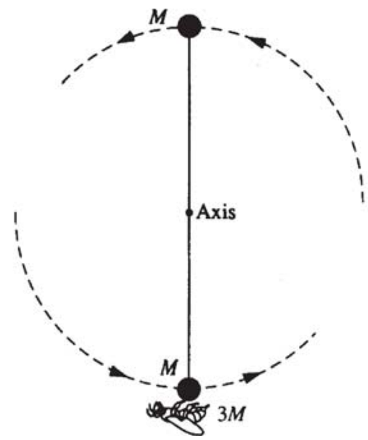
a. Determine the angular velocity of the bar immediately after the collision.
b. Determine the speed v of the l-kilogram object immediately after the collision.
c. Determine the magnitude of the angular momentum of the object about the pivot just before the collision.
d. Determine the angle θ.



1992M2. Two identical spheres, each of mass M and negligible radius, are fastened to opposite ends of a rod of negligible mass and length 2*l*. This system is initially at rest with the rod horizontal, as shown above, and is free to rotate about a frictionless, horizontal axis through the center of the rod and perpendicular to the plane of the page. A bug, of mass 3M*,* lands gently on the sphere on the left. Assume that the size of the bug is small compared to the length of the rod. Express your answers to all parts of the question in terms of M, *l*, and physical constants.

a. Determine the torque about the axis immediately after the bug lands on the sphere.

b. Determine the angular acceleration of the rod-spheres-bug system immediately after the bug lands.

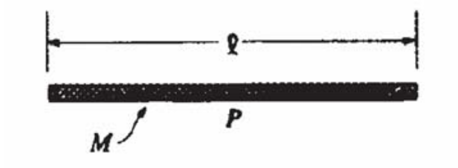


The rod-spheres-bug system swings about the axis. At the instant that the rod is vertical, as shown above, determine each of the following.

c. The angular speed of the bug

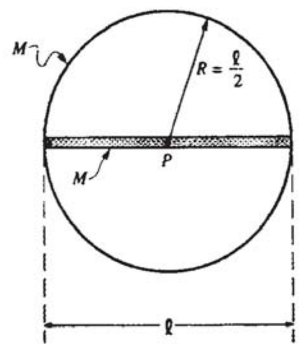
d. The angular momentum of the system

e. The magnitude and direction of the force that must be exerted on the bug by the sphere to keep the bug from being thrown off the sphere



1996M3. Consider a thin uniform rod of mass M and length l, as shown above.

a. Show that the rotational inertia of the rod about an axis through its center and perpendicular to its length is $Ml^2/12$.



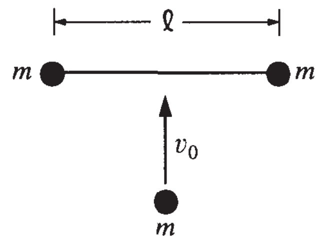
The rod is now glued to a thin hoop of mass M and radius R/2 to form a rigid assembly, as shown above. The centers of the rod and the hoop coincide at point P. The assembly is mounted on a horizontal axle through point P and perpendicular to the page.

b. What is the rotational inertia of the rod-hoop assembly about the axle?

Several turns of string are wrapped tightly around the circumference of the hoop. The system is at rest when a cat, also of mass M, grabs the free end of the string and hangs vertically from it without swinging as it unwinds, causing the rod-hoop assembly to rotate. Neglect friction and the mass of the string.

c. Determine the tension T in the string.
d. Determine the angular acceleration a of the rod-hoop assembly.
e. Determine the linear acceleration of the cat.
f. After descending a distance H = $5l/3$, the cat lets go of the string. At that instant, what is the angular momentum of the cat about point P?

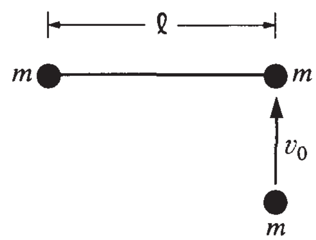
1998M2. A space shuttle astronaut in a circular orbit around the Earth has an assembly consisting of two small dense spheres, each of mass m, whose centers are connected by a rigid rod of length l and negligible mass. The astronaut also has a device that will launch a small lump of clay of mass m at speed v_0 . Express your answers in terms of m, v_0 l. and fundamental constants.



a. Initially, the assembly is "floating" freely at rest relative to the cabin, and the astronaut launches the clay lump so that it perpendicularly strikes and sticks to the midpoint of the rod, as shown above.

i. Determine the total kinetic energy of the system (assembly and clay lump) after the collision.

ii. Determine the change in kinetic energy as a result of the collision.



b. The assembly is brought to rest, the clay lump removed, and the experiment is repeated as shown above, with the clay lump striking perpendicular to the rod but this time sticking to one of the spheres of the assembly.

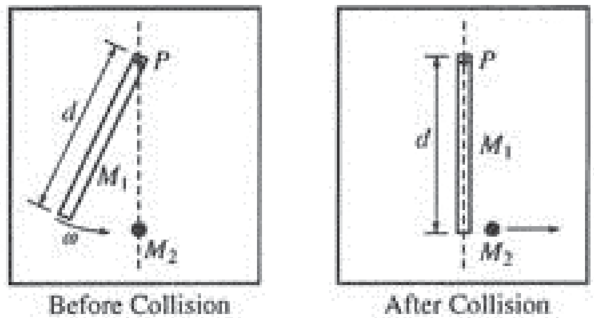
i. Determine the distance from the left end of the rod to the center of mass of the system (assembly and clay lump) immediately after the collision. (Assume that the radii of the spheres and clay lump are much smaller than the separation of the spheres.)

ii. On the figure above, indicate the direction of the motion of the center of mass immediately after the collision.

iii. Determine the speed of the center of mass immediately after the collision.

iv. Determine the angular speed of the system (assembly and clay lump) immediately after the collision.

v. Determine the change in kinetic energy as a result of the collision.



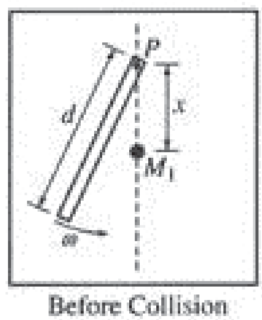
TOP VIEWS

2005M3. A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3} M_1 d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

a. Derive an expression for the angular momentum of the rod about point P before the collision.

b. Derive an expression for the speed v of the ball after the collision.

c. Assuming that this collision is elastic, calculate the numerical value of the ratio M_1 / M_2



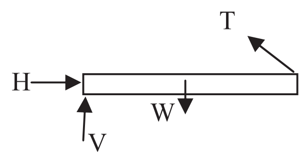
d. A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

SECTION A – Torque and Statics

Solution — Answer

1. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed — **A**

2. FBD — **B**



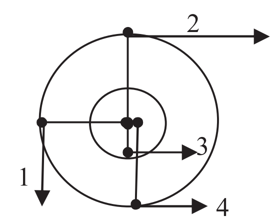
Since the rope is at an angle it has x and y components of force.
Therefore, H would have to exist to counteract T_x.
Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar

3. Applying rotational equilibrium to each diagram gives — **D**

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$
$L_1 = M_1(L_2)/m$
(sub this L_1) into the Diagram 2 eqn, and solve.

DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$
$M_2(L_1) = m(L_2)$

4. Find the torques of each using proper signs and add up. — **B**
$+(1) - (2) + (3) + (4)$
$+F(3R) - (2F)(3R) + F(2R) + F(3R) = 2FR$



5. Simply apply rotational equilibrium — **B**
$(m_1g) \cdot r_1 = (m_2g) \cdot r_2$
$m_1a = m_2b$

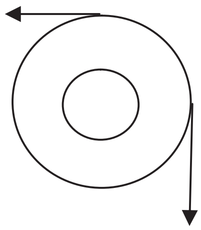
SECTION B – Rotational Kinematics and Dynamics

1. $I_{tot} = \Sigma I = I_0 + I_M = I_0 + M(\frac{1}{2}L)^2$ — **A**
2. $\Sigma\tau = I\alpha$ where $\Sigma\tau = (3M_0)(l) - (M_0)(2l) = M_0l$ and $I = (3M_0)(l)^2 + (M_0)(2l)^2 = 7M_0l^2$ — **A**
3. $\tau_X = Fl$; $\tau_O = F_OL_O \sin\theta$, solve for the correct combination of F_O and L_O — **C**
4. Just as the tension in a rope is greatest at the bottom of as vertical circle, the force needed to maintain circular motion in any vertical circle is greatest at the bottom as the applied force must balance the weight of the object and additionally provide the necessary centripetal force — **C**
5. $\Sigma F_{bottom} = F_{adhesion} - mg = F_{centripetal} = m\omega^2r$ — **D**
6. For one complete revolution $\theta = 2\pi$; $\omega^2 = \omega_0^2 + 2\alpha\theta$ — **C**

7. $\tau = \Delta L/\Delta t = (I\omega_f - 0)/T$ D

8. $P_{avg} = \tau\omega_{avg} = (I\omega_f/T)(\frac{1}{2}\omega_f)$ or $P_{avg} = \Delta K/T$ B

9. C



$\Sigma\tau = T_2R - T_1R = I\alpha$

10. If the cylinder is "suspended in mid air" (i.e. the linear acceleration is zero) then $\Sigma F = 0$ D

11. $\Sigma\tau = TR = I\alpha = \frac{1}{2} MR^2\alpha$ which gives $\alpha = 2T/MR$ and since $\Sigma F = 0$ then $T = Mg$ so $\alpha = 2g/R$ the acceleration of the person's hand is equal to the linear acceleration of the string around the rim of the cylinder $a = \alpha R = 2g$ B

12. In order that the mass not slide down $f = \mu F_N \geq mg$ and $F_N = m\omega^2R$ solving for μ gives $\mu \geq g/\omega^2R$ A

SECTION C – Rolling

1. $K_{tot} = K_{rot} + K_{trans} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2 = \frac{1}{2} (2/5)MR^2\omega^2 + \frac{1}{2} Mv^2 = (1/5)Mv^2 + \frac{1}{2} Mv^2 = (7/10)Mv^2 = MgH$, solving gives $H = 7v^2/10g$ D

2. $Mgh = K_{tot} = K_{rot} + K_{trans}$, however without friction, there is no torque to cause the sphere to rotate so $K_{rot} = 0$ and $Mgh = \frac{1}{2} Mv^2$ A

3. $Mgh = K_{tot} = K_{rot} + K_{trans} = \frac{1}{2} I\omega^2 + \frac{1}{2} Mv^2$; substituting v/r for ω gives $Mgh = \frac{1}{2}(I/r^2 + M)v^2$ and solving for v gives $v^2 = 2Mgh/(I/r^2 + M)$, multiplying by r^2/r^2 gives desired answer D

4. The first movement of the point of contact of a rolling object is vertically upward as there is no side to side (sliding) motion for the point in contact A

SECTION D – Angular Momentum

1. $L_i = L_f$ so $I_i\omega_I = I_f\omega_f$ and since $I_f < I_i$ (mass more concentrated near axis), then $\omega_f > \omega_i$ The increase in ω is in the same proportion as the decrease in I, and the kinetic energy is proportional to $I\omega^2$ so the increase in ω results in an overall increase in the kinetic energy. Alternately, the skater does work to pull their arms in and this work increases the KE of the skater — C

2. $L = mvr_\perp$ where $r_\perp$ is the perpendicular line joining the origin and the line along which the particle is moving — B

3. $L = I\omega$ and since ω is uniform the ratio $L_{upper}/L_{lower} = I_{upper}/I_{lower} = 2mL^2/2(2m)(2L)^2 = 1/8$ — D

4. Since it is a perfectly inelastic (sticking) collision, KE is not conserved. As there are no external forces or torques, both linear and angular momentum are conserved — D

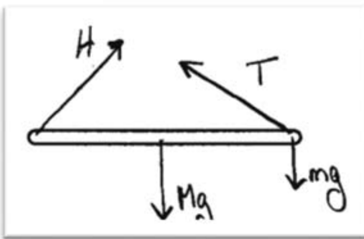
5. As there are no external forces or torques, both linear and angular momentum are conserved. As the type of collision is not specified, we cannot say kinetic or mechanical energy *must* be the same. — A/B

WARNING: These are AP Physics C Free Response Practice – Rotation – ANSWERS
Use with caution!

SECTION A – Torque and Statics

2008M2

a.



b. $\Sigma\tau = 0$
About the hinge: TL sin 30º – mgL – Mg(L/2) = 0 gives T = 29 N

c. $I_{total} = I_{rod} + I_{block}$ where $I_{rod,end} = I_{cm} + MD^2 = ML^2/12 + M(L/2)^2 = ML^2/3$
$I_{total} = ML^2/3 + mL^2 = 0.42$ kg-m^2

d. $\Sigma\tau = I\alpha$
$mgL + MgL/2 = I\alpha$ gives $\alpha = 21$ rad/s^2

SECTION B – Rotational Kinematics and Dynamics

1973M3

a. Define a coordinate system with the x-axis directed to the vertical rod and the y-axis directed upwards and perpendicular to the first. Let T_1 be the tension in the horizontal string. Let T_2 be the tension in the string tilted upwards.
Applying Newton's Second Law: $\Sigma F_x = T_1 + T_2 \sin\theta = m\omega^2 R$; $\Sigma F_y = T_2 \cos\theta - mg = 0$
Solving yields: $T_2 = mg/\cos\theta$ and $T_1 = m(\omega^2 R - g \tan\theta)$

b. Let $T_1 = 0$ and solving for ω gives $\omega = (g \tan\theta/R)^{1/2}$

1976M2

a.

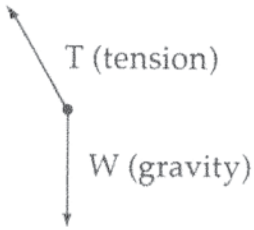
b. $\Sigma\tau = I\alpha$ (about center of mass) (one could also choose about the point at which the tape comes off the cylinder)
$TR = ½ MR^2 \times (a/R)$
$T = ½ Ma$

$\Sigma F = ma$
$Mg - T = Ma$
$Mg = 3Ma/2$
$a = 2g/3$

c. As there are no horizontal forces, the cylinder moves straight down.

<u>1978M1</u>

a.



b. $\Sigma F = ma$; $T \cos\theta = mg$ and $T \sin\theta = m\omega^2 r = m\omega^2(A + \ell \sin\theta)$

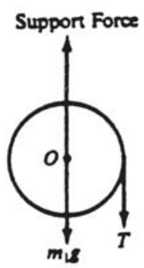
$$\omega = \sqrt{\frac{g \tan\theta}{A + l \sin\theta}}$$

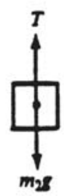
c. $W = \Delta E = \Delta K + \Delta U = \frac{1}{2} mv^2 + mg\ell(1 - \cos\theta)$ for each rider

$W = 6(\frac{1}{2} mv^2 + mg\ell(1 - \cos\theta))$

<u>1983M2</u>

a.





b. i./ii. On the disk: $\Sigma\tau = I\alpha = TR = \frac{1}{2} m_1R^2\alpha$

For the block $a = \alpha R$ so $\alpha = a/R$ and $\Sigma F = m_2g - T = m_2a$

Solving yields

$$a = \frac{2m_2g}{2m_2 + m_1} \text{ and } T = \frac{m_1m_2g}{2m_2 + m_1}$$

1985M3

a.

b. i. $\Sigma F = ma$; $T_1 - mg = ma$ and $2mg - T_2 = 2ma$
ii. $\Sigma\tau = I\alpha$; $(T_2 - T_1)r = I\alpha$

c. $\alpha = a/r$
Combining equations from b.i. gives $T_2 - T_1 = mg - 3ma$
Substituting for $(T_2 - T_1)$ into torque equation gives $a = 2g/9$

d. $T_1 = m(g + a) = 11mg/9$

e. $F_N = 7mg + T_1 + T_2$ (the table counters all the downward forces on the *apparatus*)
$T_2 = 2m(g - a) = 14mg/9$
$F_N = 88mg/9$

1988M3

a. I is proportional to mR^2; masses are equal and R becomes 2R
$I_{2R} = 4I$

b. The disks are coupled by the chain along their rims, which means the linear motion of the rims have the same displacement, velocity and acceleration.
$v_R = v_{2R}$; $R\omega_R = 2R\omega_{2R}$; $R\alpha t = 2R\alpha_{2R}t$ gives $\alpha_{2R} = \alpha/2$

c. $\tau_{2R} = T(2R) = I_{2R}\alpha_{2R} = (4I)(\alpha/2) = 2I\alpha$ giving $T = I\alpha/R$

d. $\Sigma\tau = \tau_{student} - TR = I\alpha$
$\tau_{student} = I\alpha + TR = I\alpha + (I\alpha/R)R = 2I\alpha$

e. $K = ½ I\omega^2 = ½ I(\alpha t)^2$

1989M2

a. $\Sigma F = ma$; $2Mg - T_v = 2Ma$ so $T_v = 2M(g - a)$

b. $\Sigma\tau = T_vR - T_hR = I\alpha = 3MR^2(a/R)$
$T_h = (T_vR - 3MRa)/R = 2M(g - a) - 3Ma = 2Mg - 5Ma$

c. $F_f = \mu F_N = \mu(4Mg)$
$T_h - F_f = 3Ma$
$2Mg - 5Ma - 4\mu Mg = 3Ma$
$4\mu Mg = 2Mg - 8Ma$
$\mu = (2g - 8a)/4g$
plugging in given values gives $\mu = 0.1$

d. $F_f = 4\mu Mg = ma_C = 4Ma_C$
$a_C = 1\ m/s^2$

1991M2

a. $\Sigma\tau = 0$; $m_2gr_2 = m_1gr_1$; $m_2 = m_1r_1/r_2 = 6.67$ kg

b./c. $\tau = I\alpha$; $Tr_1 = (45\ kg\text{-}m^2)\alpha$
$\Sigma F = ma$; $(20\ kg)g - T = (20\ kg)a$
Combining with $a = \alpha r$ gives $\alpha = 2\ rad/s^2$ and $T = 180$ N

d. $mgh = ½ mv^2 + ½ I\omega^2 = ½ mv^2 + ½ I(v^2/r^2)$ giving $v = 1.4$ m/s

1993M3

a. $\Sigma\tau = F_a\ell - Mg\ell/2 = 0$ giving $F_a = Mg/2$

b. $\Sigma\tau = Mg\ell/2 = I\alpha = (M\ell^2/3)\alpha$; $\alpha = 3g/2\ell$

c. $a = \alpha r$ where $r = \ell/2$

$a = (3g/2\ell)(\ell/2) = 3g/4$

d. $\Sigma F = Ma$; $Mg - F_a = Ma = M(3g/4)$

$F_a = Mg/4$

e. $\Delta U = \Delta K_{rot}$

$mgh = mg(\ell/2)\sin\theta = \frac{1}{2} I\omega^2 = \frac{1}{2}(M\ell^2/3)\omega^2$

solving gives $\omega = (3g\sin\theta/\ell)^{1/2}$

1999M3

a. $\Sigma\tau = 0$ so $\tau_{cw} = \tau_{ccw}$ and $\tau_{cw} = TR$ (from the string) so we just need to find τ_{ccw} as the sum of the torques from the various parts of the system

$\Sigma\tau_{ccw} = \tau_{rod} + \tau_{block} = mgR\sin\theta_0 + 2mg(2R)\sin\theta_0 = 5mgR\sin\theta_0 = TR$ so $T = 5mg\sin\theta_0$

b. i. $I = I_{disk} + I_{rod} + I_{block} = 3mR^2/2 + 4mR^2/3 + 2m(2R)^2 = 65mR^2/6$

$\alpha = \tau/I = (5mgR\sin\theta_0)/(65mR^2/6) = 6g\sin\theta_0/13R$

ii. $a = \alpha r$ where $r = 2R$ so $a = 12g\sin\theta_0/13$

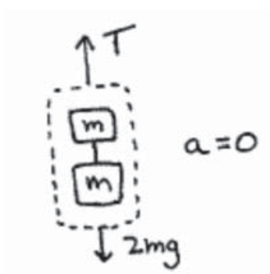
c. ΔU (from each component) $= K = \frac{1}{2} I\omega^2$

$mgR\cos\theta_0 + 2mg(2R)\cos\theta_0 = \frac{1}{2}(65mR^2/6)\omega^2$

$\omega = (12g\cos\theta_0/13R)^{1/2}$ and $v = \omega r = \omega(2R) = 4(3gR\cos\theta_0/13)^{1/2}$

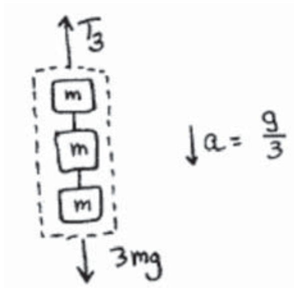
2000M3

a.



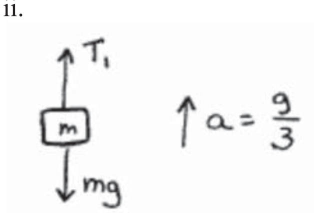
$\Sigma F = ma = 0$ so $T = 2mg$

b. i.



$\Sigma F = ma$
$3mg - T_3 = 3m(g/3)$
$T_3 = 2mg$

ii.



$\Sigma F = ma$
$T_1 - mg = m(g/3)$
$T_1 = 4mg/3$

iii. $\Sigma\tau = (T_3 - T_1)R_1 = I\alpha$ and $\alpha = a/R_1 = g/3R_1$
$(2mg - 4mg/3)R_1 = I_1(g/3R_1)$
$I_1 = 2mR_1^2$

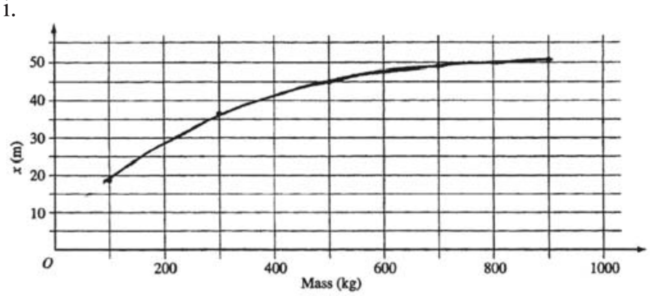
c. i. Tangential speeds are equal; $\omega_1 R_1 = \omega_2 R_2 = \omega_2(2R_1)$ therefore $\omega_2 = \omega_1/2$
ii. $L = I\omega = (16I_1)(\omega_1/2) = 8I_1\omega_1$
iii. $K = ½ I_1\omega_1^2 + ½ I_2\omega_2^2 = (5/2)I_1\omega_1^2$

2001M3

a. $I = \Sigma mr^2 = mL^2 + mL^2 = 2mL^2$
b. $\Sigma F = ma$; $4mg - T = 4ma$
$\Sigma\tau = I\alpha$; $Tr = I\alpha$; $T = I\alpha/r = 4mg - 4ma$ and $\alpha = a/r$, solving gives $a = 2gr^2/(L^2 + 2r^2)$
c. Equal, total energy is conserved
d. Less, the small blocks rise and gain potential energy. The total energy available is still 4mgD, therefore the kinetic energy must be less than in part c.

2003M3

a. i.



ii. x = 33 m

b. i. $y = ½ gt^2$; $t = (2y/g)^{1/2} = 1.75$ s
ii. $U_{initial} = U_{bucket} + U_{projectile}$ = M(9.8 m/s^2)(3 m) + (10 kg)(9.8 m/s^2)(3 m) = 29.4M + 294
iii. $U_{initial} = U_{final} + K$ where U_{final} = Mg(1 m) + (10 kg)g(15 m) = 9.8M + 1470
$K_{projectile} = ½\ 10v_x^2$ and $K_{bucket} = ½\ Mv_b^2$ where $v_b = v_x/6$
putting it all together gives $29.4M + 294 = 9.8M + 1470 + 5v_x^2 + (M/72)v_x^2$

$$v_x = \sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

c. i. $x = v_x t$

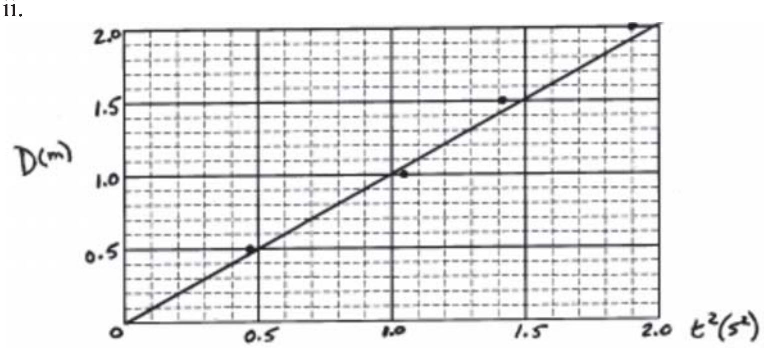
$$x = 1.75\sqrt{\frac{19.6M - 1176}{5 + M/72}}$$

d. x(300 kg) = 39.7 m (greater than the experimental value)
possible reasons include friction at the pivot, air resistance, neglected masses not negligible

2004M2

a. $x = v_0t + ½\ at^2$
x = D and $v_0 = 0$ so $D = ½\ at^2$ and $a = 2D/t^2$

b. i. graph D vs. t^2 (as an example)
ii.



iii. a = 2(slope) = 2.04 m/s^2

c. $\Sigma\tau = TR = I\alpha$ and $\alpha = a/R$ so $I = TR^2/a$
$\Sigma F = mg - T = ma$ so $T = m(g - a)$
$I = m(g - a)R^2/a = mR^2((g/a) - 1)$

d. The string was wrapped around the pulley several times, causing the effective radius at which the torque acted to be larger than the radius of the pulley used in the calculation.

The string slipped on the pulley, allowing the block to accelerate faster than it would have otherwise, resulting in a smaller experimental moment of inertia.

Friction is not a correct answer, since the presence of friction would make the experimental value of the moment of inertia too large

SECTION C – Rolling

NOTE: Rolling problems may be solved considering rotation about the center of mass or the point of contact. The solutions below will only show one of the two methods, but for most, if not all cases, the other method is applicable. When considering rotation about the point of contact, remember to use the parallel axis theorem for the moment of inertia of the rolling object.

1974M2

a. Torque provided by friction; at minimum μ, $F_f = \mu F_N = \mu Mg \cos\theta$
   $\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma = \mu Mg \cos\theta$ giving $a = (5/2)\mu g \cos\theta$
   $\Sigma F = Ma$; $Mg \sin\theta - \mu Mg \cos\theta = Ma = (5/2)\mu Mg \cos\theta$ giving $\mu = (2/7)\tan\theta$
b. Energy at the bottom is the same in both cases, however with $\mu = 0$, there is no torque and no energy in rotation, which leaves more (all) energy in translation and velocity is higher

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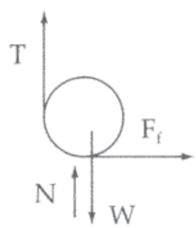
1977M2

a.



b. $\Sigma F_y = 0$; $T + N = W$; $N = W - T = Mg - (3/5)Mg = (2/5)Mg$
   $\Sigma F_x = ma$; $F_f = ma$; $\mu N = ma$; $½ (2/5)Mg = Ma$; $a = g/5$
c. $\Sigma\tau = I\alpha$; $(T - F_f)R = ½ MR^2\alpha$
   $(3/5)Mg - (1/5)Mg = ½ MR\alpha$
   $(2/5)g = ½ R\alpha$
   $\alpha = 4g/5R$
d. The cylinder is slipping on the surface and does not meet the condition for pure rolling

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1980M3

a. $\Sigma F = ma$; $F_f = \mu F_N$;
   $-\mu Mg = Ma$
   $a = -\mu g$
   $v = v_0 + at$
   $v = v_0 - \mu gt$
b. $\tau = I\alpha$ where the torque is provided by friction $F_f = \mu Mg$
   $\mu MgR = (2MR^2/5)\alpha$
   $\alpha = (5\mu g/2R)$
   $\omega = \omega_0 + \alpha t = (5\mu g/2R)t$
c. Slipping stops when the tangential velocity si equal to the velocity of the center of mass, or the condition for pure rolling has been met: $v(t) = \omega(t)R$
   $v_0 - \mu gt = R(5g/2R)t$, which gives $T = (2/7)(v_0/\mu g)$
d. Since the line of action of the frictional force passes through P, the net torque about point P is zero. Thus, the time rate of change of the angular momentum is zero and the angular momentum is constant.

1986M2

a. $U = K$

$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$ and $\omega = v/R$

$Mgh = \frac{1}{2} Mv^2 + \frac{1}{2} (2/5)MR^2(v/R)^2 = \frac{1}{2} Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$

i. $K_{trans} = \frac{1}{2} Mv^2 = (5/7)Mgh$

ii. $K_{rot} = \frac{1}{2} I\omega^2 = (2/7)Mgh$ (or $Mgh - K_{trans}$)

b. i. $\tau = F_fR = I\alpha = I(a/R)$

$F_fR = (2/5)MR^2(a/R)$

$F_f = (2/5)Ma$

$\Sigma F = ma$

$Mg \sin\theta - F_f = Ma$

$Mg \sin\theta - (2/5)Ma = Ma$

$g \sin\theta = (7/5)\, a$

$a = (5/7)\, g \sin\theta$

ii. $F_f = (2/5)Ma = (2/5)M(5/7)g \sin\theta = (2/7)Mg \sin\theta$

c. $K_{tot} = Mgh$

d. Greater, the moment of inertia of the hollow sphere is greater and will be moving slower at the bottom of the incline. Since the translational speed is less, the translational KE is taking a smaller share of the same total energy as the solid sphere.

1990M2

a. $K = U$

$\frac{1}{2} mv_0^2 = mgH$; $H = v_0^2/2g$

b. $K + W_f = U$ where $W_f = -F_fd$ and $F_f = \mu mg \cos\theta$ and $d = h/\sin\theta$

$\frac{1}{2} mv_0^2 - (\mu mg \cos\theta)(h/\sin\theta) = mgh$

$\frac{1}{2} mv_0^2 = mgh(\mu \cot\theta + 1)$

$h = v_0^2/(2g(\mu \cot\theta + 1)) = H/(\mu \cot\theta + 1)$

c. $K_{trans} + K_{rot} = U$ where $K_{rot} = \frac{1}{2} I\, \omega^2 = \frac{1}{2} (mR^2)(v/R)^2 = \frac{1}{2} mv_0^2$

$\frac{1}{2} mv_0^2 + \frac{1}{2} mv_0^2 = mgh'$

$h' = v_0^2/g = 2H$

d. Rotational energy will not change therefore $\frac{1}{2} mv_0^2 = mgh''$ and $h'' = v_0^2/2g = H$

1994M2

a. $K_{tot} = K_{trans} + K_{rot} = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$ and $\omega = v/R$

$K_{tot} = \frac{1}{2} Mv^2 + \frac{1}{2} (2/5)MR^2(v/R)^2 = \frac{1}{2} Mv^2 + (1/5)Mv^2 = 7Mv^2/10 = 1750$ J

b. i. $K_{total,bottom} = K_{top} + U_{top} = 7Mv_{top}^2/10 + Mgh$; $v_{top} = 7.56$ m/s

ii. It is directed parallel to the incline: 25º

c. $y = y_0 + v_{oy}t + \frac{1}{2} a_yt^2$

$0 \text{ m} = 3 \text{ m} + (7.56 \text{ m/s})(\sin 25^\circ)t + \frac{1}{2} (-9.8 \text{ m/s}^2)t^2$ which gives $t = 1.16$ s (positive root)

$x = v_xt = (7.56 \text{ m/s})(\cos 25^\circ)(1.16 \text{ s}) = 7.93$ m

d. The speed would be less than in b.

The gain in potential energy is entirely at the expense of the translational kinetic energy as there is no torque to slow the rotation.

1997M3

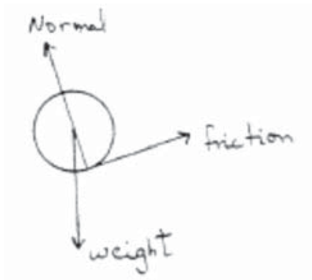
a. $U = K$

$MgH = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$ and $\omega = v/R$

$MgH = \frac{1}{2} Mv^2 + \frac{1}{2} (\frac{1}{2})MR^2(v/R)^2 = \frac{1}{2} Mv^2 + \frac{1}{4} Mv^2 = 3Mv^2/4$

$v = (4gH/3)^{1/2}$

b.



c. For a change of pace, we can use kinematics:
$v_f^2 = v_i^2 + 2ad$
$4gH/3 = 0 + 2a(H/\sin\theta)$
$a = (2/3)g \sin\theta$

d. $\Sigma F = Ma$
$Mg \sin\theta - F_f = Ma = M(2/3)g \sin\theta$
$Mg \sin\theta - \mu Mg \cos\theta = (2/3)Mg \sin\theta$
$\mu \cos\theta = (1/3) \sin\theta$
$\mu = (1/3) \tan\theta$

e. i. The translational speed is greater, less energy is transferred to the rotational motion so more goes into the translational motion. Additionally, with a smaller frictional force, the translational acceleration is greater.
ii. Total kinetic energy is less. Energy is dissipated as heat due to friction.

<u>2002M2</u>

a. For each tire: $I = ½ ML^2 = ½ (m/4)r^2$
$I_{total} = 4 \times I = ½ mr^2$

b. U = K; total mass = 2m
$2mgh = ½ (2m)v^2 + ½ I\omega^2$ and $\omega = v/R$
$2mgh = mv^2 + ½ (½)mr^2(v/r)^2 = ½ mv^2 + (¼)mv^2 = 5mv^2/4$
$v = (8gh/5)^{1/2}$

c. $U_g = U_s$
$2mgh = ½ kx_m^2$; $x_m = 2(mgh/k)^{1/2}$

d. In an inelastic collision, energy is lost. With less energy after the collision, the spring is compressed less.

<u>2006M3</u>

a. $\Sigma\tau = I\alpha$
$F_fR = I\alpha = MR^2(a/R)$; $F_f = Ma$
$\Sigma F = ma$
$Mg \sin\theta - F_f = Ma$
$Mg \sin\theta - Ma = Ma$
$a = ½ g \sin\theta$

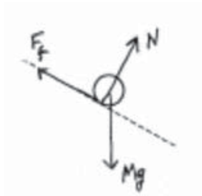
b. $v_f^2 = 2aL = gL \sin\theta$
$v_f = (gL \sin\theta)^{1/2}$

c. $H = ½ gt^2$; $t = (2H/g)^{1/2}$
$d = v_xt = (gL \sin\theta)^{1/2}(2H/g)^{1/2} = (2LH \sin\theta)^{1/2}$

d. Greater. A disk will have smaller rotational inertia and will therefore have a greater rotational velocity. This will lead to a greater translational velocity, and a greater distance *x*.

2010M2

a.



b. Torque provided by friction; $F_f = \mu F_N = \mu Mg\cos\theta$

$\tau = F_f R = I\alpha = (2/5)MR^2(a/R)$; $F_f = (2/5)Ma$; $Ma = (5/2)F_f$

$\Sigma F = Ma$

$Mg\sin\theta - F_f = (5/2)F_f$

$F_f = (2/7)Mg\sin\theta = 8.4$ N

c. $Mgh = ½ Mv^2 + ½ I\omega^2$ and $\omega = v/R$

$Mgh = ½ Mv^2 + ½ (2/5)MR^2(v/R)^2 = ½ Mv^2 + (1/5)Mv^2 = 7Mv^2/10$

$v^2 = 10gh/7$; $v = 5.3$ m/s

d. The horizontal speed of the wagon is due to the horizontal component of the ball in the collision:

$M_i v_{ix} = M_f v_{fx}$; where $M_f = M_{ball} + M_{wagon} = 18$ kg

(6 kg)(5.3 m/s)(cos 30º) = (18 kg)v_f

$v_f = 1.5$ m/s

SECTION D – Angular Momentum

1975M2

a. $L_i = L_f$

$m_0 v_0 R\sin\theta = I\omega$

$\omega = m_0 v_0 R\sin\theta/I$; $I = (M + m_0)R^2$

$\omega = m_0 v_0 \sin\theta/(M + m_0)R$

b. $K_i = ½ m_0 v_0^2$

$K_f = ½ I\omega^2 = ½ (M + m_0)R^2(m_0 v_0 \sin\theta/(M + m_0)R)^2$

$K_f/K_i = m_0 \sin^2\theta/(M + m_0)$

1978M2

a. $L_i = L_f$

$I\omega = mvr$

$(1/3)M_1\ell^2\omega = M_2 v\ell$

$v = M_1\ell\omega/3M_2$

b. $p_{system} = p_{cm\ of\ rod} = M_1 v_{cm} = M_1\omega(\ell/2)$

c. $P_f = M_2 v_f = M_1\omega\ell/3M_2$

d. There is a net external force on the system from the axis at point P.

e. Since the net external force acts at point P (the pivot), the net torque about point P is zero, hence angular momentum is conserved.

1981M3

a. $m_2 v = m_2(-v/2) + M_1 v'$

$v' = 3m_2 v/2M_1$

b. $L_i = L_f$

$m_2 v(L/3) = m_2(-v/2)(L/3) + (1/12)M_1 L^2\omega$

$\omega = 6m_2 v/M_1 L$

c. $\Delta K = K_f - K_i = ½\, m_2(-v/2)^2 + ½\, M_1 v'^2 + ½\, I\omega^2 - ½\, m_2 v^2$
$= -3m_2v^2/8 + 21m_2^2v^2/8M_1$

<u>1982M3</u>

a. $L = I\omega$ where $I = \Sigma mr^2 = (2m)\ell^2 + m(2\ell)^2 = 6m\ell^2$
$L = 6m\ell^2\omega$

b. $F_f = \mu mg$
$\Sigma\tau = -(\mu(2m)g\ell + \mu mg(2\ell)) = -4\mu mg\ell$

c. $\alpha = \tau/I = -4\mu mg\ell/6m\ell^2 = -2\mu g/3\ell$
$\omega = \omega_0 + \alpha t$; setting $\omega = 0$ and solving for T gives $T = 3\omega_0\ell/2\mu g$

<u>1987M3</u>

a. $K = U$
$½\, I\omega^2 = mgh_{cm}$
$½\, (m\ell^2/3)\omega^2 = mg(\ell/2)$ which gives $\omega = 5$ rad/s

b. $K_i = K_f$
$½\, m_0v_0^2 = ½\, m_0v^2 + ½\, I\omega^2$
$v = 8$ m/s

c. $L = mvr = (1 \text{ kg})(10 \text{ m/s})(1.2 \text{ m}) = 12 \text{ kg-m}^2/\text{s}$

d. $L_i = L_f$
$12 \text{ kg-m}^2/\text{s} = m_0(v_\perp)\ell + I\omega = m_0(v \cos\theta)\ell + I\,\omega$
$\theta = 60°$

<u>1992M2</u>

a. $\Sigma\tau = (3M + M)g\ell - Mg\ell = 3Mg\ell$

b. $I = \Sigma mr^2 = 4M\ell^2 + M\ell^2 = 5m\ell^2$
$\alpha = \tau/I = 3Mg\ell/5m\ell^2 = 3g/5\ell$

c. $\Delta U_{bug} + \Delta U_{left\ sphere} + \Delta U_{right\ sphere} = \Delta K_{rot}$
since $\Delta U_{left\ sphere} = -\Delta U_{right\ sphere}$, we only need to consider ΔU_{bug}
$3Mg\ell = ½\, I\omega^2 = ½\, (5M\ell^2)\omega^2$
$\omega = (6g/5\ell)^{1/2}$

d. $L = I\omega = 5M\ell^2(6g/5\ell)^{1/2} = (30M^2g\ell^3)^{1/2}$

e. Let T be the force we are looking for
$\Sigma F = ma_c$
$T - 3Mg = M\omega^2\ell$
$T = 3Mg + 3M(6g/5\ell)\ell = 33Mg/5$

<u>1996M3</u>

a.

$$I = \int r^2 dm$$

$$dm = \frac{M}{l}dr$$

$$I = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{M}{l} r^2 dr$$

$$I = \frac{M}{l}\frac{r^3}{3}\Big|_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

b. $I = \Sigma I = M\ell^2/12 + M(\ell/2)^2 = M\ell^2/3$

c./d./e.

$\Sigma F = ma$

for cat: $Mg - T = Ma$

$\Sigma\tau = I\alpha$ where $\alpha = a/r = a/(\ell/2)$

for hoop: $T\ell/2 = (M\ell^2/3)(a/(\ell/2))$ which gives $a = 3T/4M$

substituting gives $Mg - T = 3T/4$

$T = 4Mg/7$

$\alpha = T\ell/2I = 6g/7\ell$

$a = \alpha(\ell/2) = 3g/7$

f. $L = Mv(\ell/2)$ where v is found from $v^2 = v_0^2 + 2aH = 2(3g/7)(5\ell/3) = 10g\ell/7$

$L = ½ M\ell(10g\ell/7)^{1/2}$

1998M2

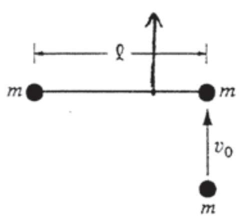
a. i. $mv_0 = (3m)v_f$; $v_f = v_0/3$

$K_f = ½ (3m)(v_0/3)^2 = mv_0^2/6$

ii. $\Delta K = K_f - K_i = mv_0^2/6 - ½ mv_0^2 = -mv_0^2/3$

b. i. $r_{cm} = \Sigma m_i r_i/\Sigma m = m(0) + 2m(\ell)/(m + 2m) = (2/3)\ell$

ii.



iii. $p_i = p_f$

$mv_0 = (3m)v_f$; $v_f = v_0/3$

iv. $L_i = L_f$

$mv_0R \sin\theta = mv_0(\ell/3) = I\omega$ where $I = \Sigma mr^2 = m(2\ell/3)^2 + 2m(\ell/3)^2 = (2/3)m\ell^2$

solving yields $\omega = v_0/2\ell$

v. $K_i = ½ mv_0^2$

$K_f = ½ mv_f^2 + ½ I\omega^2 = ½ (3m)(v_0/3)^2 + ½ (2/3)m\ell^2(v_0/2\ell)^2 = ¼ mv_0^2$

$\Delta K = -¼ mv_0^2$

2005M3

a. $L = I\omega = (1/3)M_1d^2\omega$

b. $L_f = L_i$

$M_2vd = (1/3)M_1d^2\omega$

$v = M_1d\omega/3M_2$

c. $K_f = K_i$

$½ M_2v^2 = ½ I\omega^2$

$M_2v^2 = I\omega^2$

$M_2(M_1d\omega/3M_2) = (1/3)M_1d^2\omega^2$ $\quad M_2(1/9)(M_1/M_2)^2d^2\omega^2 = (1/3)M_1d^2\omega^2$ $\quad (1/9)(M_1^2/M_2) = M_1/3$

$M_1/M_2 = 3$

d. $L_f = L_i$

$M_1vx = (1/3)M_1d^2\omega$

$v = d^2\omega/3x$

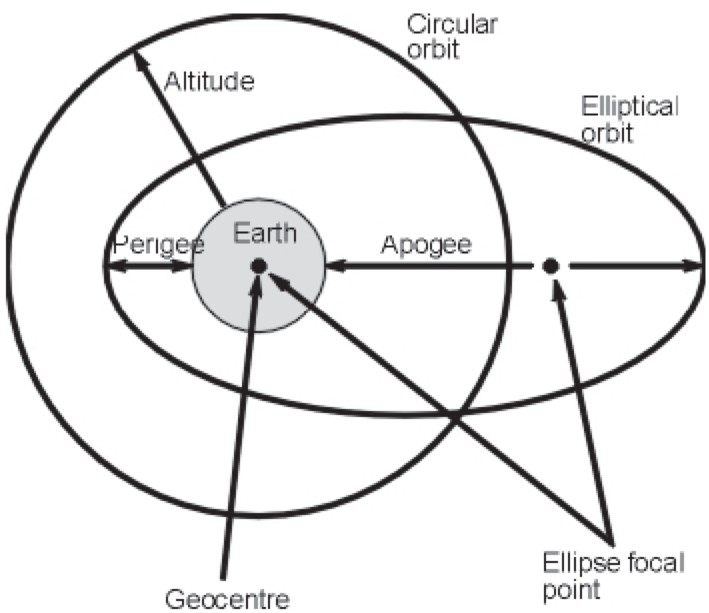
$½ M_1v^2 = ½ I\omega^2$ $\quad M_1v^2 = I\omega^2 = (1/3)M_1d^2\omega^2$

$v^2 = d^2\omega^2/3$ $\quad$ substituting from above $\quad (d^2\omega/3x)^2 = d^2\omega^2/3$

solving for x gives $x = d/\sqrt{3}$

Chapter 5

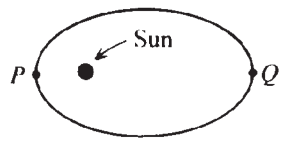
Gravitation



AP Physics Multiple Choice Practice – Gravitation

1. Each of five satellites makes a circular orbit about an object that is much more massive than any of the satellites. The mass and orbital radius of each satellite are given below. Which satellite has the greatest speed?

	Mass	Radius
(A)	½m	R
(B)	m	½R
(C)	m	R
(D)	m	2R

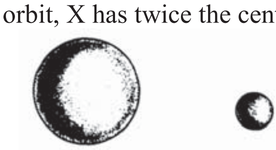


2. An asteroid moves in an elliptic orbit with the Sun at one focus as shown above. Which of the following quantities increases as the asteroid moves from point P in its orbit to point Q?
(A) Speed (B) Angular momentum (C) Kinetic energy (D) Potential energy

3. A person weighing 800 newtons on Earth travels to another planet with twice the mass and twice the radius of Earth. The person's weight on this other planet is most nearly
(A) 400 N (B) $\frac{800}{\sqrt{2}}$ N (C) $800\sqrt{2}$ (D) 1,600 N

4. Mars has a mass 1/10 that of Earth and a diameter 1/2 that of Earth. The acceleration of a falling body near the surface of Mars is most nearly
(A) 0.25 m/s^2 (B) 0.5 m/s^2 (C) 2 m/s^2 (D) 4 m/s^2

5. **Multiple correct:** If Spacecraft X has twice the mass of Spacecraft Y, then true statements about X and Y include which of the following? Select two answers.
(A) On Earth, X experiences twice the gravitational force that Y experiences.
(B) On the Moon, X has twice the weight of Y.
(C) The weight of the X on Earth will always be equal to the weight of Y on the Moon.
(D) When both are in the same circular orbit, X has twice the centripetal acceleration of Y



6. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?
(A) Acceleration (B) Kinetic energy (C) Displacement from the center of mass (D) Gravitational force

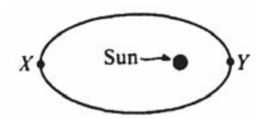
7. An object has a weight *W* when it is on the surface of a planet of radius *R*. What will be the gravitational force on the object after it has been moved to a distance of *4R* from the center of the planet?
(A) 16W (B) 4W (C)1/4W (D) 1/16 W

8. A new planet is discovered that has twice the Earth's mass and twice the Earth's radius. On the surface of this new planet, a person who weighs 500 N on Earth would experience a gravitational force of
(A) 125 N (B) 250 N (C) 500 N (D) 1000 N

9. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are the same. (B) Both are longer.
(C) The period of the mass on the spring is shorter, that of the pendulum is the same.
(D) The period of the pendulum is shorter; that of the mass on the spring is the same.

10. A satellite of mass m and speed v moves in a stable, circular orbit around a planet of mass M. What is the radius of the satellite's orbit?

(A) $\frac{Gv}{mM}$ (B) $\frac{GM}{v^2}$ (C) $\frac{GmM}{v}$ (D) $\frac{GmM}{v^2}$

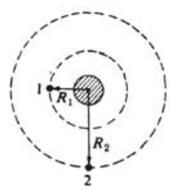


11. A satellite travels around the Sun in an elliptical orbit as shown above. As the satellite travels from point X to point Y. which of the following is true about its speed and angular momentum?

	Speed	Angular Momentum
(A)	Increases	Increases
(B)	Decreases	Decreases
(C)	Increases	Remains constant
(D)	Decreases	Remains constant

12. A newly discovered planet has a mass that is 4 times the mass of the Earth. The radius of the Earth is R_e. The gravitational field strength at the surface of the new planet is equal to that at the surface of the Earth if the radius of the new planet is equal to

(A) ½R_e (B) 2R_e (C) $\overline{R_e}$ (D) R_e^2



13. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations a_2 and a_1 of the two satellites are related by which of the following?

(A) $a_2 = 4a_1$ (B) $a_2 = 2a_1$ (C) $a_2 = a_1/2$ (D) $a_2 = a_1/4$

14. A satellite moves in a stable circular orbit with speed v_o at a distance R from the center of a planet. For this satellite to move in a stable circular orbit a distance $2R$ from the center of the planet, the speed of the satellite must be

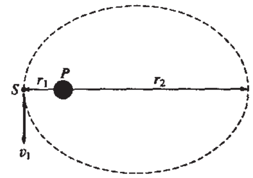
(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) $\sqrt{2v_0}$ (D) $2v_o$

15. If F_1 is the magnitude of the force exerted by the Earth on a satellite in orbit about the Earth and F_2 is the magnitude of the force exerted by the satellite on the Earth, then which of the following is true?

(A) F_1 is much greater than F_2. (B) F_1 is slightly greater than F_2.
(C) F_1 is equal to F_2. (D) F_2 is slightly greater than F_1

16. A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface. The radius of the new planet in terms of the radius R of Earth is

(A) ½R (B) $\frac{\sqrt{2}}{2}$ R (C) $\sqrt{2}$ R (D) 2R



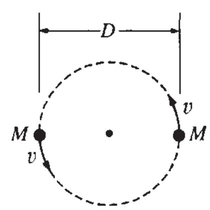
17. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?
   (A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $\frac{r_1+r_2}{2}v_1$ (D) $\frac{r_2-r_1}{r_1+r_2}v_1$

Questions 18-19

A ball is tossed straight up from the surface of a small, spherical asteroid with no atmosphere. The ball rises to a height equal to the asteroid's radius and then falls straight down toward the surface of the asteroid.

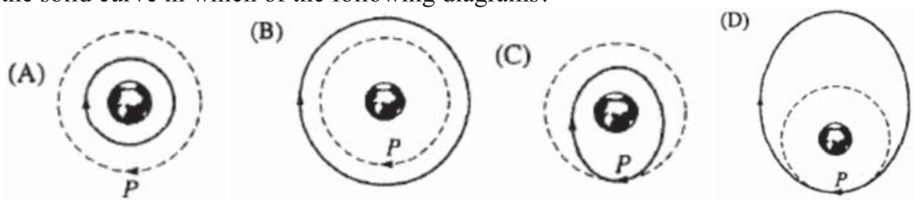
18. What forces, if any, act on the ball while it is on the way up?
   (A) Only a decreasing gravitational force that acts downward
   (B) Only a constant gravitational force that acts downward
   (C) Both a constant gravitational force that acts downward and a decreasing force that acts upward
   (C) No forces act on the ball.
19. The acceleration of the ball at the top of its path is
   (A) equal to the acceleration at the surface of the asteroid
   (B) equal to one-half the acceleration at the surface of the asteroid
   (C) equal to one-fourth the acceleration at the surface of the asteroid
   (D) zero

20. **Multiple Correct:** A satellite of mass M moves in a circular orbit of radius R with constant speed v. True statements about this satellite include which of the following? Select two answers.
   (A) Its angular speed is v/R.
   (B) The gravitational force does work on the satellite.
   (C) The magnitude and direction of its centripetal acceleration is constant.
   (D) Its tangential acceleration is zero.



21. Two identical stars, a fixed distance D apart, revolve in a circle about their mutual center of mass, as shown above. Each star has mass M and speed *v*. G is the universal gravitational constant. Which of the following is a correct relationship among these quantities?
   (A) $v^2 = GM/D$ (B) $v^2 = GM/2D$ (C) $v^2 = GM/D^2$ (D) $v^2 = 2GM^2/D$

22. A spacecraft orbits Earth in a circular orbit of radius R, as shown above. When the spacecraft is at position P shown, a short burst of the ship's engines results in a small increase in its speed. The new orbit is best shown by the solid curve in which of the following diagrams?



23. The escape speed for a rocket at Earth's surface is v_e . What would be the rocket's escape speed from the surface of a planet with twice Earth's mass and the same radius as Earth?

(A) $2v_e$ (B) $\sqrt{2}v_e$ (C) v_e (D) $\frac{v_e}{\sqrt{2}}$

24. A hypothetical planet orbits a star with mass one-half the mass of our sun. The planet's orbital radius is the same as the Earth's. Approximately how many Earth years does it take for the planet to complete one orbit?
(A) ½ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) 2

25. Two artificial satellites, 1 and 2, are put into circular orbit at the same altitude above Earth's surface. The mass of satellite 2 is twice the mass of satellite 1. If the period of satellite 1 is T, what is the period of satellite 2?
(A) $T/2$ (B) T (C) $2T$ (D) $4T$

26. A planet has a radius one-half that of Earth and a mass one-fifth the Earth's mass. The gravitational acceleration at the surface of the planet is most nearly
(A) 4.0 m/s^2 (B) 8.0 m/s^2 (C) 12.5 m/s^2 (D) 25 m/s^2

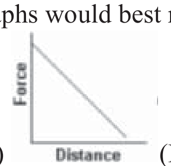
27. In the following statements, the word "weight" refers to the force a scale registers. If the Earth were to stop rotating, but not change shape,
(A) the weight of an object at the equator would increase.
(B) the weight of an object at the equator would decrease.
(C) the weight of an object at the north pole would increase.
(D) the weight of an object at the north pole would decrease.

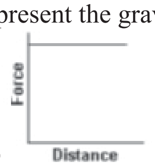
28. What happens to the force of gravitational attraction between two small objects if the mass of each object is doubled and the distance between their centers is doubled?
(A) It is doubled (B) It is quadrupled (C) It is halved (D) It remains the same

29. One object at the surface of the Moon experiences the same gravitational force as a second object at the surface of the Earth. Which of the following would be a reasonable conclusion?
(A) both objects would fall at the same acceleration
(B) the object on the Moon has the greater mass
(C) the object on the Earth has the greater mass
(D) both objects have identical masses

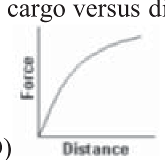
30. Consider an object that has a mass, *m*, and a weight, *W*, at the surface of the moon. If we assume the moon has a nearly uniform density, which of the following would be closest to the object's mass and weight at a distance halfway between Moon's center and its surface?
(A) ½ *m* & ½ *W* (B) ¼ *m* & ¼ *W* (C) 1 *m* & ½ *W* (D) 1 *m* & ¼ *W*

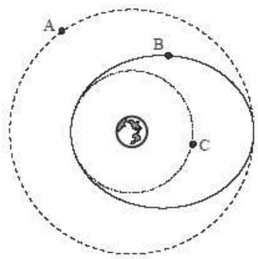
31. As a rocket blasts away from the earth with a cargo for the international space station, which of the following graphs would best represent the gravitational force on the cargo versus distance from the surface of the Earth?

(A) 

(B) 

(C) 

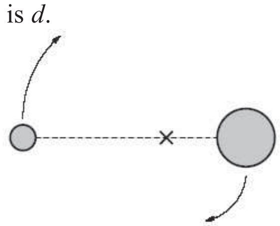
(D) 



32. Three equal mass satellites *A*, *B*, and *C* are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B, and L_C. Which of the following statements is correct?
(A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

<u>Questions 33-34</u>

Two stars orbit their common center of mass as shown in the diagram below. The masses of the two stars are 3*M* and *M*. The distance between the stars is *d*.



33. What is the value of the gravitational potential energy of the two star system?
(A) $-\frac{GM^2}{d}$ (B) $\frac{3GM^2}{d}$ (C) $-\frac{GM^2}{d^2}$ (D) $-\frac{3GM^2}{d}$

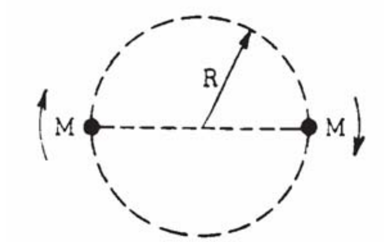
34. Determine the period of orbit for the star of mass 3*M*.
(A) $\pi\sqrt{\frac{d^3}{GM}}$ (B) $\pi\sqrt{\frac{d^3}{3GM}}$ (C) $2\pi\sqrt{\frac{d^3}{GM}}$ (D) $\frac{\pi}{4}\sqrt{\frac{d^3}{GM}}$

35. Two iron spheres separated by some distance have a minute gravitational attraction, *F*. If the spheres are moved to one half their original separation and allowed to rust so that the mass of each sphere increases 41%, what would be the resulting gravitational force?
(A) 2*F* (B) 4*F* (C) 6*F* (D) 8*F*

36. A ball thrown upward near the surface of the Earth with a velocity of 50 m/s will come to rest about 5 seconds later. If the ball were thrown up with the same velocity on Planet X, after 5 seconds it would be still moving upwards at nearly 31 m/s. The magnitude of the gravitational field near the surface of Planet X is what fraction of the gravitational field near the surface of the Earth?
(A) 0.16 (B) 0.39 (C) 0.53 (D) 0.63

37. Two artificial satellites I and II have circular orbits of radii R and $2R$, respectively, about the same planet. The orbital velocity of satellite I is v. What is the orbital velocity of satellite II?
(A) $\frac{v}{2}$ (B) $\frac{v}{\sqrt{2}}$ (C) $\sqrt{2}v$ (D) $2v$

AP Physics Free Response Practice – Gravitation



*1977M3. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

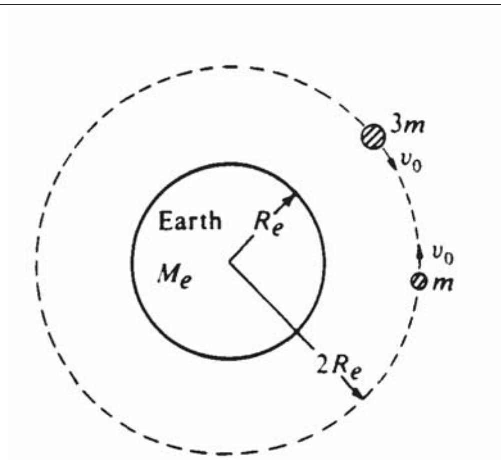
a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

b. Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.

d. Find the ratio of the speeds, v_{2M}/v_{M}.

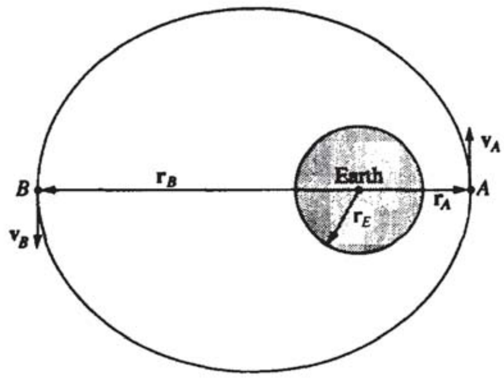


1984M2. Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



*1992M3. A spacecraft of mass 1,000 kilograms is in an elliptical orbit about the Earth, as shown above. At point A the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .

a. The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

b. The magnitude of the angular momentum of the spacecraft about the center of the Earth.

Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.

c. Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth. Determine each of the following.

d. The speed of the spacecraft if it is in a circular orbit around the Earth

e. The minimum speed of the spacecraft at point A if it is to escape completely from the Earth

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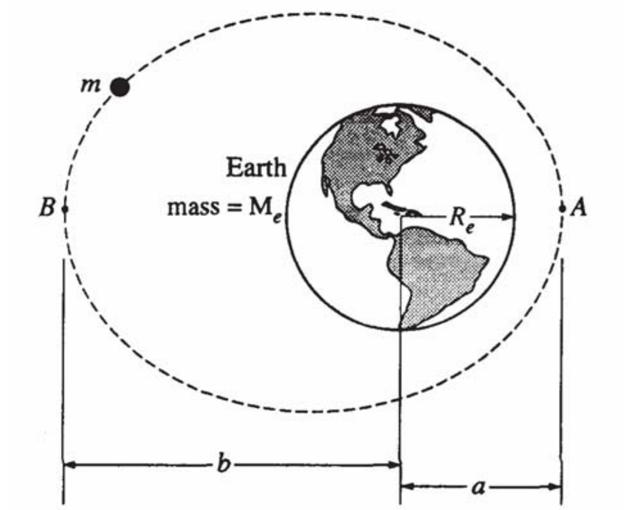
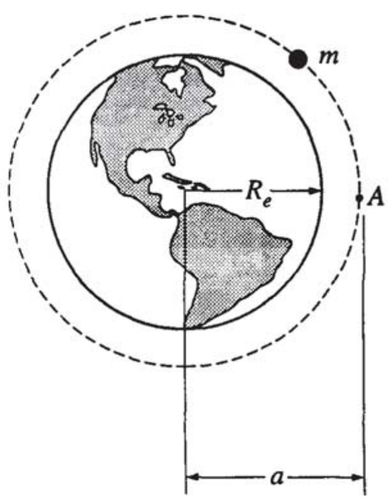


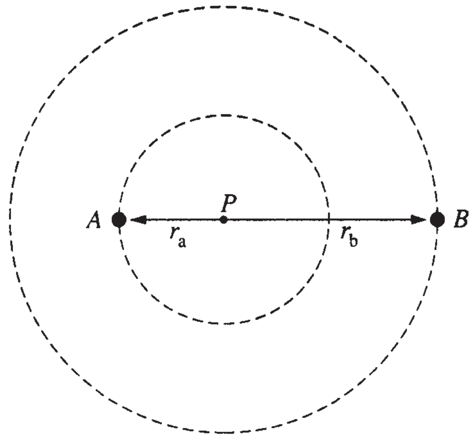
*1994M3 (modified) A satellite of mass *m* is in an elliptical orbit around the Earth, which has mass M_e and radius R_e. The orbit varies from a closest approach of distance *a* at point A to maximum distance of b from the center of the Earth at point B. At point A, the speed of the satellite is v_o. Assume that the gravitational potential energy $U_g = 0$ when masses are an infinite distance apart. Express your answers in terms of *a*, b, *m*, M_e, R_e, v_o, and G.

a. Determine the total energy of the satellite when it is at A.
b. What is the magnitude of the angular momentum of the satellite about the center of the Earth when it is at A?
c. Determine the velocity of the satellite as it passes point B in its orbit.



As the satellite passes point A, a rocket engine on the satellite is fired so that its orbit is changed to a circular orbit of radius a about the center of the Earth.

d. Determine the speed of the satellite for this circular orbit.
e. Determine the work done by the rocket engine to effect this change.



*1995M3 (modified) Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.
Determine expressions for the following three quantities in terms of r_a, r_b, T, and fundamental constants.

a. The centripetal acceleration of star A
b. The mass M_b of star B
c. The mass M_a of star A
d. Determine an expressions for the angular momentum of the system about the center of mass in terms of M_a, M_b, r_a, r_b, T, and fundamental constants.

2007M2. In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a. Calculate the radius of the GS orbit.
b. Calculate the mass of Mars.
c. Calculate the total mechanical energy of the GS in this orbit.
d. If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_________Greater than _________ Less than

Justify your answer.

e. In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

2001M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by

$$v = \sqrt{\frac{GM_J}{R}}$$

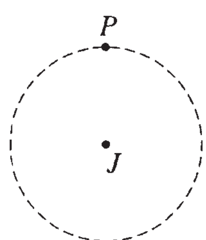
ii. The period of the orbit is given by

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

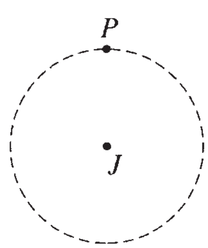
b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly faster than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly slower than the correct speed for a circular orbit of that radius.



2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

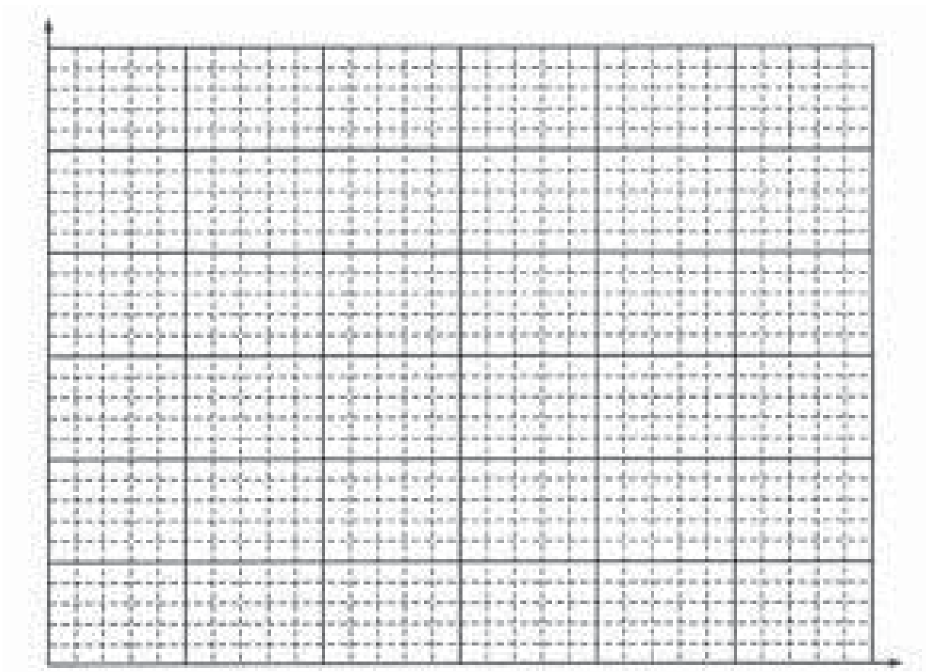
a. Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

ANSWERS - AP Physics Multiple Choice Practice – Gravitation

	Solution	Answer
1.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed. The smallest radius of orbit will be the fastest satellite.	B
2.	As a satellite moves farther away, it slows down, also decreasing its angular momentum and kinetic energy. The total energy remains the same in the absence of resistive or thrust forces. The potential energy becomes less negative, which is an increase.	D
3.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	A
4.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10	D
5.	The gravitational force on an object *is* the weight, and is proportional to the mass. In the same circular orbit, it is only the mass of the body being orbited and the radius of the orbit that contributes to the orbital speed and acceleration.	A,B
6.	Newton's third law	D
7.	Force is inversely proportional to distance between the centers squared. R × 4 = F ÷ 16	D
8.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	B
9.	A planet of the same size and twice the mass of Earth will have twice the acceleration due to gravity. The period of a mass on a spring has no dependence on g, while the period of a pendulum is inversely proportional to g.	D
10.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$	B
11.	Kepler's second law (Law of areas) is based on conservation of angular momentum, which remains constant. In order for angular momentum to remain constant, as the satellite approaches the sun, its speed increases.	C
12.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 4 = g × 4 and if the net effect is $g = g_{Earth}$ then r must be twice that of Earth.	B
13.	$a = g = \frac{GM}{r^2}$, if $R_2 = 2R_1$ then $a_2 = ¼\ a_1$	D

14. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. If r is doubled, v decreases by $\sqrt{2}$ B

15. Newton's third law C

16. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and if the net effect is $g = g_{Earth}$ then r must be $\sqrt{2}$ times that of Earth C

17. From conservation of angular momentum $v_1r_1 = v_2r_2$ A

18. As the ball moves away, the force of gravity decreases due to the increasing distance. A

19. $g = \frac{GM}{r^2}$ At the top of its path, it has doubled its original distance from the center of the asteroid. C

20. Angular speed (in radians per second) is v/R. Since the satellite is not changing speed, there is no tangential acceleration and v^2/r is constant. A,D

21. The radius of each orbit is ½ D, while the distance between them is D. This gives B

$$\frac{GMM}{D^2} = \frac{Mv^2}{D/2}$$

22. An burst of the ships engine produces an increase in the satellite's energy. Now the satellite is moving at too large a speed for a circular orbit. The point at which the burst occurs must remain part of the ship's orbit, eliminating choices A and B. The Earth is no longer at the focus of the ellipse in choice E. D

23. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is B

$\frac{1}{2}mv^2 = \left|-\frac{GMm}{r}\right|$ which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$

24. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 7 = g × 7 and r × 2 = g ÷ 4, so the net effect is g × 7/4 C

25. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed or period. B

26. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 5 = g ÷ 5 and r ÷ 2 = g × 4, so the net effect is g × 4/5 B

27. Part of the gravitational force acting on an object at the equator is providing the necessary centripetal force to move the object in a circle. If the rotation of the earth were to stop, this part of the gravitational force is no longer required and the "full" value of this force will hold the object to the Earth. A

28. $F = \frac{GMm}{r^2}$. F is proportional to each mass and inversely proportional to the distance between their centers squared. If each mass is doubled, F is quadrupled. If r is doubled F is quartered. D

29. Since the acceleration due to gravity is less on the surface of the moon, to have the same gravitational force as a second object on the Earth requires the object on the Moon to have a larger mass. B

30. The mass of an object will not change based on its location. As one digs into a sphere of uniform density, the acceleration due to gravity (and the weight of the object) varies directly with distance from the center of the sphere. C

31. $F = \frac{GMm}{r^2}$ so F is proportional to $1/r^2$. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration due to gravity is only slightly smaller in orbit compared to the surface of the Earth. C

32. The angular momentum of each satellite is conserved independently so we can compare the orbits at any location. Looking at the common point between orbit A and B shows that satellite A is moving faster at that point than satellite B, showing $L_A > L_B$. A similar analysis at the common point between B and C shows $L_B > L_C$ A

33. $U = -\frac{GMm}{r}$ D

34. Since they are orbiting their center of mass, the larger mass has a radius of orbit of $\frac{1}{4}d$. The speed can be found from $\frac{G(3M)M}{d^2} = \frac{(3M)v^2}{d/4}$ which gives $v = \sqrt{\frac{GM}{4d}} = \frac{2\pi(d/4)}{T}$ A

35. $F = \frac{GMm}{r^2}$; If r ÷ 2, F × 4. If each mass is multiplied by 1.41, F is doubled (1.41×1.41) D

36. g = Δv/t = (31 m/s – 50 m/s)/(5 s) = –3.8 m/s^2 B

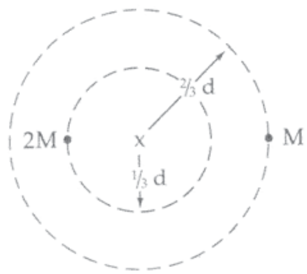
37. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. B

1977M3

a. $F_g = F_c$ gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b. E = PE + KE = $-\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$

c.



d. $F_{g2} = F_{g1} = F_c$

$\frac{(2M){v_2}^2}{{}^1\!/_3 d} = \frac{M{v_1}^2}{{}^2\!/_3 d}$ gives $v_2/v_1 = ½$

1984M2

a. $F_g = F_c$ gives $\frac{GM_e m}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives $(3m)v_0 – mv_0 = (4m)v'$ giving $v' = ½ v_0$

c. E = PE + KE = $-\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_e m}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_e m}{4R_e}$

1992M3

a. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2 = –8.1 \times 10^9$ J

b. L = mvr = 8.5×10^{13} kg-m²/s

c. Angular momentum is conserved so $mv_a r_a = mv_b r_b$ giving $v_b = 2.4 \times 10^3$ m/s

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{GM/R} = 5.8 \times 10^3$ m/s

e. Escape occurs when E = PE + KE = 0 giving $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0$ and $v_{esc} = \sqrt{2GM/R} = 8.2 \times 10^3$ m/s

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1994M3

a. E = PE + KE = $-\frac{GM_e m}{a} + \frac{1}{2}m{v_0}^2$

b. L = mvr = $mv_0 a$

c. Conservation of angular momentum gives $mv_0 a = mv_b b$, or $v_b = v_0 a/b$

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ and $v = \sqrt{GM_e/a}$

e. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2}m\left(\frac{GM_e}{a} - {v_0}^2\right)$

1995M3

a. $v = \frac{2\pi r}{T}$ *and* $a = \frac{v^2}{r} = \frac{4\pi^2 r_a}{T^2}$

b. The centripetal force on star A is due to the gravitational force exerted by star B.
$M_a a_a = \frac{GM_a M_b}{(r_a + r_b)^2}$ and substituting part (a) gives $M_b = \frac{4\pi^2 r_a (r_a + r_b)^2}{GT^2}$

c. The same calculations can be performed with the roles of star A and star B switched.
$M_a = \frac{4\pi^2 r_b (r_a + r_b)^2}{GT^2}$

d. $L_{total} = M_a v_a r_a + M_b v_b r_b = M_a \frac{2\pi r_a}{T} r_a + M_b \frac{2\pi r_b}{T} r_b = \frac{2\pi}{T}(M_a r_a{}^2 + M_b r_b{}^2)$

2007M2

a. $v = 2\pi R/T$ gives $R = 3.83 \times 10^6$ m

b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ *and* $M = \frac{v^2 R}{G} = 6.64 \times 10^{23}$ kg

c. $E = PE + KE = -\frac{GMm}{R} + \frac{1}{2}mv^2 = -5.38 \times 10^9$ J

d. From Kepler's third law r^3/T^2 = constant so if r decreases, then T must also.

e. Conservation of angular momentum gives $mv_1r_1 = mv_2r_2$ so $v_2 = r_1v_1/r_2$, but the distances *above the surface* are given so the radius of Mars must be added to the given distances before plugging them in for each r. This gives $v_2 = 3.34 \times 10^3$ m/s.

2001M2

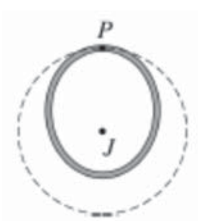
a. i. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ *and* $v = \sqrt{GM_J/R}$

ii. $v = d/T = 2\pi R/T$ giving $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM_J/R}} = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$

b. Plugging numerical values into a.ii. above gives $R = 1.59 \times 10^8$ m

c. i.

ii.



2005M2

a. $F = \frac{GM_S m}{R^2}$

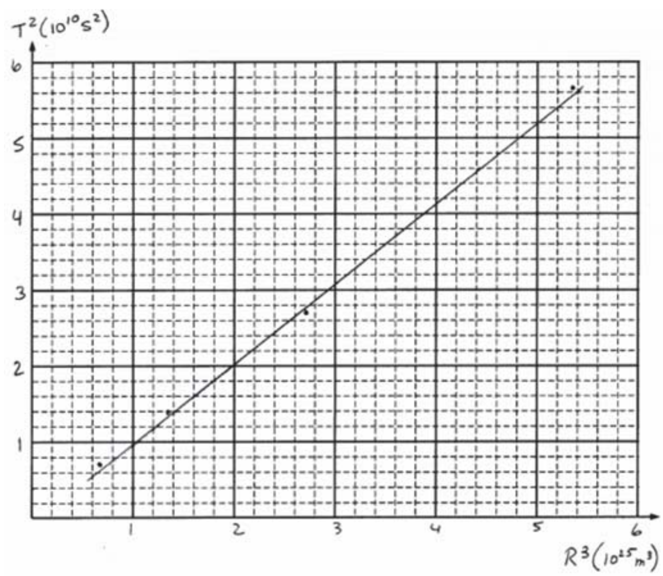
b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

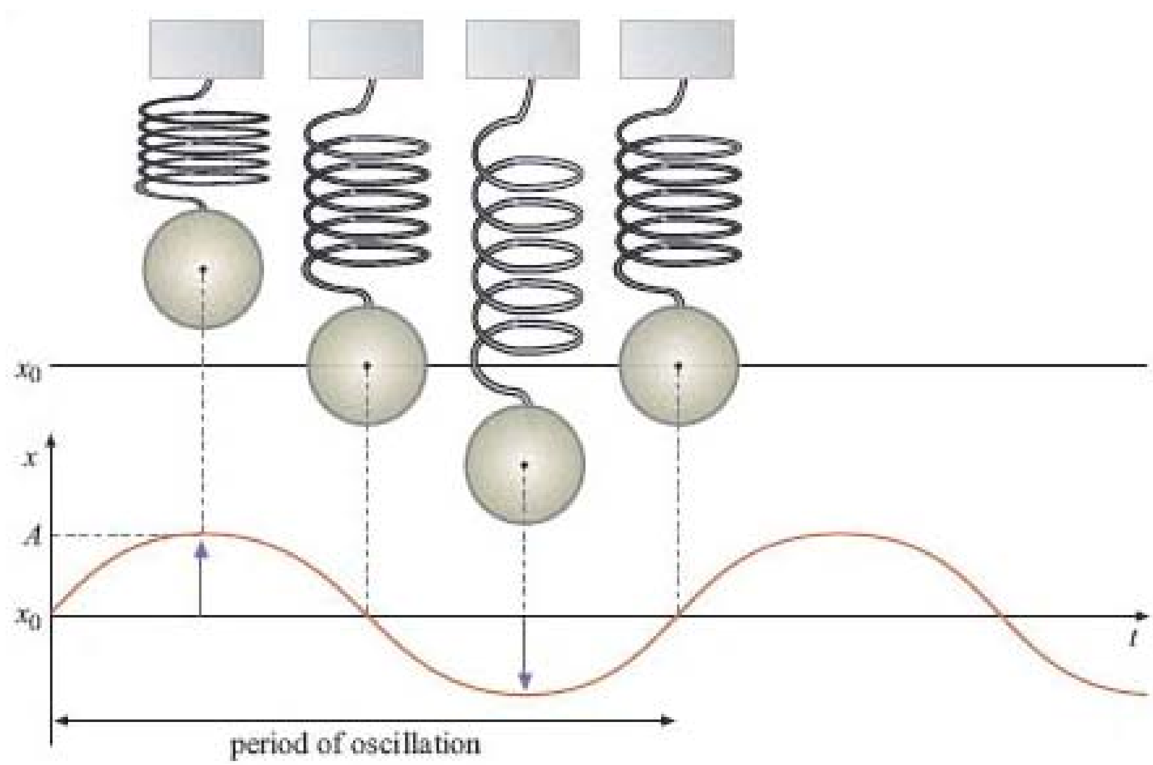
e.



f. From part c. we have an expression for the slope of the line.
Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg

Chapter 8

Oscillations



AP Physics Multiple Choice Practice – Oscillations

1. A mass m, attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the mass's speed as it passes through its equilibrium position?

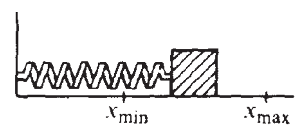
$$(A)\ A\sqrt{\frac{k}{m}} \quad (B)\ A\sqrt{\frac{m}{k}} \quad (C)\ \frac{1}{A}\sqrt{\frac{k}{m}} \quad (D)\ \frac{1}{A}\sqrt{\frac{m}{k}}$$

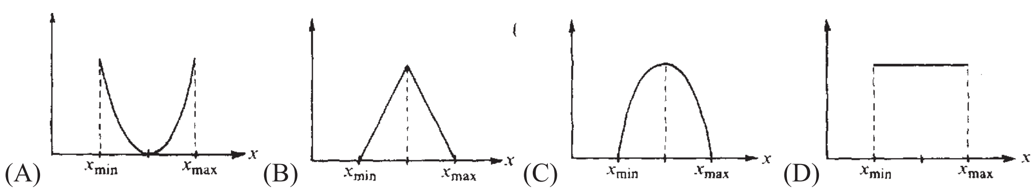
2. A mass m is attached to a spring with a spring constant k. If the mass is set into simple harmonic motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to the equilibrium position?

$$(A)\ v=\sqrt{\frac{md}{k}} \quad (B)\ v=\sqrt{\frac{kd}{m}} \quad (C)\ v=\sqrt{\frac{kd}{mg}} \quad (D)\ v=d\sqrt{\frac{k}{m}}$$

3. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The kinetic and potential energies are both constant.
(C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.

Questions 4-5: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.





4. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C (D) D

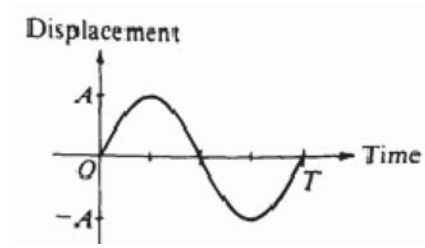
5. Which graph can represent the kinetic energy of the block as a function of x ?
(A) A (B) B (C) C (D) D

6. An object swings on the end of a cord as a simple pendulum with period T. Another object oscillates up and down on the end of a vertical spring also with period T. If the masses of both objects are doubled, what are the new values for the Periods?

	Pendulum	Mass on Spring
(A)	$\frac{T}{\sqrt{2}}$	$T\sqrt{2}$
(B)	T	$T\sqrt{2}$
(C)	$T\sqrt{2}$	T
(D)	$T\sqrt{2}$	$\frac{T}{\sqrt{2}}$

7. An object is attached to a spring and oscillates with amplitude A and period T, as represented on the graph. The nature of the velocity v and acceleration a of the object at time T/4 is best represented by which of the following?

(A) $v > 0, a > 0$ (B) $v > 0, a < 0$
(C) $v > 0, a = 0$ (D) $v = 0, a < 0$

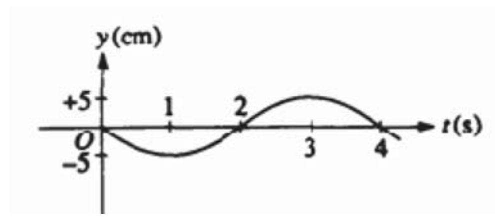


8. When an object oscillating in simple harmonic motion is at its maximum displacement from the equilibrium position. Which of the following is true of the values of its speed and the magnitude of the restoring force?

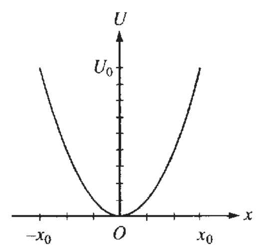
Speed	Restoring Force
(A) Zero	Maximum
(B) Zero	Zero
(C) Maximum	½ maximum
(D) Maximum	Zero

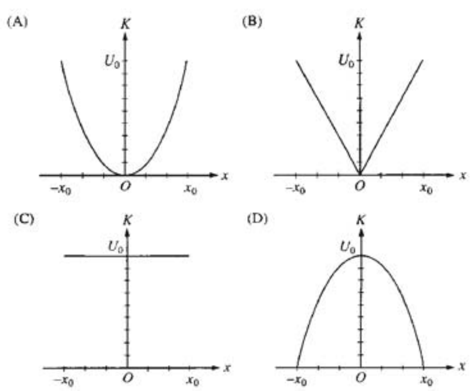
9. A particle oscillates up and down in simple harmonic motion. Its height y as a function of time t is shown in the diagram. At what time t does the particle achieve its maximum positive acceleration?

(A) 1 s (B) 2 s (C) 3 s (D) 4 s

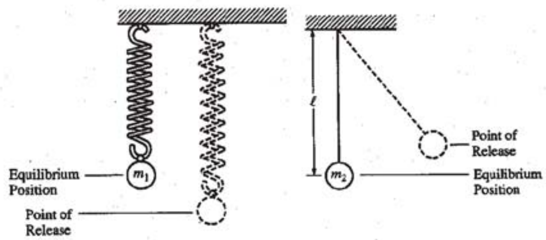


10. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?





Questions 11-12

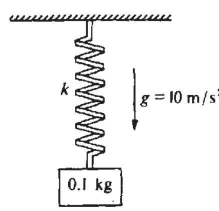


A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion

11. Which of the following is true for both spheres?
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position
(B) The maximum kinetic energy is attained as the sphere reaches its point of release.
(C) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(D) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(E) The maximum total energy is attained only as the sphere passes through its equilibrium position.

12. If both spheres have the same period of oscillation, which of the following is an expression for the spring constant
(A) $L / m_1 g$
(B) $g / m_2 L$
(C) $m_2 g / L$
(D) $m_1 g / L$

13. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are shorter.
(B) Both are the same.
(C) The period of the mass on the spring is shorter; that of the pendulum is the same.
(D) The period of the pendulum is shorter; that of the mass on the spring is the same



Questions 14-15
A 0.1 -kilogram block is attached to an initially unstretched spring of force constant k = 40 newtons per meter as shown above. The block is released from rest at time t = 0.

14. What is the amplitude, in meters, of the resulting simple harmonic motion of the block?

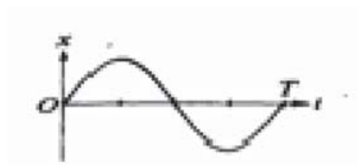
(A) $\frac{1}{40}m$ (B) $\frac{1}{20}m$ (C) $\frac{1}{4}m$ (D) $\frac{1}{2}m$

15. What will the resulting period of oscillation be?

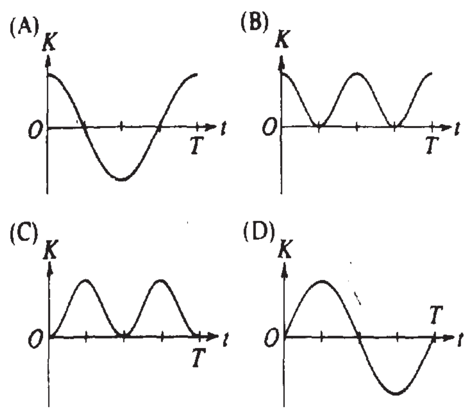
(A) $\frac{\pi}{40}s$ (B) $\frac{\pi}{20}s$ (C) $\frac{\pi}{10}s$ (D) $\frac{\pi}{4}s$

16. A ball is dropped from a height of 10 meters onto a hard surface so that the collision at the surface may be assumed elastic. Under such conditions the motion of the ball is
(A) simple harmonic with a period of about 1.4 s
(B) simple harmonic with a period of about 2.8 s
(C) simple harmonic with an amplitude of 5 m
(D) periodic with a period of about 2.8 s but not simple harmonic

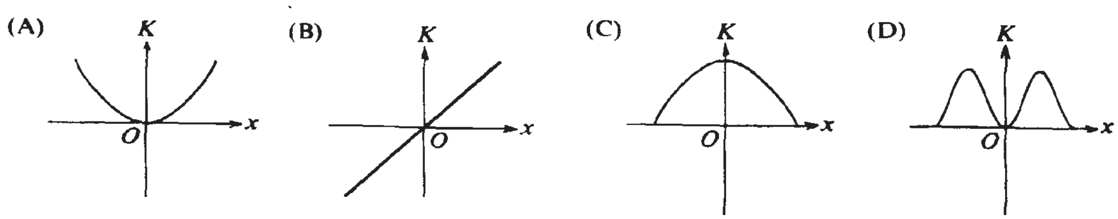
Questions 17-18 refer to the graph below of the displacement x versus time t for a particle in simple harmonic motion.



17. Which of the following graphs shows the kinetic energy K of the particle as a function of time t for one cycle of motion?



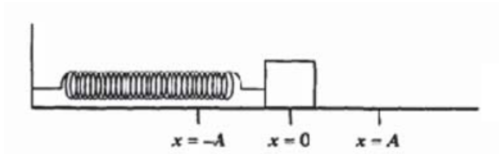
18. Which of the following graphs shows the kinetic energy K of the particle as a function of its displacement x ?



19. A mass m is attached to a vertical spring stretching it distance d. Then, the mass is set oscillating on a spring with an amplitude of A, the period of oscillation is proportional to

(A) $\sqrt{\frac{d}{g}}$ (B) $\sqrt{\frac{g}{d}}$ (C) $\sqrt{\frac{d}{mg}}$ (D) $\sqrt{\frac{m^2 g}{d}}$

20. Two objects of equal mass hang from independent springs of unequal spring constant and oscillate up and down. The spring of greater spring constant must have the
(A) smaller amplitude of oscillation (B) larger amplitude of oscillation
(C) shorter period of oscillation (D) longer period of oscillation



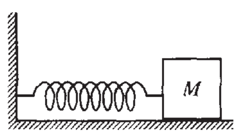
Questions 21-22. A block on a horizontal frictionless plane is attached to a spring, as shown above. The block oscillates along the x-axis with simple harmonic motion of amplitude A.

21. Which of the following statements about the block is correct?
(A) At x = 0, its acceleration is at a maximum. (B) At x = A, its displacement is at a maximum.
(C) At x = A, its velocity is at a maximum. (D) At x = A, its acceleration is zero.

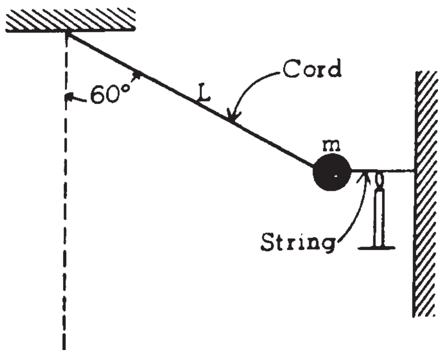
22. Which of the following statements about energy is correct?
(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.

23. A simple pendulum consists of a l.0 kilogram brass bob on a string about 1.0 meter long. It has a period of 2.0 seconds. The pendulum would have a period of 1.0 second if the
(A) string were replaced by one about 0.25 meter long
(B) string were replaced by one about 2.0 meters long
(C) bob were replaced by a 0.25 kg brass sphere
(D) bob were replaced by a 4.0 kg brass sphere

24. A pendulum with a period of 1 s on Earth, where the acceleration due to gravity is g, is taken to another planet, where its period is 2 s. The acceleration due to gravity on the other planet is most nearly
(A) g/4 (B) g/2 (C) 2g (D) 4g



25. An ideal massless spring is fixed to the wall at one end, as shown above. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is
(A) $\frac{Mg}{A}$ (B) $\frac{Mgv_m}{2A}$ (C) $\frac{Mv_m^2}{2A}$ (D) $\frac{Mv_m^2}{A^2}$



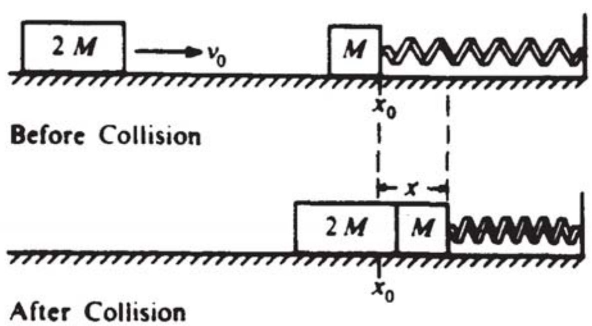
1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.

c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

d. The motion of the pendulum after the string is burned is periodic. Is it also simple harmonic? Why, or why not?

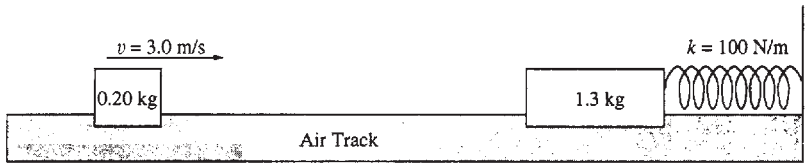


1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact

b. x, the maximum distance the spring is compressed

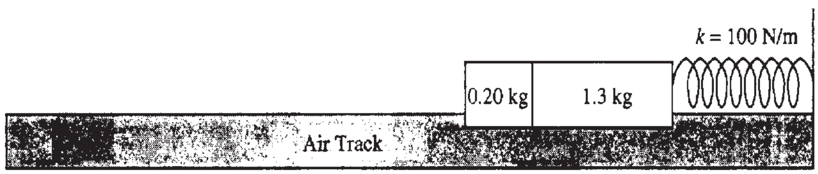
c. T, the period of the subsequent simple harmonic motion



1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

a. Determine the following for the 0.20-kilogram mass immediately before the impact.
  i. Its linear momentum ii. Its kinetic energy

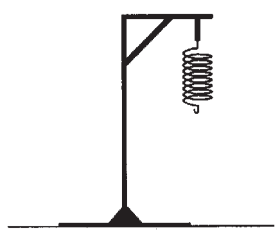
b. Determine the following for the combined masses immediately after the impact.
  i. The linear momentum ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

c. Determine the amplitude of the harmonic motion.

d. Determine the period of the harmonic motion.



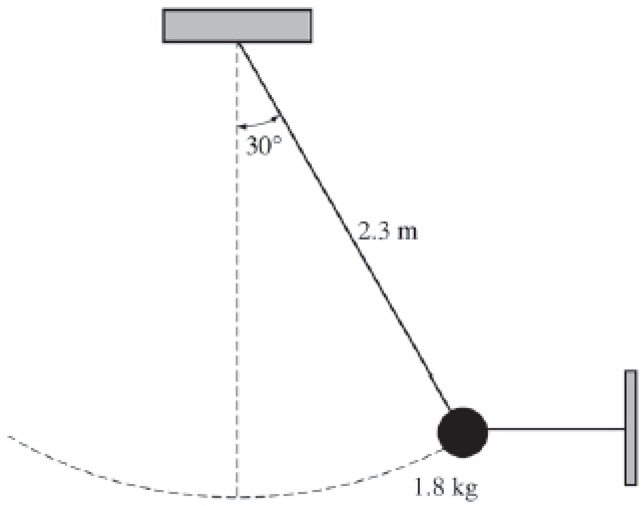
1996B2. A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
  i. What additional, commonly available equipment would you need?
  ii. What measurements would you make?
  iii. How would k be determined from these measurements?

b. Assume that the spring constant is determined to be 500 N/m. A 2.0-kg mass is attached to the lower end of the spring and released from rest. Determine the frequency of oscillation of the mass.

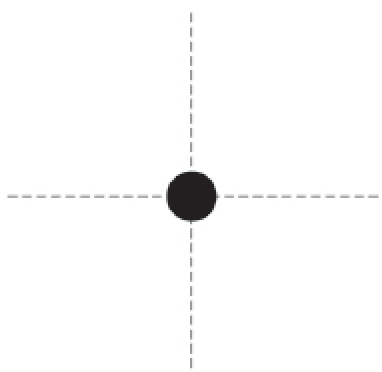
c. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
  i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
  ii. Explain how you would make the determination.

2005B2



A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



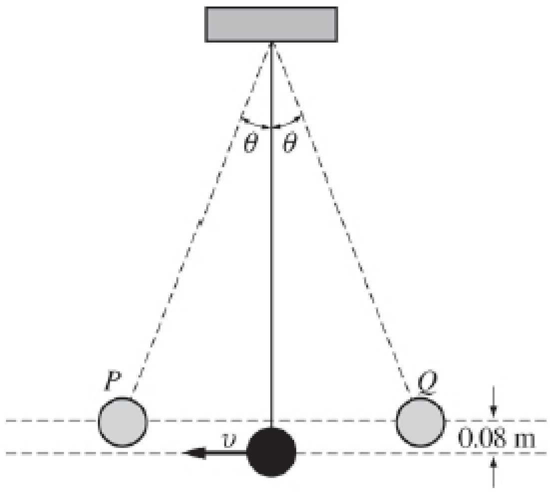
(b) Calculate the tension in the horizontal string.

(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

(d) How long will it take the bob to reach the lowest position for the first time?

2005B2B

A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

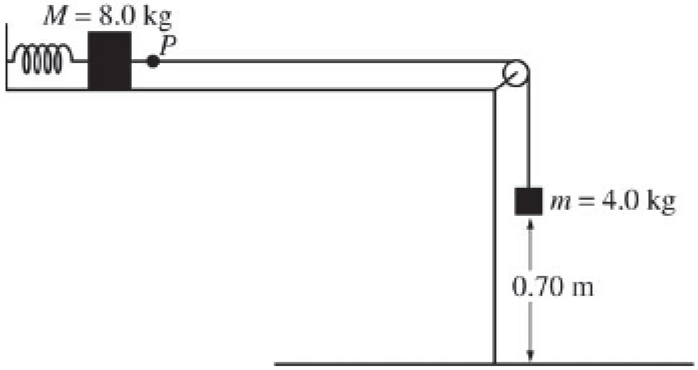
ii. When it is in motion at its lowest position

(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

(d) Describe one modification that could be made to double the period of oscillation.

2006B1



An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.

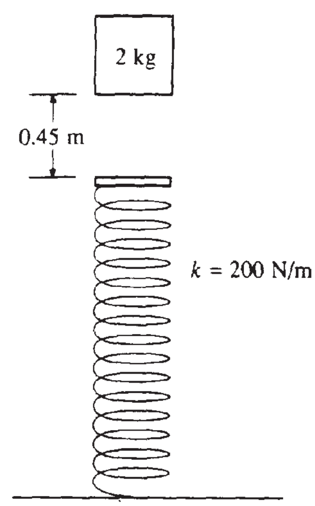
(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.

M = 8.0 kg m = 4.0 kg

(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the frequency of oscillation of the 8.0 kg block.
(f) Calculate the maximum speed attained by the 8.0 kg block.

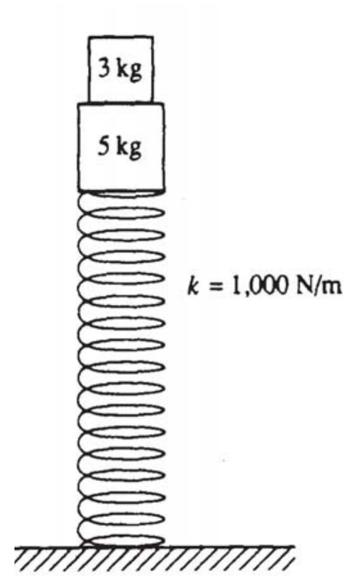
C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.



a. Determine the speed of the block at the instant it hits the end of the spring
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Determine the resulting amplitude of the oscillation that ensues
f. Is the speed of the block a maximum at the equilibrium position, explain.
g. Determine the period of the simple harmonic motion that ensues

1990M3. A 5-kilogram block is fastened to an ideal vertical spring that has an unknown spring constant. A 3-kilogram block rests on top of the 5-kilogram block, as shown above.

a. When the blocks are at rest, the spring is compressed to its equilibrium position a distance of $\Delta x_1 = 20$ cm, from its original length. Determine the spring constant of the spring



The 3 kg block is then raised 50 cm above the 5 kg block and dropped onto it.

b. Determine the speed of the combined blocks after the collision

c. Setup, plug in known values, but do not solve an equation to determine the amplitude Δx_2 of the resulting oscillation

d. Determine the resulting frequency of this oscillation.

e. Where will the block attain its maximum speed, explain.

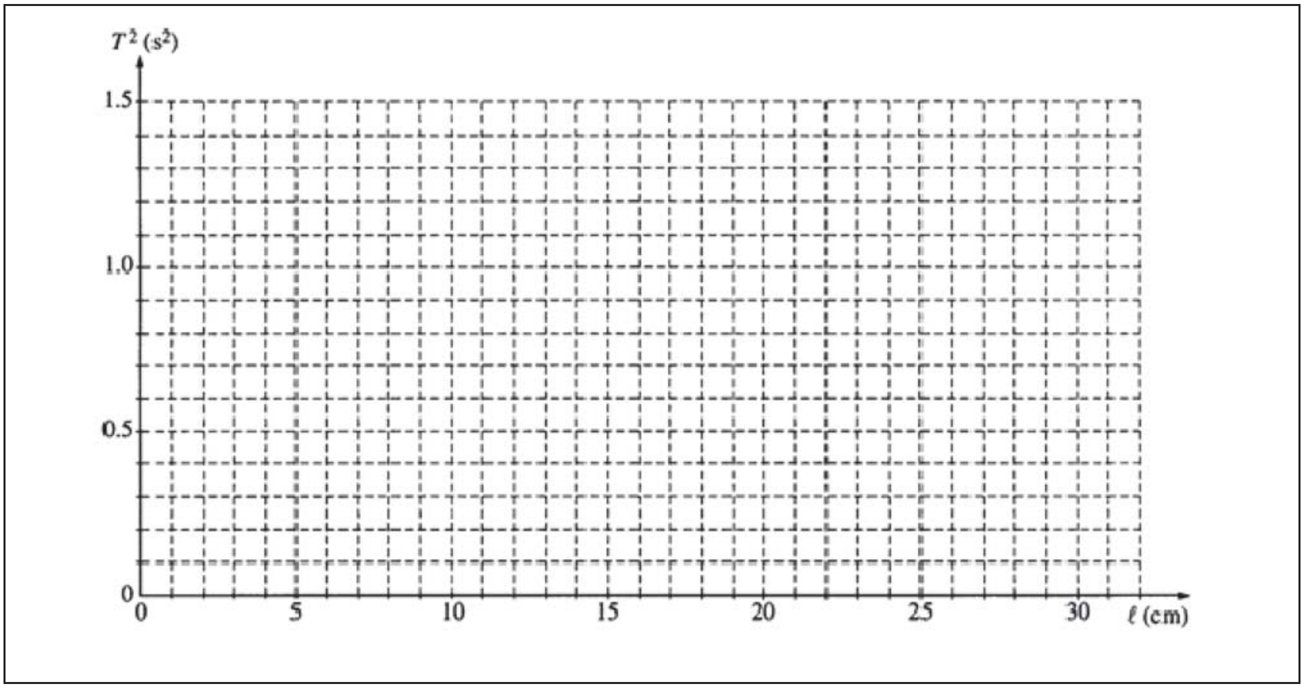
f. Is this motion simple harmonic.

(2000 M1) You are conducting an experiment to measure the acceleration due to gravity g_u at an unknown location. In the measurement apparatus, a simple pendulum swings past a photogate located at the pendulum's lowest point, which records the time t_{10} for the pendulum to undergo 10 full oscillations. The pendulum consists of a sphere of mass m at the end of a string and has a length *l*. There are four versions of this apparatus, each with a different length. All four are at the unknown location, and the data shown below are sent to you during the experiment.

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62		
18	8.89		
21	10.09		
32	12.08		

a. For each pendulum, calculate the period T and the square of the period. Use a reasonable number of significant figures. Enter these results in the table above.

b. On the axes below, plot the square of the period versus the length of the pendulum. Draw a best-fit straight line for this data.

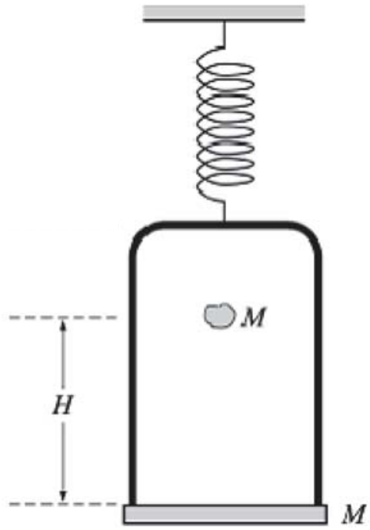


c. Assuming that each pendulum undergoes small amplitude oscillations, from your fit, determine the experimental value g_{exp} of the acceleration due to gravity at this unknown location. Justify your answer.

d. If the measurement apparatus allows a determination of g_u that is accurate to within 4%, is your experimental value in agreement with the value 9.80 m/s^2 ? Justify your answer.

e. Someone informs you that the experimental apparatus is in fact near Earth's surface, but that the experiment has been conducted inside an elevator with a constant acceleration *a*. If the elevator is moving down, determine the direction of the elevator's acceleration, justify your answer.

C2003M2.



An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

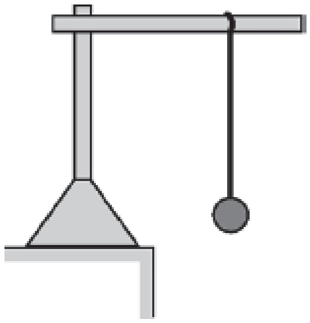
(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the clay and pan just after the clay strikes it.

(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

(d) Determine the resulting period of oscillation

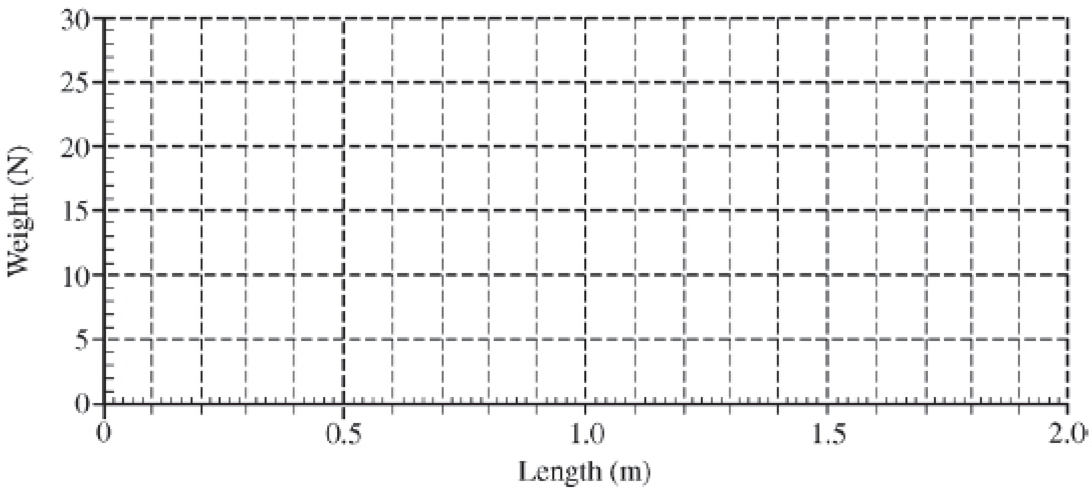
C2008M3



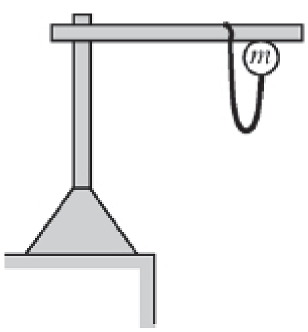
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant k of the cord.



The student now attaches an object of unknown mass m to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass m of the object.

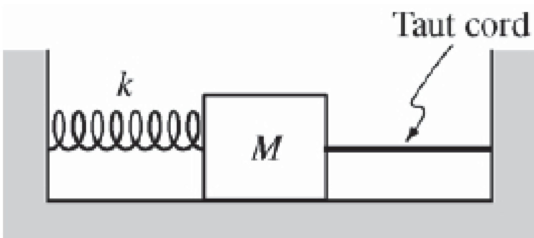
(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

iii. Calculate the speed of the object at the equilibrium position

iv. Is the speed in part iii above the maximum speed, explain your answer.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Calculate the time after the cord breaks until the block first reaches its position furthest to the left.
(e) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Oversillations

	Solution	Answer
1.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} mv^2$	A
2.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k d^2 = \frac{1}{2} mv^2$	D
3.	Energy is conserved here and switches between kinetic and potential which have maximums at different locations	D
4.	Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates	D
5.	The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the KE gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph.	C
6.	Pendulum is unaffected by mass. Mass-spring system has mass causing the T to change proportional to $\sqrt{m}$ so since the mass is doubled the period is changed by $\sqrt{2}$	B
7.	At T/4 the mass reaches maximum + displacement where the restoring force is at a maximum and pulling in the opposite direction and hence creating a negative acceleration. At maximum displacement the mass stops momentarily and has zero velocity	D
8.	See #7 above	A
9.	+ Acceleration occurs when the mass is at negative displacements since the force will be acting in the opposite direction of the displacement to restore equilibrium. Based on $F=k\Delta x$ the most force, and therefore the most acceleration occurs where the most displacement is	A
10.	As the object oscillates, its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D	D
11.	For the spring, equilibrium is shown where the maximum transfer of kinetic energy has occurred and likewise for the pendulum the bottom equilibrium position has the maximum transfer of potential energy into spring energy.	A
12.	Set period formulas equal to each other and rearrange for k	D
13.	In a mass-spring system, both mass and spring constant (force constant) affect the period.	D
14.	At the current location all of the energy is gravitational potential. As the spring stretches to its max location all of that gravitational potential will become spring potential when it reaches its lowest position. When the box oscillates back up it will return to its original location converting all of its energy back to gravitational potential and will oscillate back and forth between these two positions. As such the maximum stretch bottom location represents twice the amplitude so simply halving that max Δx will give the amplitude. Finding the max stretch: → The initial height of the box h and the stretch Δx have the same value ($h=\Delta x$) $U = U_{sp}$ $mg(\Delta x_1) = \frac{1}{2} k\Delta x_1^2$ $mg = \frac{1}{2} k \Delta x_1$ $\Delta x_1 = .05$ m. This is 2A, so the amplitude is 0.025 m or 1/40 m. Alternatively, we could simply find the equilibrium position measured from the initial top position based on the forces at equilibrium, and this equilibrium stretch measured from the top will be the amplitude directly. To do this:	A

$F_{net} = 0$ $F_{sp} = mg$ $k\Delta x_2 = mg$ $\Delta x_2 = 0.025$ m, which is the amplitude

15. Plug into period for mass-spring system $T = 2\pi \sqrt{(m/k)}$ C

16. Based on free fall, the time to fall down would be 1.4 seconds. Since the collision with the ground is elastic, all of the energy will be returned to the ball and it will rise back up to its initial height completing 1 cycle in a total time of 2.8 seconds. It will continue doing this oscillating up and down. However, this is not simple harmonic because to be simple harmonic the force should vary directly proportional to the displacement but that is not the case in this situation D

17. Energy will never be negative. The max kinetic occurs at zero displacement and the kinetic energy become zero when at the maximum displacement B

18. Same reasoning as above, it must be C C

19. First use the initial stretch to find the spring constant. $F_{sp} = mg = k\Delta x$ $k = mg / d$ D

    Then plug that into $T = 2\pi \sqrt{(m/k)}$ $T = 2\pi\sqrt{\frac{m}{(\frac{mg}{d})}}$

20. Based on $T = 2\pi \sqrt{(m/k)}$ the larger spring constant makes a smaller period C

21. Basic fact about SHM. Amplitude is max displacement B

22. Basic fact about SHM. Spring potential energy is a min at x=0 with no spring stretch D

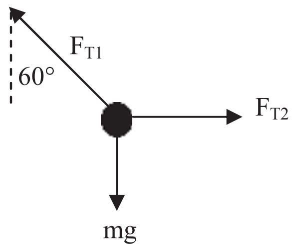
23. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ the length equates to ½ the period A

24. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ g would double the period A

25. Using energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$ solve for k D

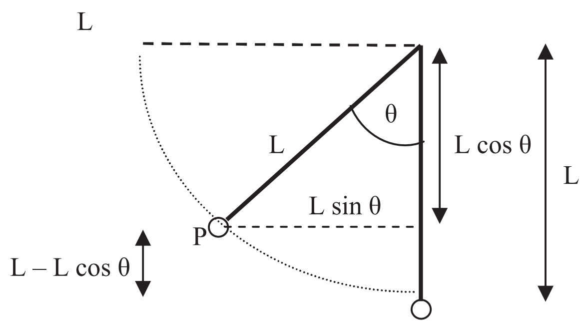
1975B7.

(a)



(b) $F_{NET(Y)} = 0$
$F_{T1} \cos \theta = mg$
$F_{T1} = mg / \cos(60) = 2mg$

(c) When the string is cut it swings from top to bottom, similar to the diagram for 1974B1 from work-energy problems with θ on the opposite side as shown below



$U_{top} = K_{bot}$
$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

(d) This motion is not simple harmonic because the restoring force, $(F_{gx}) = mg \sin \theta$, is not directly proportional to the displacement due to the sin function. For small angles of θ the motion is approximately SHM, though not exactly, but in this example the larger value of θ creates and even larger disparity.

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $\frac{1}{2}(3M)(2/3\ v_o)^2 = \frac{1}{2}\ k\ \Delta x^2$ $\sqrt{\frac{4Mv_o^{\ 2}}{3k}}$

c) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3m}{k}}$

1995 B1.

a) i) $p = mv$ $= (0.2)(3) = 0.6$ kg m/s
ii) $K = \frac{1}{2}mv^2$ $= \frac{1}{2}(0.2)(3)^2 = 0.9$ J

b) i.) Apply momentum conservation $p_{before} = p_{after} = 0.6$ kg m/s
ii) First find the velocity after, using the momentum above
$0.6 = (1.3+0.2)\ v_f$ $v_f = 0.4$ m/s, then find K, $K = \frac{1}{2}(m_1+m_2)\ v_f^2 = \frac{1}{2}(1.3+0.2)(0.4)^2 = 0.12$ J

c) Apply energy conservation $K = U_{sp}$ $0.12\text{ J} = \frac{1}{2}\ k\Delta x^2 = \frac{1}{2}(100)\ \Delta x^2$ $\Delta x = 0.05$ m

d) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.5}{100}} = 0.77s$

1996B2.

a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k\ \Delta x$ and solve for k

b) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{500}} = 0.4s$

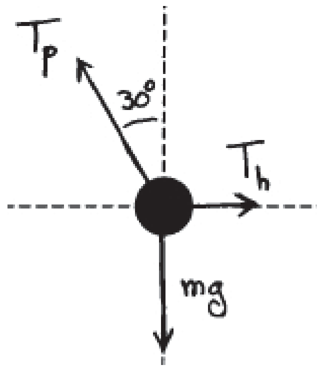
… then the frequency is given by $f = 1/T = 2.5$ Hz

c) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan\theta$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} - \mu_s\ mg$, Giving $mg = F_{spring} / \mu$.
Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid mechanics unit.

2005B2.

a) FBD



b) Apply $F_{net(X)} = 0$ $\quad$ $F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $\quad$ $T_P \sin 30 = T_H$

$T_P = 20.37$ N $\quad$ $T_H = 10.18$ N

c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$

$g(L - L\cos\theta) = ½\, v^2$

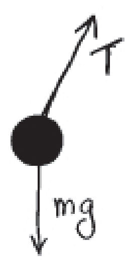
$(10)(2.3 - 2.3\cos 30) = ½\, v^2$ $\quad$ $v_{bottom} = 2.5$ m/s

d) The bob will reach the lowest position in ¼ of the period.

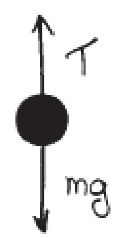
$$T = \frac{1}{4}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\pi}{2}\sqrt{\frac{2.3}{9.8}} = 0.76\ s$$

B2005B2.

FBD $\quad$ i)



ii)



b) Apply energy conservation?

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$ $\quad$ $(9.8)(.08) – ½\, v^2$ $\quad$ $v = 1.3$ m/s

c) $F_{net(c)} = mv^2/r$

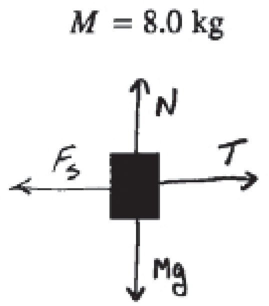
$F_t - mg = mv^2/r$ $\quad$ $F_t = mv^2/r + mg$ $\quad$ $(0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $\quad$ $F_t = 0.93$ N

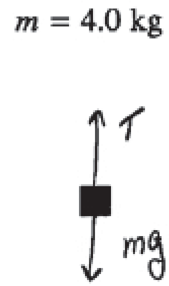
d) "g" and "L" are the two factors that determine the pendulum period based on $\left(T = 2\pi\sqrt{\frac{L}{g}}\right)$

To double the value of T, L should be increased by 4x or g should be decreased by ¼. The easiest modification would be simply to increase the length by 4 x

2006B1.

a) FBD





b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

c) Tension in the string is uniform throughout, now looking at the 8 kg mass,

$F_{sp} = F_t = k\Delta x$ $39 = k\,(0.05)$ $k = 780$ N/m

d) 4 kg mass is in free fall. $D = v_i t + \frac{1}{2} g t^2$ $-0.7 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.38$ sec

e) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{780}} = 0.63s$

… then the frequency is given by $f = 1/T = 1.6$ Hz

f) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy

$U_{sp} = K$ $\frac{1}{2} k \Delta x^2 = \frac{1}{2} mv^2$ $\frac{1}{2}(780)(0.05)^2 = \frac{1}{2}(8)v^2$ $v = 0.49$ m/s

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C1989M3.

a) Apply energy conservation from top to end of spring using h=0 as end of spring.

$U = K$ $mgh = \frac{1}{2} m v^2$ $(9.8)(0.45) = \frac{1}{2} v^2$ $v = 3$ m/s

b) At equilibrium the forces are balanced $F_{net} = 0$ $F_{sp} = mg = (2)(9.8) = 19.6$ N

c) Using the force from part b, $F_{sp} = k\,\Delta x$ $19.6 = 200\,\Delta x$ $\Delta x = 0.098$ m

d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the top position is now increased by the amount of Δx found in part c $h_{new} = h + \Delta x = 0.45 + 0.098 = 0.548$ m

$U_{top} = U_{sp} + K_{\text{(at equil)}}$

$mgh_{new} = \frac{1}{2} k\,\Delta x^2 + \frac{1}{2} mv^2$ $(2)(9.8)(0.548) = \frac{1}{2}(200)(0.098)^2 + \frac{1}{2}(2)(v^2)$ $v = 3.13$ m/s

e) Use the turn horizontal trick. Set equilibrium position as zero spring energy then solve it as a horizontal problem where $K_{equil} = U_{sp\text{(at max amp.)}}$ $\frac{1}{2} mv^2 = \frac{1}{2} k\Delta x^2$ $\frac{1}{2}(2)(3.13)^2 = \frac{1}{2}(200)(A^2)$ $A = 0.313$ m

f) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

g) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 0.63s$

C1990M3.

a) Equilibrium so $F_{net} = 0$, $F_{sp} = mg$ $k\Delta x = mg$ $k(0.20) = (8)(9.8)$ $k = 392$ N/m

b) First determine the speed of the 3 kg block prior to impact using energy conservation

$U = K$ $mgh = \frac{1}{2} m v^2$ $(9.8)(0.50) = \frac{1}{2} v^2$ $v = 3.13$ m/s

Then solve perfect inelastic collision. $p_{before} = p_{after}$ $m_1 v_{1i} = (m_1+m_2) v_f$ $(3)(3.13)=(8)v_f$ $v_f = 1.17$ m/s

c) Since we do not know the speed at equilibrium nor do we know the amplitude Δx_2 the turn horizontal trick would not work initially. If you first solve for the speed at equilibrium as was done in 1989M3 first, you could then use the turn horizontal trick. However, since this question is simply looking for an equation to be solved, we will use energy conservation from the top position to the lowest position where the max amplitude is reached. For these two positions, the total distance traveled is equal to the distance traveled to equilibrium + the distance traveled to the max compression $(\Delta x_1 + \Delta x_2) = (0.20 + \Delta x_2)$ which will serve as both the initial height as well as the total compression distance. We separate it this way because the distance traveled to the maximum compression from equilibrium is the resulting amplitude Δx_2 that the question is asking for.

Apply energy conservation

$U_{top} + K_{top} = U_{sp(max\text{-}comp)}$

$mgh + \frac{1}{2} m v^2 = \frac{1}{2} k \Delta x_2^2$ $(8)(9.8)(0.20 + \Delta x_2) + \frac{1}{2}(8)(1.17)^2 = \frac{1}{2}(392)(0.20 + \Delta x_2)^2$

The solution of this quadratic would lead to the answer for Δx_2 which is the amplitude.

d) First find period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{392}} = 0.90s$ Then find frequency f = 1/T = 1.11 Hz

e) The maximum speed will occur at equilibrium because the net force is zero here and the blocks stop accelerating in the direction of motion momentarily. Past this point, an upwards net force begins to exist which will slow the blocks down as they approach maximum compressions and begin to oscillate.

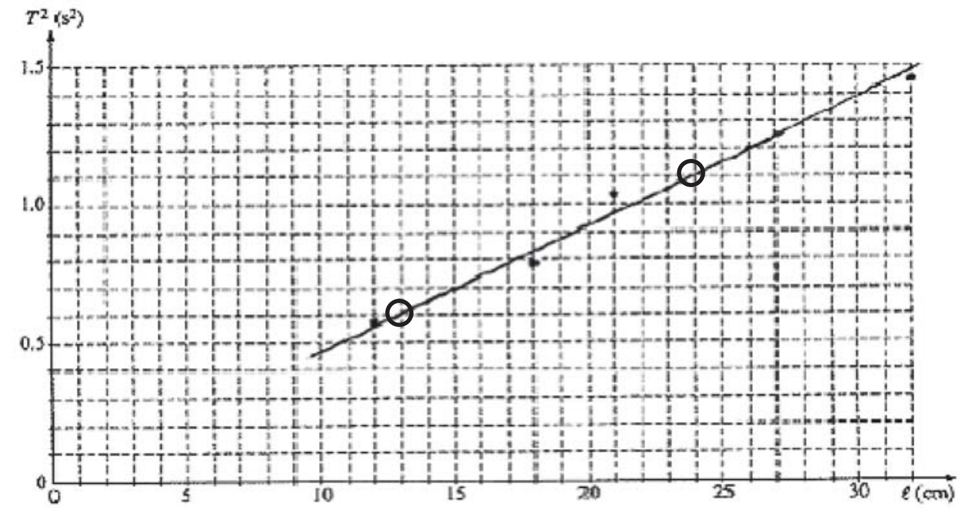
f) This motion is simple harmonic because the force acting on the masses is given by $F=k\Delta x$ and is therefore directly proportional to the displacement meeting the definition of simple harmonic motion

C2000M1.

a)

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62	0.762	0.581
18	8.89	0.889	0.790
21	10.09	1.009	1.018
32	12.08	1.208	1.459

b)



c) We want a linear equation of the form y = mx.

Based on $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\qquad T^2 = 2^2\pi^2\dfrac{L}{g}$ $\qquad T^2 = \dfrac{4\pi^2}{g}L$

y = m x

This fits our graph with y being T^2 and x being L. Finding the slope of the line will give us a value that we can equate to the slope term above and solve it for g. Since the points don't fall on the line we pick random points as shown circled on the graph and find the slope to be = 4.55. Set this = to $4\pi^2$ / g and solve for g = 8.69 m/s^2

d) A +/- 4% deviation of the answer (8.69) puts its possible range in between 8.944 – 8.34 so this result does not agree with the given value 9.8

e) Since the value of g is less than it would normally be (you feel lighter) the elevator moving down would also need to be **accelerating down** to create a lighter feeling and smaller F_n. Using down as the positive direction we have the following relationship, $F_{net} = ma$ $\qquad mg - F_n = ma$ $\qquad F_n = mg - ma$
For F_n the be smaller than usual, a would have to be + which we defined as down.

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ $mgh = ½ mv^2$ $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\, v_f$ $M\left(\sqrt{2gH}\right) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Again we cannot use the turn horizontal trick because we do not know information at the equilibrium position. While the tray was initially at its equilibrium position, its collision with the clay changed where this location would be.

Even though the initial current rest position immediately after the collision has an unknown initial stretch to begin with due to the weight of the tray and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

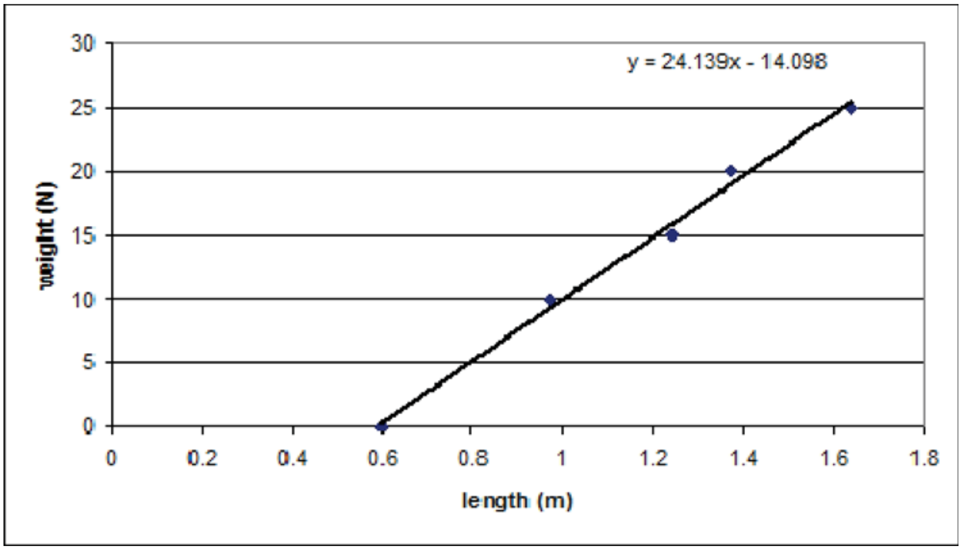
Apply energy conservation $K + U = U_{sp\,(gained)}$ $½ mv^2 + mgh = ½ k\, \Delta x^2$
Plug in mass (2m), h = H/2 and Δx = H/2 ➔ $½ (2m)v^2 + (2m)g(H/2) = ½ k(H/2)^2$
plug in v_f from part b $m(2gH/4) + mgH = kH^2/8$ ….

Both sides * (1/H) ➔ mg/2 + mg = kH/8 ➔ 3/2 mg = kH/8 k = 12mg / H

d) Based on $T = 2\pi\sqrt{\dfrac{2M}{\frac{12Mg}{H}}} = 2\pi\sqrt{\dfrac{H}{6g}}$

C2008M3

(a)



(b) The slope of the line is $F / \Delta x$ which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

$mgh = ½ k \Delta x^2$ $m(9.8)(1.5) = ½ (24)(0.9)^2$ $m = 0.66$ kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $F_{sp} = (0.66)(9.8)$ $F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $6.5 = (24) \Delta x$ $\Delta x = 0.27$ m

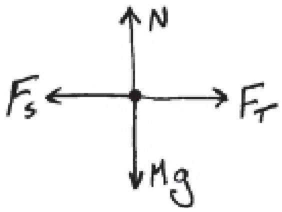
iii) Measured from the starting position of the mass, the equilibrium position would be located at the location marked by the unstretched cord length + the stretch found above. 0.6+0.27 = 0.87m. Set this as the h=0 location and equate the U_{top} to the U_{sp} + K here.

$mgh = ½ k \Delta x^2 + 1/2\, mv^2$ $(0.66)(9.8)(0.87) = ½ (24)(0.27)^2 + ½ (0.66) v^2$ $v = 3.8$ m/s

iv) This is the maximum speed because this is the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the mass down until it reaches its maximum compression and stops momentarily.

Supplemental.

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k\, \Delta x^2 = \frac{1}{2} k\Delta {x_2}^2 + \frac{1}{2} mv^2$
$k\, (\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$ $$v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$$

(d) To reach the position from the far left will take ½ of a period of oscillation.

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad t = \frac{1}{2}2\pi\sqrt{\frac{M}{k}} \qquad = \pi\sqrt{\frac{M}{k}}$$

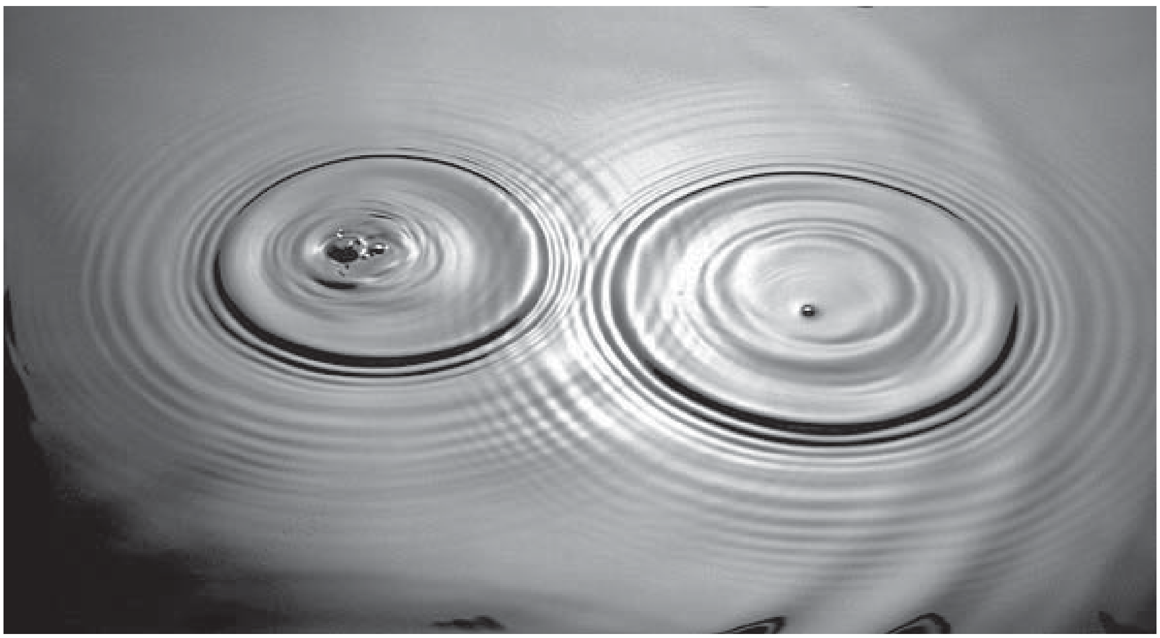
(e) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get

$F_{net} = ma$ $F_{sp} - f_k = ma$

From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$

Chapter 9

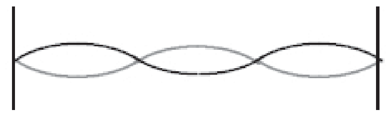
Waves and Sound



AP Physics Multiple Choice Practice –Waves and Optics

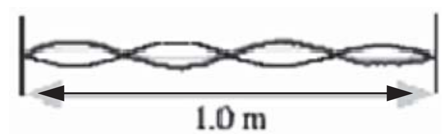
SECTION A – Waves and Sound

1. A string is firmly attached at both ends. When a frequency of 60 Hz is applied, the string vibrates in the standing wave pattern shown. Assume the tension in the string and its mass per unit length do not change. Which of the following frequencies could NOT also produce a standing wave pattern in the string?
   A) 30 Hz B) 40 Hz C) 80 Hz D) 180 Hz

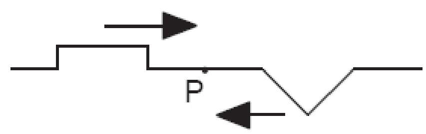


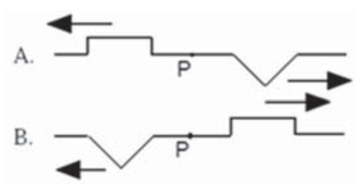
2. If the frequency of sound wave is doubled, the wavelength:
   A) halves and the speed remains unchanged.
   B) doubles and the speed remains unchanged.
   C) halves and the speed halves.
   D) doubles and the speed doubles.

3. The standing wave pattern diagrammed to the right is produced in a string fixed at both ends. The speed of waves in the string is 2 m/s. What is the frequency of the standing wave pattern?
   A) 0.25 Hz B) 1 Hz C) 2 Hz D) 4 Hz



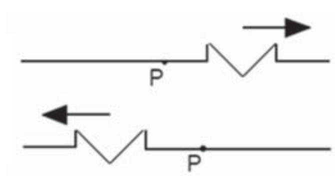
4. Two waves pulses approach each other as seen in the figure. The wave pulses overlap at point P. Which diagram best represents the appearance of the wave pulses as they leave point P?





C.

D.



5. If the speed of sound in air is 340 m/s, the length of the organ pipe, open at both ends, that can resonate at the fundamental frequency of 136 Hz, would be:
   A) 0.40 m B) 0.80 m C) 1.25 m D) 2.5 m

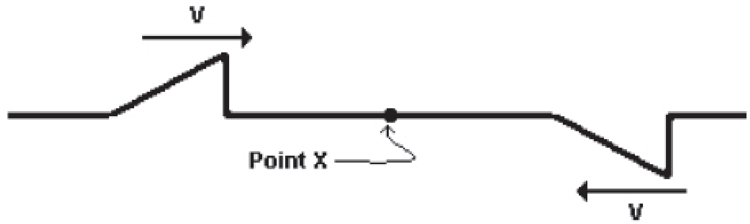
6. As sound travels from steel into air, both its speed and its:
   A) wavelength increase B) wavelength decrease C) frequency increase D) frequency remain unchanged

7. A pipe that is closed at one end and open at the other resonates at a fundamental frequency of 240 Hz. The next lowest/highest frequency it resonates at is most nearly.
   A) 80 Hz B) 120 Hz C) 480 Hz D) 720 Hz

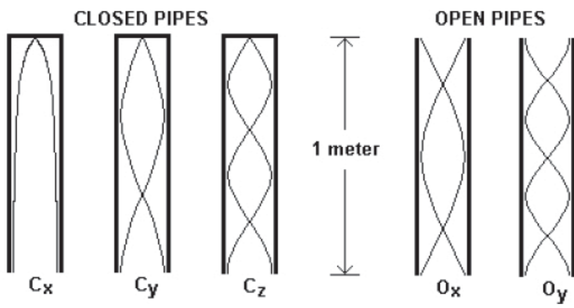
8. Assume that waves are propagating in a uniform medium. If the frequency of the wave source doubles then
   A) the wavelength of the waves halves. B) the wavelength of the waves doubles.
   C) the speed of the waves halves. D) the speed of the waves doubles.

9. Assume the speed of sound is 340 m/s. One stereo loudspeaker produces a sound with a wavelength of 0.68 meters while the other speaker produces sound with a wavelength of 0.65 m. What would be the resulting beat frequency?
   A) 3 Hz B) 23 Hz C) 511.5 Hz D) 11,333 Hz

10. The diagram shows two transverse pulses moving along a string. One pulse is moving to the right and the second is moving to the left. Both pulses reach point x at the same instant. What would be the resulting motion of point x as the two pulses pass each other?



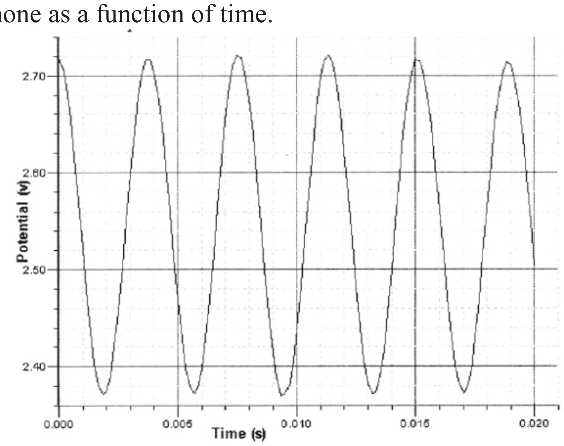
A) down, up, down
B) up then down
C) up, down, up
D) there would be no motion, the pulses cancel one another



11. **Multiple Correct.** The diagrams above represent 5 different standing sound waves set up inside of a set of organ pipes 1 m long Which of the following statements correctly relates the frequencies of the organ pipes shown? Select two answers.

A) C_y is twice the frequency of C_x. B) C_z is five times the frequency of C_x.
C) O_y is twice the frequency of O_x. D) O_x is twice the frequency of C_x.

Questions 12-13: The graph below was produced by a microphone in front of a tuning fork. It shows the voltage produced from the microphone as a function of time.



12. The frequency of the tuning fork is (approximately)
A) 0.004 s B) 0.020 s C) 50Hz D) 250 Hz

13. In order to calculate the speed of sound from the graph, you would also need to know
A) pitch B) wavelength C) frequency D) volume

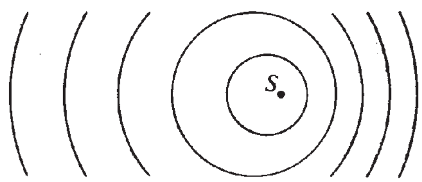
14. A tube is open at both ends with the air oscillating in the 4^{th} harmonic. How many displacement nodes are located within the tube?
A) 2 B) 3 C) 4 D) 5

15. A person vibrates the end of a string sending transverse waves down the string. If the person then doubles the rate at which he vibrates the string while maintaining the same tension , the speed of the waves
A) is unchanged while the wavelength is halved.
B) is unchanged while the wavelength is doubled.
C) doubles while the wavelength doubled.
D) doubles while the wavelength is halved.

16. A tube of length L_1 is open at both ends. A second tube of length L_2 is closed at one end and open at the other end. This second tube resonates at the same fundamental frequency as the first tube. What is the value of L_2 ?
A) $4L_1$ B) $2L_1$ C) L_1 D) ½ L_1

17. For a standing wave mode on a string fixed at both ends, adjacent antinodes are separated by a distance of 20 cm. Waves travel on this string at a speed of 1200 cm/s. At what frequency is the string vibrated to produce this standing wave?
(A) 120 *Hz* (B) 60 *Hz* (C) 40 *Hz* (D) 30 *Hz*

18. What would be the wavelength of the fundamental and first two overtones produced by an organ pipe of length L that is closed at one end and open at the other?
A) L, ½ L, ¼ L B) ½ L, ¼ L, 1/6 L C) 4L, 4/3 L, 4/5 L D) 4L, 2L, L

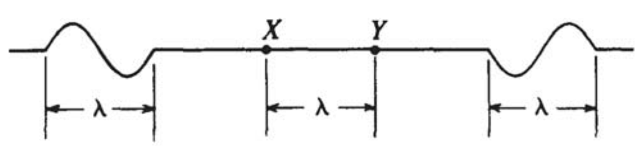


19. A small vibrating object S moves across the surface of a ripple tank producing the wave fronts shown above. The wave fronts move with speed v. The object is traveling in what direction and with what speed relative to the speed of the wave fronts produced?

Direction	Speed
(A) To the right	Equal to v
(B) To the right	Less than v
(C) To the left	Less than v
(D) To the left	Greater than v

20. A vibrating tuning fork sends sound waves into the air surrounding it. During the time in which the tuning fork makes one complete vibration, the emitted wave travels
(A) one wavelength
(B) about 340 meters
(C) a distance directly proportional to the square root of the air density
(D) a distance inversely proportional to the square root of the pressure

21. Two wave pulses, each of wavelength λ, are traveling toward each other along a rope as shown. When both pulses are in the region between points X and Y, which are a distance λ apart, the shape of the rope is



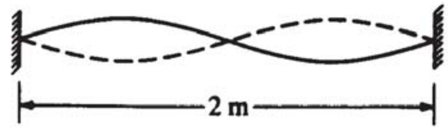
(A) 

(B) 

(C) 

(D) 

Questions 22-23

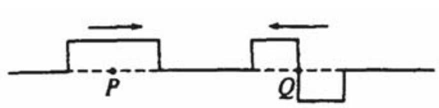


A standing wave of frequency 5 hertz is set up on a string 2 meters long with nodes at both ends and in the center, as shown above.

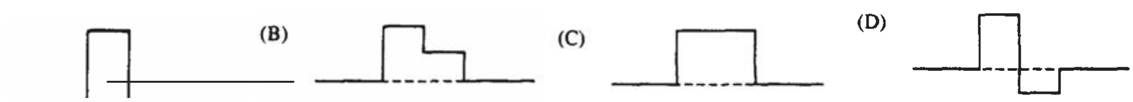
22. The speed at which waves propagate on the string is
    A) 0.4 m/s B) 2.5 m/s C) 5 m/s D) 10 m/s

23. The fundamental frequency of vibration of the string is
    A) 1 Hz B) 2.5 Hz C) 5 Hz D) 10 Hz

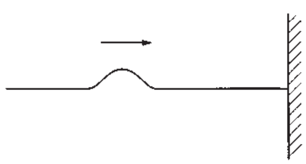
24. **Multiple correct:** In the Doppler Effect for sound waves, factors that affect the frequency that the observer hears include which of the following? Select two answers.
    A) the loudness of the sound
    B) the speed of the source
    C) the speed of the observer
    D) the phase angle



25. The figure above shows two wave pulses that are approaching each other. Which of the following best shows the shape of the resultant pulse when the centers of the pulses, points P and Q coincide?

(A) 

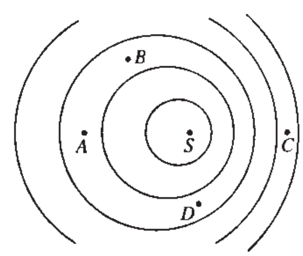
26. **Multiple Correct:** One end of a horizontal string is fixed to a wall. A transverse wave pulse is generated at the other end, moves toward the wall as shown and is reflected at wall. Properties of the reflected pulse include which of the following? Select two answers:



(A) It has a greater speed than that of the incident pulse.
(B) It has a greater amplitude than that of the incident pulse.
(C) It is on the opposite side of the string from the incident pulse.
(D) It has a smaller amplitude than that of the incident pulse.

27. A small vibrating object on the surface of a ripple tank is the source of waves of frequency 20 Hz and speed 60 cm/s. If the source *S* is moving to the right, as shown, with speed 20 cm/s, at which of the labeled points will the frequency measured by a stationary observer be greatest?

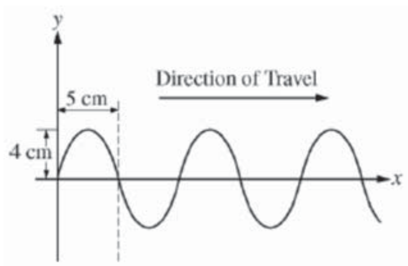
    (A) A (B) B (C) C (D) D



28. The frequencies of the first two overtones (second and third harmonics) of a vibrating string are f and 3f/2 .What is the fundamental frequency of this string?
    A) f/3 B) f/2 C) f D) 2f

29. **Multiple Correct:** Two fire trucks have sirens that emit waves of the same frequency. As the fire trucks approach a person, the person hears a higher frequency from truck X than from truck Y. Which of the following statements about truck X can be correctly inferred from this information? Select two answers.
   A) It is traveling faster than truck Y.
   B) It is closer to the person than truck Y.
   C) It is speeding up, and truck Y is slowing down.
   D) Its wavefronts are closer together than truck Y.

<u>Questions 30-31:</u>



The figure above shows a transverse wave traveling to the right at a particular instant of time. The period of the wave is 0.2 s.

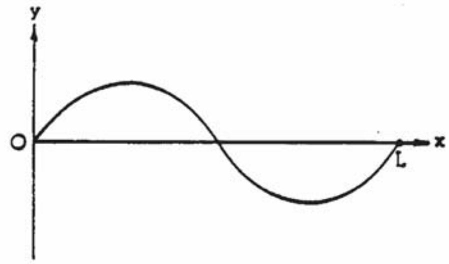
30. What is the amplitude of the wave?
   A) 4 cm B) 5 cm C) 8 cm D) 10 cm

31. What is the speed of the wave?
   A) 4 cm/s B) 25 cm/s C) 50 cm/s D) 100 cm/s

32. **Multiple Correct:** A standing wave pattern is created on a guitar string as a person tunes the guitar by changing the tension in the string. Which of the following properties of the waves on the string will change as a result of adjusting only the tension in the string? Select two answers.
   A) the speed of the traveling wave that creates the pattern
   B) the wavelength of the standing wave
   C) the frequency of the standing wave
   D) the amplitude of the standing wave

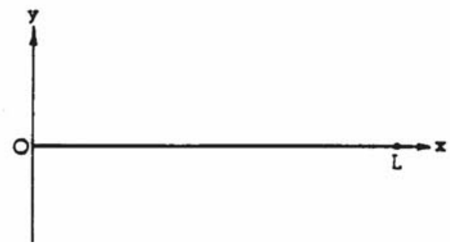
1980B4. In the graphs that follow, a curve is drawn in the first graph of each pair. For the other graph in each pair, sketch the curve showing the relationship between the quantities labeled on the axes. Your graph should be consistent with the first graph in the pair.

(c) y = Displacement of a String of Length L, Fixed at Both Ends, Vibrating at a Frequency f = 100 hertz



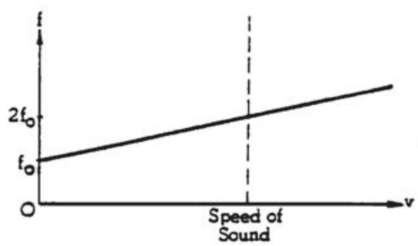
x = Distance from One End of the String

y = Displacement of a String of Length L, Fixed at Both Ends, Vibrating at a Frequency f = 150 hertz



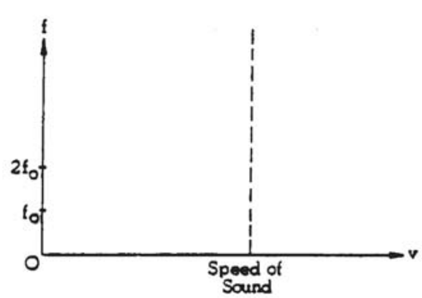
x = Distance from One End of the String

(d) f = Observed Frequency When Observer Moves Toward Stationary Source Emitting Sound of Frequency f_o



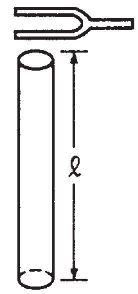
v = Speed of Moving Observer

f = Observed Frequency When Source Emitting Sound of Frequency f_o Moves Toward Stationary Observer

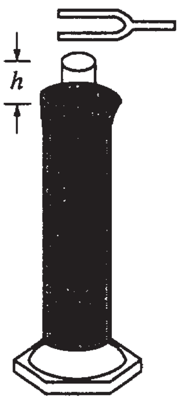


v = Speed of Moving Source

1995B6. A hollow tube of length L open at both ends as shown, is held in midair. A tuning fork with a frequency f_o vibrates at one end of the tube and causes the air in the tube to vibrate at its fundamental frequency. Express your answers in terms of L and f_o



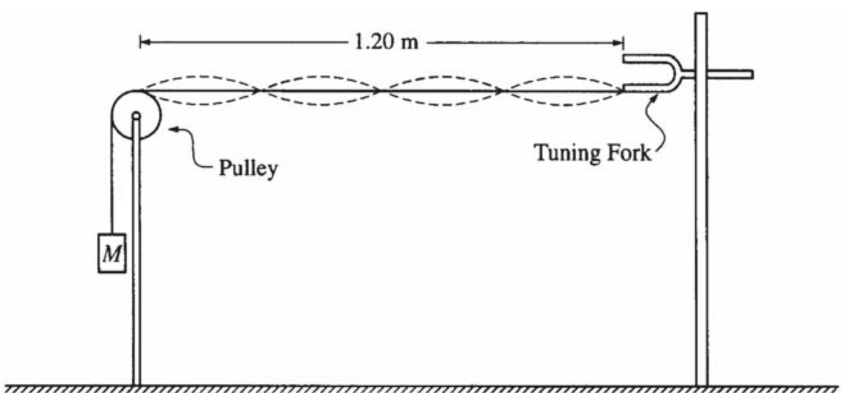
a. Determine the wavelength of the sound.
b. Determine the speed of sound in the air inside the tube.
c. Determine the next higher frequency at which this air column would resonate.



Note: Figure not drawn to scale.

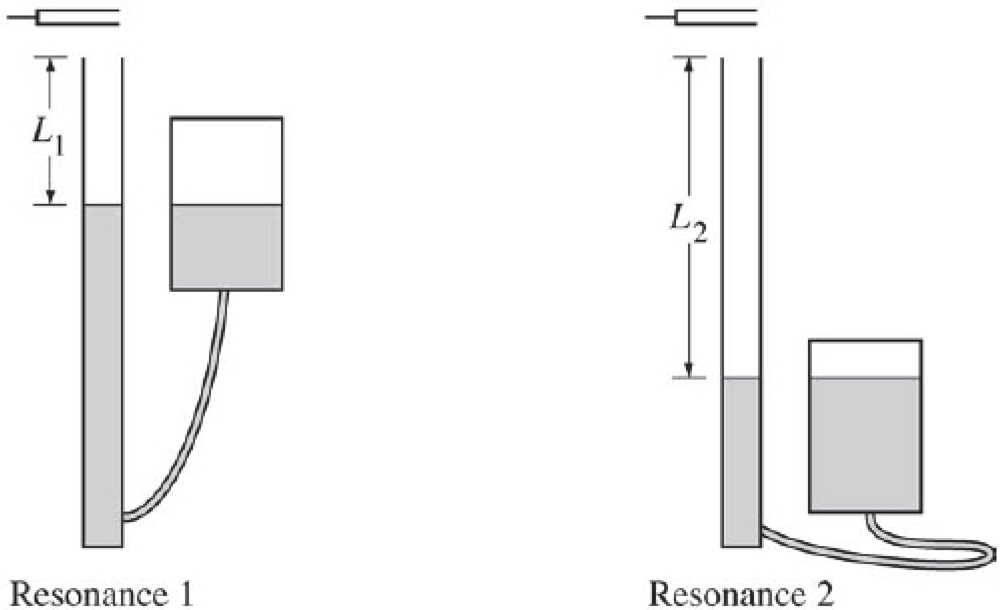
The tube is submerged in a large, graduated cylinder filled with water. The tube is slowly raised out of the water and the same tuning fork, vibrating with frequency f_o, is held a fixed distance from the top of the tube.

d. Determine the height h of the tube above the water when the air column resonates for the first time. Express your answer in terms of L.



1998B5. To demonstrate standing waves, one end of a string is attached to a tuning fork with frequency 120 Hz. The other end of the string passes over a pulley and is connected to a suspended mass M as shown in the figure above. The value of M is such that the standing wave pattern has four "loops." The length of the string from the tuning fork to the point where the string touches the top of the pulley is 1.20 m. The linear density of the string is 1.0×10^{-4} kg/m, and remains constant throughout the experiment.

a. Determine the wavelength of the standing wave.
b. Determine the speed of transverse waves along the string.
c. The speed of waves along the string increases with increasing tension in the string. Indicate whether the value of M should be increased or decreased in order to double the number of loops in the standing wave pattern. Justify your answer.
d. If a point on the string at an antinode moves a total vertical distance of 4 cm during one complete cycle, what is the amplitude of the standing wave?



Note: Figure not drawn to scale.

B2004B3. A vibrating tuning fork is held above a column of air, as shown in the diagrams above. The reservoir is raised and lowered to change the water level, and thus the length of the column of air. The shortest length of air column that produces a resonance is L_1 = 0.25 m, and the next resonance is heard when the air column is L_2 = 0.80 m long. The speed of sound in air at 20° C is 343 m/s and the speed of sound in water is 1490 m/s.

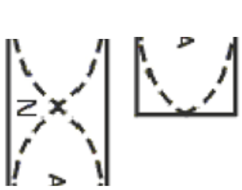
(a) Calculate the wavelength of the standing sound wave produced by this tuning fork.
(b) Calculate the frequency of the tuning fork that produces the standing wave, assuming the air is at 20° C.
(c) Calculate the wavelength of the sound waves produced by this tuning fork in the water, given that the frequency in the water is the same as the frequency in air.
(d) The water level is lowered again until a third resonance is heard. Calculate the length L_3 of the air column that produces this third resonance.
(e) The student performing this experiment determines that the temperature of the room is actually slightly higher than 20° C. Is the calculation of the frequency in part (b) too high, too low, or still correct?

_____ Too high _____ Too low _____ Still correct   Justify your answer.

AP Physics Multiple Choice Practice – Waves and Optics – ANSWERS

SECTION A – Waves and Sound

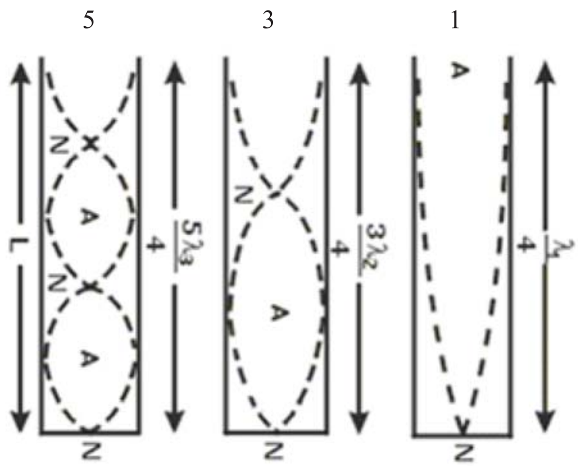
	Solution	Answer
1.	The given diagram is the 3rd harmonic at 60 Hz. That means the fundamental is 20Hz. The other possible standing waves should be multiples of 20	A
2.	Frequency and wavelength are inverses	A
3.	From diagram, wavelength = 0.5 m. Find the frequency with $v = f\lambda$	D
4.	After waves interfere they move along as if they never met	B
5.	For an open–open pipe the harmonic frequency is given by. $f_n = \frac{nv}{2L}$ with n=1	C
6.	When sound travels into less dense medium, its speed decreases (unlike light) … however, like all waves when traveling between two mediums, the frequency remains constant. Based on $v = f\lambda$, if the speed decreases and the frequency is constant then the λ must decrease also.	B
7.	Open–closed pipes only have odd multiples of harmonic so next f is 3x f_1	D
8.	For a given medium, speed is constant. Doubling the frequency halves the wavelength	A
9.	Determine each separate frequency using the speed of sound as 340 and $v = f\lambda$. Then subtract the two frequencies to get the beat frequency.	B
10.	Step the two pulses through each other a little bit at a time and use superposition to see how the amplitudes add. At first the amplitude jumps up rapidly, then the amplitude moves down as the rightmost negative pulse continues to propagate. At the very end of their passing the amplitude would be all the wave down and then once they pass the point will jump back up to equilibrium	C
11.	Wavelengths of each are (dist/cycle) … 4L, 4/3 L, 4/5 L, L, 2/3 L … Frequencies are $f = v/\lambda$. v/4L, 3v/4L, 5v/4L, v/L, 3v/2L … Oy is 2x Cy	B,C
12.	f = cycles / seconds	D
13.	To use $v = f\lambda$, you also need the λ	B
14.	To produce pipe harmonics, the ends are always antinodes. The first (fundamental) harmonic is when there are two antinodes on the end and one node in-between. To move to each next harmonic, add another node in the middle and fill in the necessary antinodes. (ex, 2nd harmonic is ANANA … So the 4th harmonic would be ANANANANA and have four nodes. *Alternative solution … if you know what the harmonics look like you can draw them and manually count the nodes.*	C
15.	Since the medium stays the same the speed remains constant. Based on $v = f\lambda$, for constant speed, f and λ change as inverses.	A
16.	We should look at the harmonic shapes open–open vs open–closed.	D



Comparing the fundamental harmonic of the open–open pipe to the closed–open pipe. The closed–open pipe should be half as long as the open–open pipe in order to fit the proper number of wavelengths of the same waveform to produce the given harmonic in each.

17. Two antinodes by definition will be ½ λ apart. So 20 cm = ½ λ, and the λ = 40 cm. Then using v = f λ we have 1200 = f (40) — D

18. This is similar to question 26, except now the length of the tube remains constant and the wave is changing within the tube to make each successive waveform (this would be like using different tuning forks each time for the same tube). The diagrams would look like this now: — C



Each λ is given by λ = dist / cycle
So
$\lambda_1 = 4L$
$\lambda_3 = 4/3\ L$
$\lambda_5 = 4/5\ L$

19. Doppler effect. The waves at right are compressed because the object is moving right. However, the waves are moving faster than the object since they are out in front of where the object is. — B

20. The time to make 1 cycle, is also the time it takes the wave to travel 1 λ. — A

21. Superpose the two waves on top of each other to get the answer. — B

22. Based on the diagram, the λ is clearly 2m. Plug into v = f λ. — D

23. The diagram shows the second harmonic in the string. Since harmonics are multiples, the first harmonic would be half of this. — B

24. A fact about the Doppler effect. Can also be seen from the Doppler equation (which is not required). — B, C

25. Use superposition and overlap the waves to see the resultant. — A

26. When hitting a fixed boundary, some of the wave is absorbed, some is reflected inverted. The reflected wave has less amplitude since some of the wave is absorbed, but since the string has not changed its properties the speed of the wave should remain unchanged. — C, D

27. Clearly at point C the waves are compressed so are more frequent. — C

28. Harmonics are multiples of the fundamental so the fundamental must be f/2. — B

29. Based on the Doppler effect, only speed matters. The faster a vehicle is moving, the closer together the sound waves get compressed and the higher the frequency. Take the case of a very fast vehicle traveling at the speed of sound; the compressions are all right on top of each other. So faster speed means closer compressions and higher frequencies. Choice I must be true because X is a higher frequency so must be going faster. Distance to the person affects the volume but not the pitch so choice II is wrong. III seems correct but its not. It doesn't matter whether you are speeding up or slowing down, it only matters who is going faster. For example, suppose truck X was going 5 mph and speeding up while truck Y was going 50 mph and slowing — A,D

down, this is an example of choice III but would not meet the requirement that X has a higher frequency because only actual speed matters, not what is happening to that speed.

30. By inspection. A

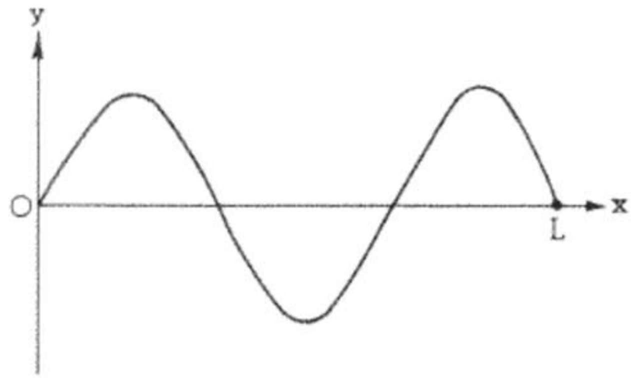
31. By inspection, the λ is 10 cm. f = 1 / T = 5, Then use v = f λ. C

32. A,C

Based on $v = \sqrt{\frac{F_t}{m/l}}$, the tension changes the speed. Then based on $f_n = \frac{nv}{2L}$, this resulting speed change will effect the frequency also. But since the frequency increases in direct proportion to the speed, and v = f λ, the λ should remain unchanged.
Note: equation of wave speed not required

1975B4.

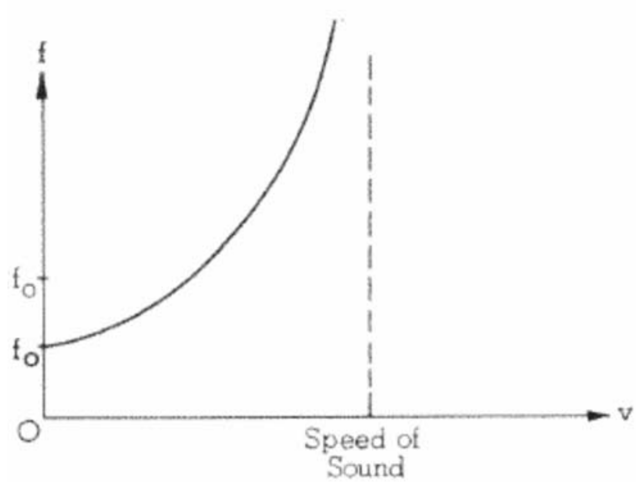
c) simple graph with 1.5x the frequency



d) Graphs are based on the Doppler equation. The graph given in the problem is for a moving observer.

Which is based on $f' = f\,\dfrac{(v_{snd} + v_{obs})}{v_{snd}}$. As the observer's velocity increases, the frequency increases linearly with it as is shown in the problem

The new graph is based on a source moving towards you. $f' = f\,\dfrac{v_{snd}}{(v_{snd} - v_{source})}$. As can be seed from this equation, as the source increases velocity, the frequency increases but when the source approaches the speed of sound, the frequency approaches ∞ and becomes undefined so has a limit to it unlike in the first graph.



1995B6.

a) The fundamental in a open–open pipe looks like this  and is ½ of a wavelength of the wave. Since this ½ wavelength fits in the length L, the total wavelength would have to be 2L.

b) Simply use $v = f\lambda \rightarrow v = 2Lf_o$

c) Harmonics are multiples of the fundamental, so the next frequency is $2f_o$

d) This is the same tuning fork so it is the same wavelength and waveform but the bottom is now closed so the wave looks like this.

The tube we had initially, fit ½ of a wavelength inside, and since its the same wavelength wave, again ½ of the wavelength of this wave would fit in length L and it would look like this.

This is impossible for a standing wave in an open–closed tube, and its also not the fundamental anyway so we have to change the length to make it look like the fundamental, Shown below. To do this, we make the length half of what it used to be.

h = L/2

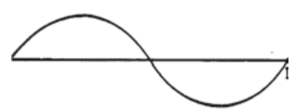
1998B5.

a) λ = dist / cycles = 1.2 m / 4 = 0.60 m

b) $v = f\lambda = (120)(0.60) = 72$ m/s

c) More 'loops' means a smaller wavelength. The frequency of the tuning fork is constant. Based on $v = f\lambda$, less speed would be required to make smaller wavelength. Since speed is based on tension, less M, makes less speed.

d) In one full cycle, a point on a wave covers 4 amplitudes … up, down, down, up. … So the amplitude is 1 cm.



B2004B3.

a) The shortest length makes the fundamental which looks like this  and is ¼ of the wavelength. This length is known to be 0.25m. So $L_1 = \frac{1}{4}\lambda$ … $\lambda = 4L_1 = 1$m.

Note: This is a real experiment, and in the reality of the experiment it is known that the antinode of the wave actually forms slightly above the top of the air column (you would not know this unless you actually performed this experiment). For this reason, the above answer is technically not correct as the tube length is slightly less than 1/4 of the wavelength. The better way to answer this question is to use the two values they give you for each consecutive harmonic. You are given the length of the first frequency (fundamental), and the length of the second frequency (third harmonic). Based on the known shapes of these harmonics, the difference in lengths between these two harmonics is equal to ½ the wavelength of the wave. Applying this →

$\Delta L = \frac{1}{2}\lambda$ … $0.8 – 0.25 = \frac{1}{2}\lambda$ $\lambda_{actual} = 1.1$ m.

Unfortunately the AP exam scored this question assuming you knew about the correction; though you received 3 out of 4 points for using the solution initially presented. We teachers, the authors of this solution guide, feel this is a bit much to ask for.

b) Using $v = f\lambda$ with the actual λ … $(340) = f(1.1)$ … $f = 312$ Hz.

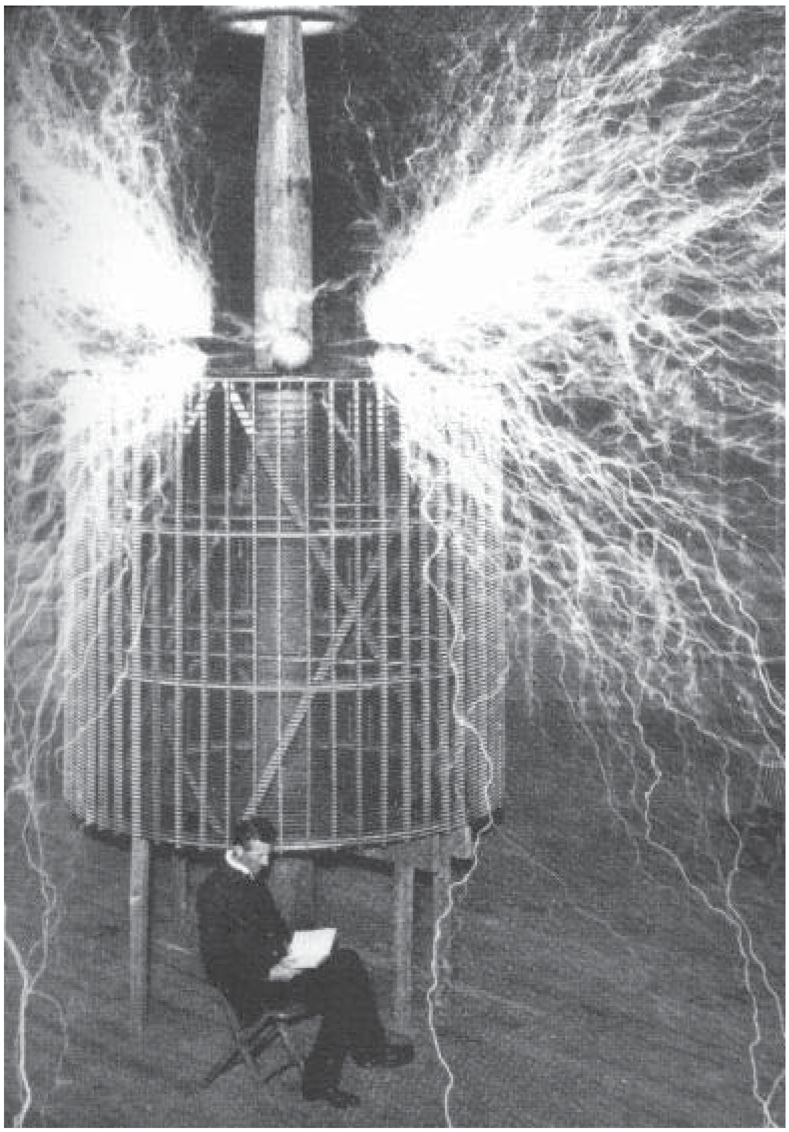
c) $v = f\lambda$ … $(1490) = (312)\lambda_{water}$ … $\lambda_{water} = 4.8$ m

d) Referring to the shapes of these harmonics is useful. The second length L_3 was the 3rd harmonic. The next harmonic (5th) will occur by adding another ½λ to the wave (based on how it looks you can see this). This will give a total length of $L_2 + \frac{1}{2}\lambda = (0.8) + \frac{1}{2}(1.1) = 1.35$ m

e) As temperature increases, the speed of sound in air increases, so the speed used in part (b) was too low. Since $f = v_{air} / \lambda$, that lower speed of sound yielded a frequency that was too low.

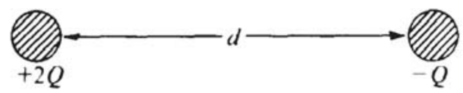
Chapter 10

Electrostatics



AP Physics Multiple Choice Practice – Electrostatics

1. A solid conducting sphere is given a positive charge Q. How is the charge Q distributed in or on the sphere?
(A) It is concentrated at the center of the sphere.
(B) It is uniformly distributed throughout the sphere.
(C) Its density increases radially outward from the center.
(D) It is uniformly distributed on the surface of the sphere only.



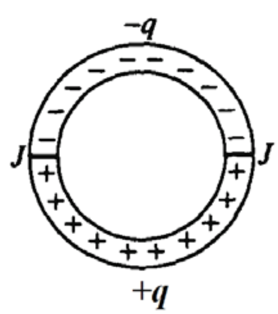
2. Two identical conducting spheres are charged to +2Q and –Q. respectively, and are separated by a distance d (much greater than the radii of the spheres) as shown above. The magnitude of the force of attraction on the left sphere is F_1. After the two spheres are made to touch and then are re-separated by distance d, the magnitude of the force on the left sphere is F_2. Which of the following relationships is correct?
(A) $2F_1 = F_2$ (B) $F_1 = F_2$ (C) $F_1 = 2F_2$ (D) $F_1 = 8 F_2$

3. Two isolated charges, + q and – 2q, are 2 centimeters apart. If F is the magnitude of the force acting on charge –2Q, what are the magnitude and direction of the force acting on charge +q?

	Magnitude	Direction
(A)	2 F	Away from charge –2q
(B)	F	Toward charge – 2q
(C)	F	Away from charge – 2q
(D)	2F	Toward charge – 2q

4. **Multiple correct:** Forces between two objects which are inversely proportional to the square of the distance between the objects include which of the following? Select two answers:
A.. Gravitational force between two celestial bodies
B. Electrostatic force between two electrons
C. Nuclear force between two neutrons
D. Magnetic force between two magnets

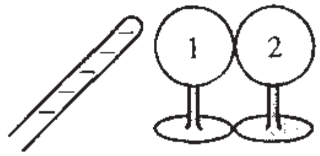
5. Two small spheres have equal charges q and are separated by a distance d. The force exerted on each sphere by the other has magnitude F. If the charge on each sphere is doubled and d is halved, the force on each sphere has magnitude
(A) F (B) 2F (C) 8F (D) 16F



6. A circular ring made of an insulating material is cut in half. One half is given a charge –q uniformly distributed along its arc. The other half is given a charge + q also uniformly distributed along its arc. The two halves are then rejoined with insulation at the junctions J, as shown above. If there is no change in the charge distributions, what is the direction of the net electrostatic force on an electron located at the center of the circle?
(A) Toward the top of the page (B) Toward the bottom of the page (C) To the right D) To the left

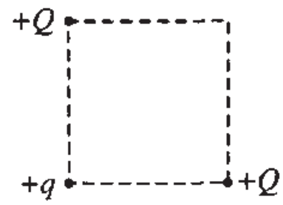
7. Two metal spheres that are initially uncharged are mounted on insulating stands, as shown above. A negatively charged rubber rod is brought close to, but does not make contact with, sphere X. Sphere Y is then brought close to X on the side opposite to the rubber rod. Y is allowed to touch X and then is removed some distance away. The rubber rod is then moved far away from X and Y. What are the final charges on the spheres?

	Sphere X	Sphere Y
A)	Negative	Negative
B)	Negative	Positive
C)	Positive	Negative
D)	Positive	Positive

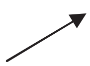
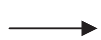
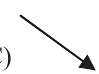
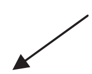


8. Two initially uncharged conductors, 1 and 2, are mounted on insulating stands and are in contact, as shown above. A negatively charged rod is brought near but does not touch them. With the rod held in place, conductor 2 is moved to the right by pushing its stand, so that the conductors are separated. Which of the following is now true of conductor 2?
(A) It is uncharged. (B) It is positively charged. (C) It is negatively charged.
(D) It is charged, but its sign cannot be predicted.

Questions 9-10

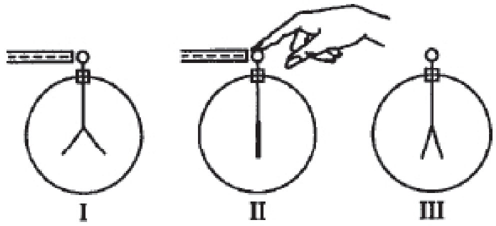


9. As shown above, two particles, each of charge +Q, are fixed at opposite corners of a square that lies in the plane of the page. A positive test charge +q is placed at a third corner. What is the direction of the force on the test charge due to the two other charges?

(A)  (B)  (C)  (D) 

10. If F is the magnitude of the force on the test charge due to only one of the other charges, what is the magnitude of the net force acting on the test charge due to both of these charges?
(A) $\frac{F}{\sqrt{2}}$ (B) F (C) $\sqrt{2}F$ (D) 2

11. Suppose that an electron (charge –e) could orbit a proton (charge +e) in a circular orbit of constant radius R. Assuming that the proton is stationary and only electrostatic forces act on the particles, which of the following represents the kinetic energy of the two–particle system?
(A) $\frac{1}{8\pi\varepsilon_0}\frac{e^2}{R}$ (B) $=\frac{1}{8\pi\varepsilon_0}\frac{e^2}{R}$ (C) $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$ (D) $=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$

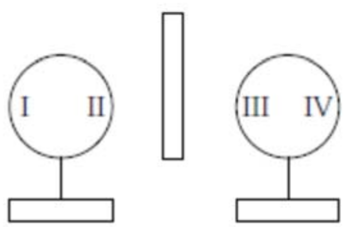


12. When a negatively charged rod is brought near, but does not touch, the initially uncharged electroscope shown above, the leaves spring apart (I). When the electroscope is then touched with a finger, the leaves collapse (II). When next the finger and finally the rod are removed, the leaves spring apart a second time (III). The charge on the leaves is
(A) positive in both I and III
(B) negative in both I and III
(C) positive in I, negative in III
(D) negative in I, positive in III

13. **Multiple Correct.** A positively charged conductor attracts a second object. Which of the following statements *could* be true? Select two answers
    A. The second object is a conductor with positive net charge.
    B. The second object is a conductor with zero net charge.
    C. The second object is an insulator with zero net charge.
    D. The second object is an insulator with positive net charge.

14. Two positive point charges repel each other with force 0.36 N when their separation is 1.5 m. What force do they exert on each other when their separation is 1.0 m?
(A) 0.81 N (B) 0.36 N (C) 0.24 N (D) 0.16 N

15. A point charge +q is placed midway between two point charges +3q and –q separated by a distance 2d. If Coulomb's constant is k, the magnitude of the force on the charge +q is:
(A) $2\dfrac{kq^2}{d^2}$ (B) $4\dfrac{kq^2}{d^2}$ (C) $6\dfrac{kq^2}{d^2}$ (D) $9\dfrac{kq^2}{d^2}$

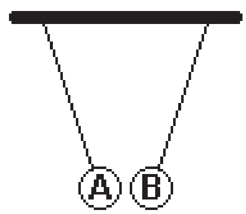


16. A charged rod is placed between two insulated conducting spheres as shown. The spheres have no net charge. Region II has the same polarity as Region
(A) I only (B) III only (C) IV only (D) I & IV only

17. When two charged point–like objects are separated by a distance *R*, the force between them is *F*. If the distance between them is quadrupled, the force between them is
(A) 16 *F* (B) 4 *F* (C) *F*/4 (D) *F*/16

18. An electroscope is given a positive charge, causing its foil leaves to separate. When an object is brought near the top plate of the electroscope, the foils separate even further. We could conclude
(A) that the object is positively charged.
(B) that the object is electrically neutral.
(C) that the object is negatively charged.
(D) only that the object is charged.

19. Four positive point charges are arranged as shown in the accompanying diagram. The force between charges 1 and 3 is 6.0 N; the force between charges 2 and 3 is 5.0 N; and the force between charges 3 and 4 is 3.0 N. The magnitude of the total force on charge 3 is most nearly
(A) 6.3 N (B) 8.0 N (C) 10 N (D) 11 N (E) 14 N



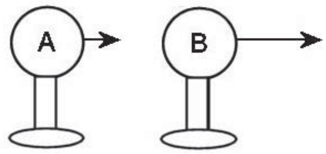
20. Two small hollow metal spheres hung on insulating threads attract one another as shown. It is known that a positively charged rod will attract ball A.
Which of the statements can be correctly concluded about the charge on the balls?
A. Ball A has a positive charge
B. Ball B has a negative charge
C. Ball A and Ball B have opposite charges
D. Ball A is neutral

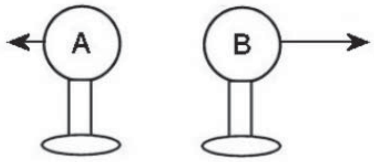
21. Two identical electrical point charges Q, separated by a distance d produce an electrical force of F on one another. If the distance is decreased to a distance of $0.40d$, what is the strength of the resulting force?
(A) $6.3F$ (B) $2.5F$ (C) $0.40F$ (D) $0.16F$

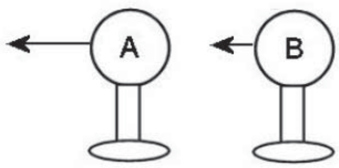
22. A positively charged object is brought near but not in contact with the top of an uncharged gold leaf electroscope. The experimenter then briefly touches the electroscope with a finger. The finger is removed, followed by the removal of the positively charged object. What happens to the leaves of the electroscope when a negative charge is now brought near but not in contact with the top of the electroscope?
(A) they remain uncharged
(B) they move farther apart
(C) they move closer together
(D) they remain negatively charged but unmoved

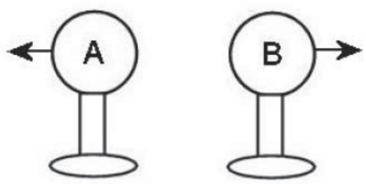
23. A positive point charge exerts a force of magnitude F on a negative point charge placed a distance x away. If the distance between the two point charges is halved, what is the magnitude of the new force that the positive point charge exerts on the negative point charge?
(A) $4F$ (B) $2F$ (C) $F/2$ (D) F/4

24. Two uniformly charged non–conducting spheres on insulating bases are placed on an air table. Sphere A has a charge $+3Q$ coulombs and sphere B has a charge $+Q$ coulombs. Which of the following correctly illustrates the magnitude and direction of the electrostatic force between the spheres when they are released?

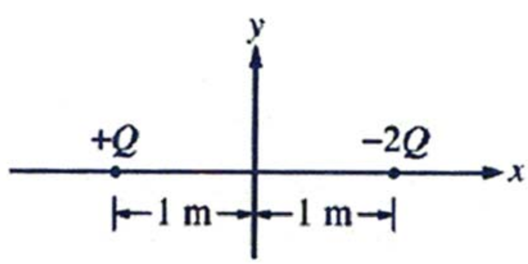
(A) 

(B) 

(C) 

(D) 

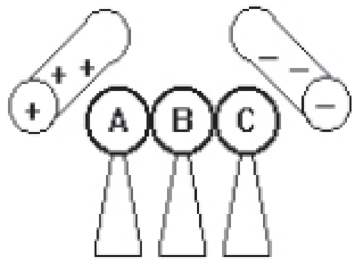
25. A person rubs a neutral comb through their hair and the comb becomes negatively charged. Which of the following is the best explanation for this phenomenon?
(A) The hair gains protons from the comb.
(B) The hair gains protons from the comb while giving electrons to the comb.
(C) The hair loses electrons to the comb.
(D) The comb loses protons to the person's hand while also gaining electrons from the hair.



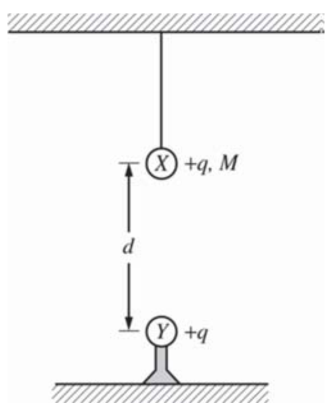
26. A charge of $+Q$ is located on the x–axis at $x = -1$ meter and a charge of $-2Q$ is held at $x = +1$ meter, as shown in the diagram above. At what position on the x–axis will a test charge of $+q$ experience a zero net electrostatic force?
(A) $-(3+\sqrt{8})$ m (B) –1/3 m (C) 1/3 m (D) $(3+\sqrt{8})$ m

27. Two point objects each carrying charge $10Q$ are separated by a distance d. The force between them is F. If half the charge on one object is transferred to the other object while at the same time the distance between them is doubled, what is the new force between the two objects?
(A) 0.19 F (B) 0.25 F (C) 4.0 F (D) no change in F

28. Two identical spheres carry identical electric charges. If the spheres are set a distance d apart they repel one another with a force F. A third sphere, identical to the other two but initially uncharged is then touched to one sphere and then to the other before being removed. What would be the resulting force between the original two spheres?
(A) ¾ F (B) 5/8 F (C) ½ F (D) 3/8 F

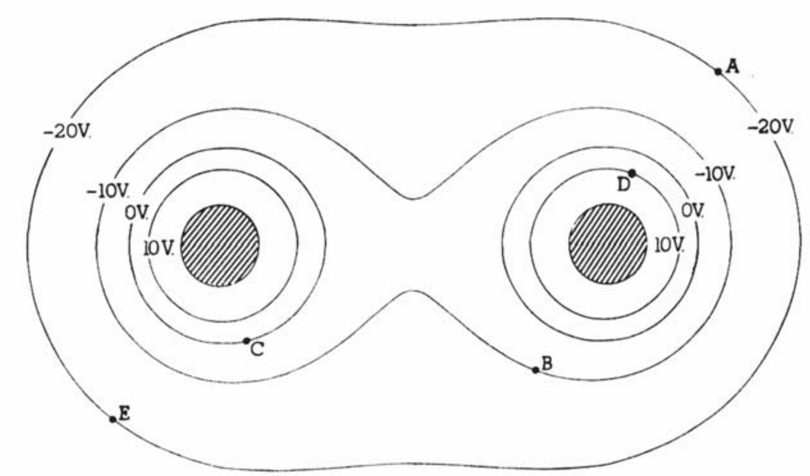


29. Three metal spheres A, B, and C are mounted on insulating stands. The spheres are touching one another, as shown in the diagram below. A strong positively charged object is brought near sphere A and a strong negative charge is brought near sphere C. While the charged objects remain near spheres A and C, sphere B is removed by means of its insulating stand. After the charged objects are removed, sphere B is first touched to sphere A and then to sphere C. The resulting charge on B would be of what relative amount and sign?
(A) the same sign but 1/2 the magnitude as originally on sphere A
(B) the opposite sign but 1/2 the magnitude as originally on sphere A
(C) the opposite sign but 1/4 the magnitude as originally on sphere A
(D) the same sign but 1/2 the magnitude as originally on sphere C



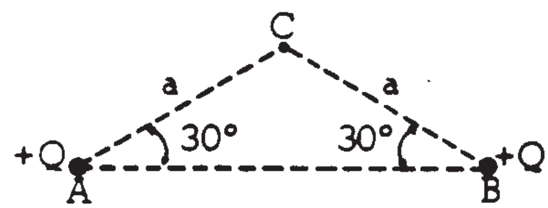
30. Sphere X of mass M and charge $+q$ hangs from a string as shown above. Sphere Y has an equal charge $+q$ and is fixed in place a distance d directly below sphere X. If sphere X is in equilibrium, the tension in the string is most nearly
(A) Mg (B) $Mg - kq/d$ (C) $Mg + kq^2/d^2$ (D) $Mg - kq^2/d^2$

AP Physics Free Response Practice – Electrostatics **WARNING: Only Electric Force is on AP Physics 1**

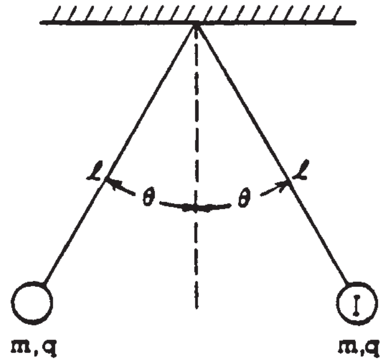


1974B5. The diagram above shows some of the equipotentials in a plane perpendicular to two parallel charged metal cylinders. The potential of each line is labeled.

a. The left cylinder is charged positively. What is the sign of the charge on the other cylinder?
b. On the diagram above, sketch lines to describe the electric field produced by the charged cylinders.
c. Determine the potential difference, $V_A - V_B$, between points A and B.
d. How much work is done by the field if a charge of 0.50 coulomb is moved along a path from point A to point E and then to point D?



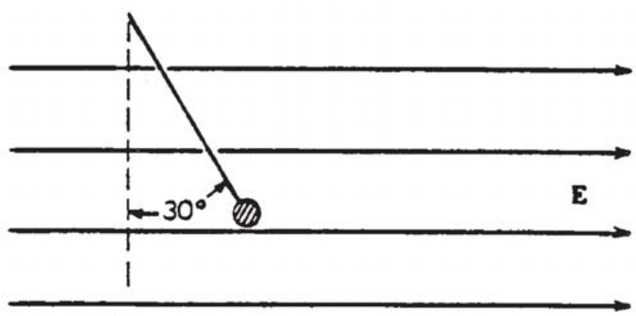
1975B2. Two identical electric charges +Q are located at two corners A and B of an isosceles triangle as shown above.
a. How much work does the electric field do on a small test charge +q as the charge moves from point C to infinity,
b. In terms of the given quantities, determine where a third charge +2Q should be placed so that the electric field at point C is zero. Indicate the location of this charge on the diagram above.



1979B7. Two small spheres, each of mass m and positive charge q, hang from light threads of lengths *l*. Each thread makes an angle θ with the vertical as shown above.

a. On the diagram draw and label all forces on sphere I.

b. Develop an expression for the charge q in terms of m, *l*, θ, g, and the Coulomb's law constant.

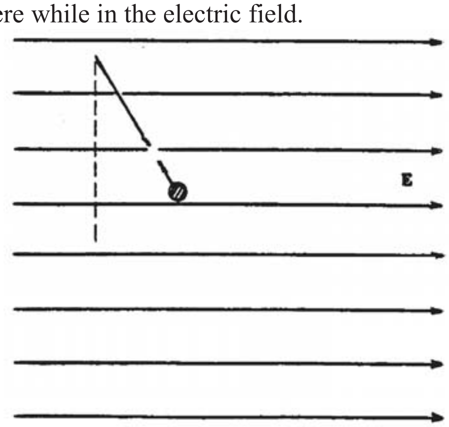


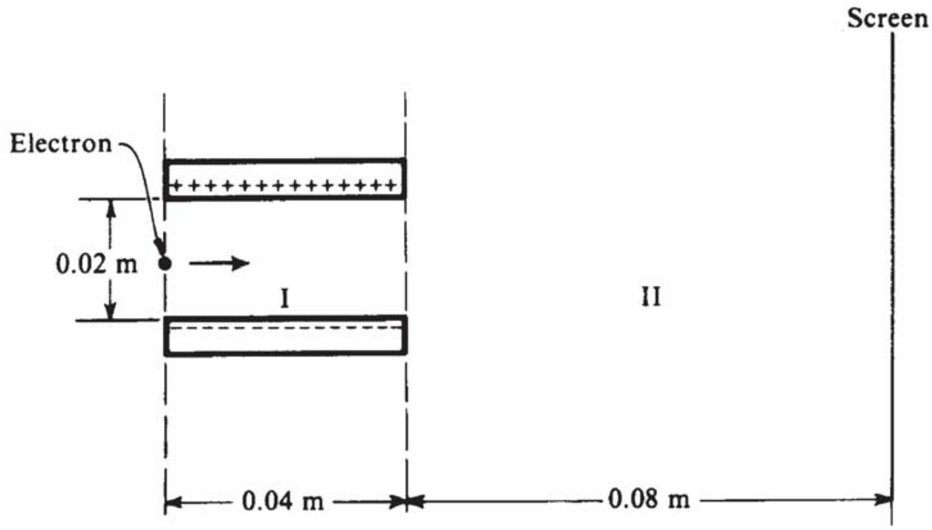
1981B3. A small conducting sphere of mass 5×10^{-3} kilogram, attached to a string of length 0.2 meter, is at rest in a uniform electric field E, directed horizontally to the right as shown above. There is a charge of 5×10^{-6} coulomb on the sphere. The string makes an angle of 30° with the vertical. Assume g = 10 meters per second squared.

a. In the space below, draw and label all the forces acting on the sphere.

b. Calculate the tension in the string and the magnitude of the electric field.

c. The string now breaks. Describe the subsequent motion of the sphere and sketch on the following diagram the path of the sphere while in the electric field.





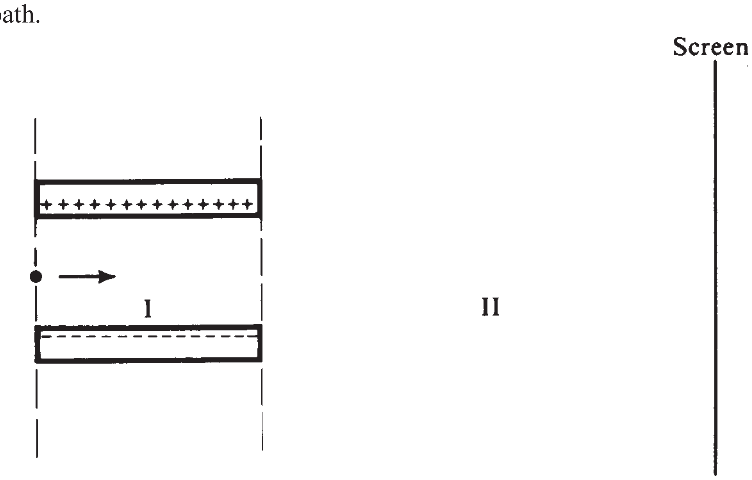
1985B3. An electron initially moves in a horizontal direction and has a kinetic energy of 2.0×10^3 electron–volts when it is in the position shown above. It passes through a uniform electric field between two oppositely charged horizontal plates (region I) and a field–free region (region II) before eventually striking a screen at a distance of 0.08 meter from the edge of the plates. The plates are 0.04 meter long and are separated from each other by a distance of 0.02 meter. The potential difference across the plates is 250 volts. Gravity is negligible.

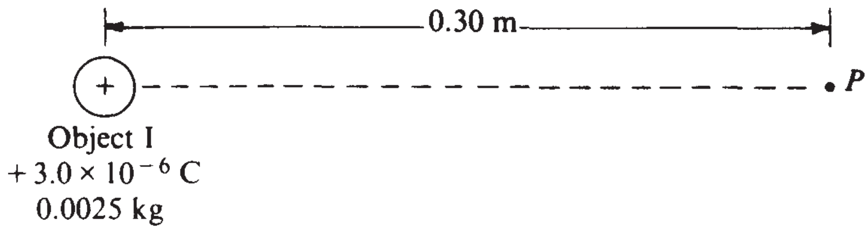
a. Calculate the initial speed of the electron as it enters region I.

b. Calculate the magnitude of the electric field E between the plates, and indicate its direction on the diagram above.

c. Calculate the magnitude of the electric force F acting on the electron while it is in region I.

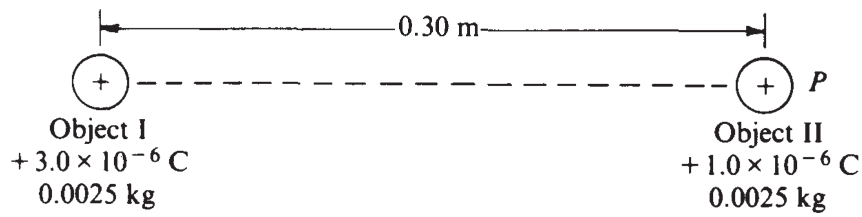
d. On the diagram below, sketch the path of the electron in regions I and II. For each region describe the shape of the path.





1987B2. Object I, shown above, has a charge of + 3 x 10^{-6} coulomb and a mass of 0.0025 kilogram.

a. What is the electric potential at point P, 0.30 meter from object I?

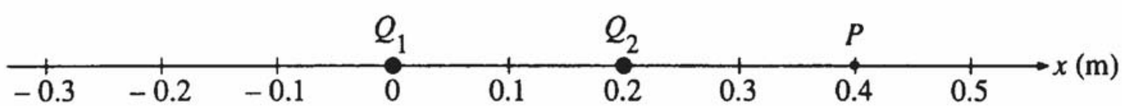


Object II, of the same mass as object I, but having a charge of + 1 x 10^{-6} coulomb, is brought from infinity to point P, as shown above.

b. How much work must be done to bring the object II from infinity to point P?
c. What is the magnitude of the electric force between the two objects when they are 0.30 meter apart?
d. What are the magnitude and direction of the electric field at the point midway between the two objects?

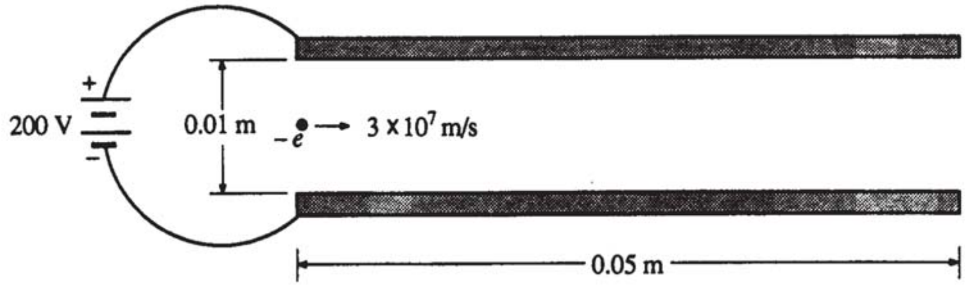
The two objects are then released simultaneously and move apart due to the electric force between them. No other forces act on the objects.

e. What is the speed of object I when the objects are very far apart?



1989B2. Two point charges, Q_1 and Q_2, are located a distance 0.20 meter apart, as shown above. Charge $Q_1 = +8.0\mu C$. The net electric field is zero at point P, located 0.40 meter from Q_1 and 0.20 meter from Q_2.

a. Determine the magnitude and sign of charge Q_2.
b. Determine the magnitude and direction of the net force on charge Q_1
c. Calculate the electrostatic potential energy of the system.
d. Determine the coordinate of the point R on the x–axis between the two charges at which the electric potential is zero.
e. How much work is needed to bring an electron from infinity to point R. which was determined in the previous part?



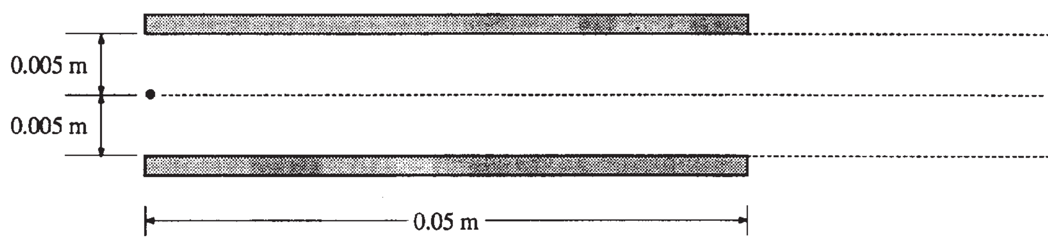
1990B2 (modified) A pair of square parallel conducting plates, having sides of length 0.05 meter, are 0.01 meter apart and are connected to a 200–volt power supply, as shown above. An electron is moving horizontally with a speed of 3×10^7 meters per second when it enters the region between the plates. Neglect gravitation and the distortion of the electric field around the edges of the plates.

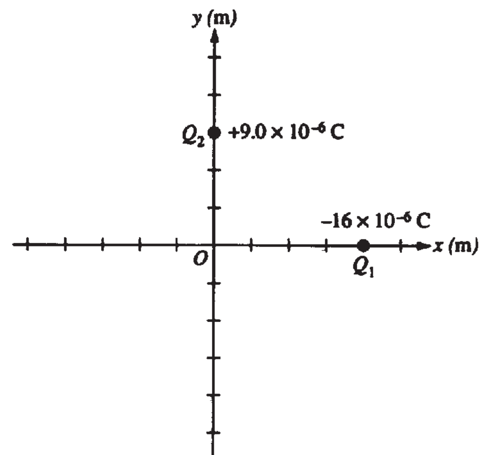
a. Determine the magnitude of the electric field in the region between the plates and indicate its direction on the figure above.

b. Determine the magnitude and direction of the acceleration of the electron in the region between the plates.

c. Determine the magnitude of the vertical displacement of the electron for the time interval during which it moves through the region between the plates.

d. On the diagram below, sketch the path of the electron as it moves through and after it emerges from the region between the plates. The dashed lines in the diagram have been added for reference only.



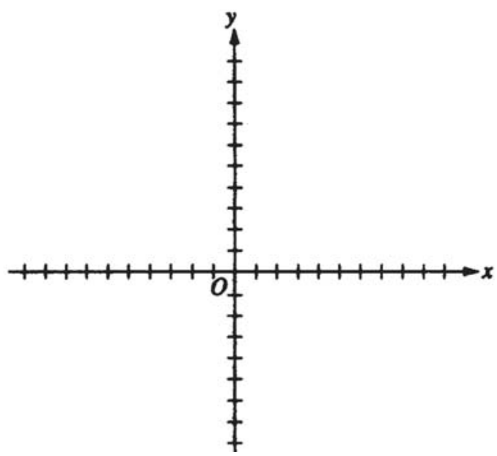


1993B2. A charge $Q_1 = -1.6 \times 10^{-6}$ coulomb is fixed on the x–axis at +4.0 meters, and a charge $Q_2 = +9 \times 10^{-6}$ coulomb is fixed on the y–axis at +3.0 meters, as shown on the diagram above.

a. i. Calculate the magnitude of the electric field E_1 at the origin O due to charge Q_1

ii. Calculate the magnitude of the electric field E_2 at the origin O due to charge Q_2.

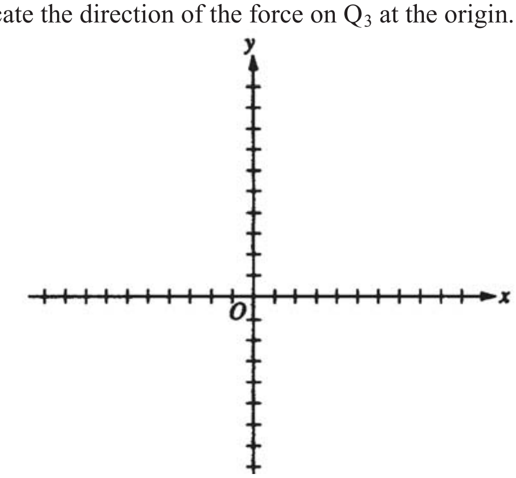
iii. On the axes below, draw and label vectors to show the electric fields E_1 and E_2 due to each charge, and also indicate the resultant electric field E at the origin.



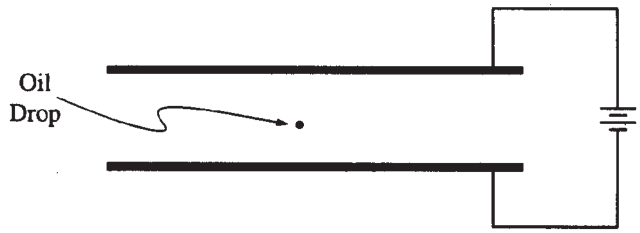
b. Calculate the electric potential V at the origin.

A charge $Q_3 = -4 \times 10^{-6}$ coulomb is brought from a very distant point by an external force and placed at the origin.

c. On the axes below, indicate the direction of the force on Q_3 at the origin.

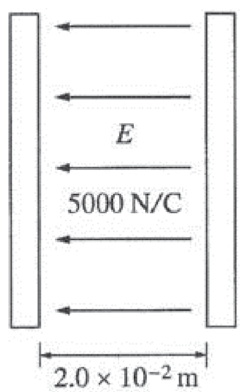


d. Calculate the work that had to be done by the external force to bring Q_3 to the origin from the distant point.



1996B6 Robert Millikan received a Nobel Prize for determining the charge on the electron. To do this, he set up a potential difference between two horizontal parallel metal plates. He then sprayed drops of oil between the plates and adjusted the potential difference until drops of a certain size remained suspended at rest between the plates, as shown above. Suppose that when the potential difference between the plates is adjusted until the electric field is 10,000 N/C downward, a certain drop with a mass of 3.27×10^{-16} kg remains suspended.

a. What is the magnitude of the charge on this drop?
b. The electric field is downward, but the electric force on the drop is upward. Explain why.
c. If the distance between the plates is 0.01 m, what is the potential difference between the plates?
d. The oil in the drop slowly evaporates while the drop is being observed, but the charge on the drop remains the same. Indicate whether the drop remains at rest, moves upward, or moves downward. Explain briefly.



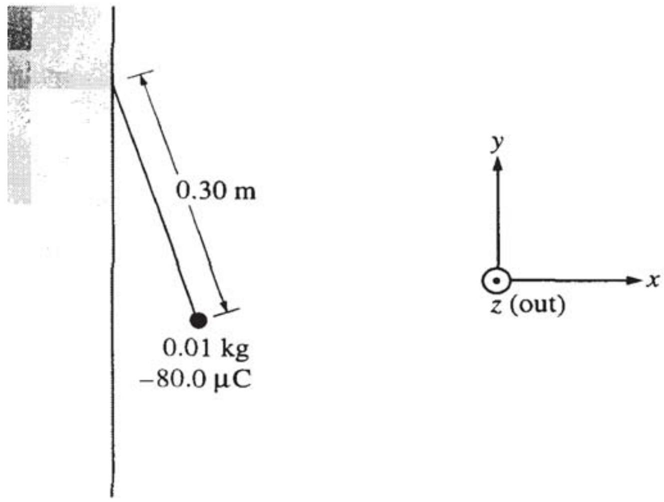
Note: Figure not drawn to scale.

2002B5B. Two parallel conducting plates, each of area 0.30 m^2, are separated by a distance of 2.0×10^{-2} m of air. One plate has charge +Q; the other has charge –Q. An electric field of 5000 N/C is directed to the left in the space between the plates, as shown in the diagram above.

a. Indicate on the diagram which plate is positive (+) and which is negative (–).
b. Determine the potential difference between the plates.
c. Determine the capacitance of this arrangement of plates.

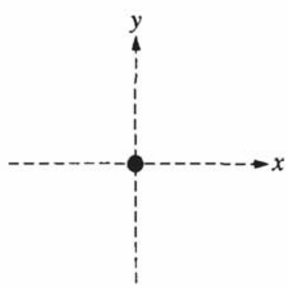
An electron is initially located at a point midway between the plates.

d. Determine the magnitude of the electrostatic force on the electron at this location and state its direction.
e. If the electron is released from rest at this location midway between the plates, determine its speed just before striking one of the plates. Assume that gravitational effects are negligible.



1998B2. A wall has a negative charge distribution producing a uniform horizontal electric field. A small plastic ball of mass 0.01 kg, carrying a charge of –80.0 μC, is suspended by an uncharged, nonconducting thread 0.30 m long. The thread is attached to the wall and the ball hangs in equilibrium, as shown above, in the electric and gravitational fields. The electric force on the ball has a magnitude of 0.032 N.

a. On the diagram below, draw and label the forces acting on the ball.



b. Calculate the magnitude of the electric field at the ball's location due to the charged wall, and state its direction relative to the coordinate axes shown.

c. Determine the perpendicular distance from the wall to the center of the ball.

d. The string is now cut.

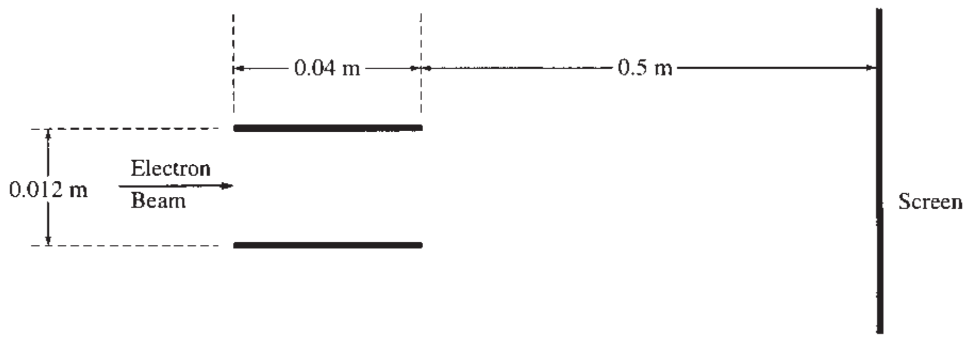
i. Calculate the magnitude of the resulting acceleration of the ball, and state its direction relative to the coordinate axes shown.

ii. Describe the resulting path of the ball.

1999B2. In a television set, electrons are first accelerated from rest through a potential difference in an electron gun. They then pass through deflecting plates before striking the screen.

a. Determine the potential difference through which the electrons must be accelerated in the electron gun in order to have a speed of 6.0×10^7 m/s when they enter the deflecting plates.

The pair of horizontal plates shown below is used to deflect electrons up or down in the television set by placing a potential difference across them. The plates have length 0.04 m and separation 0.012 m, and the right edge of the plates is 0.50 m from the screen. A potential difference of 200 V is applied across the plates, and the electrons are deflected toward the top of the screen. Assume that the electrons enter horizontally midway between the plates with a speed of 6.0×10^7 m/s and that fringing effects at the edges of the plates and gravity are negligible.



Note: Figure not drawn to scale.

b. Which plate in the pair must be at the higher potential for the electrons to be deflected upward? Check the appropriate box below.

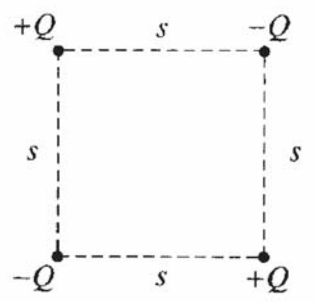
☐ Upper plate ☐ Lower plate

Justify your answer.

c. Considering only an electron's motion as it moves through the space between the plates, compute the following.
i. The time required for the electron to move through the plates
ii. The vertical displacement of the electron while it is between the plates

d. Show why it is a reasonable assumption to neglect gravity in part c.

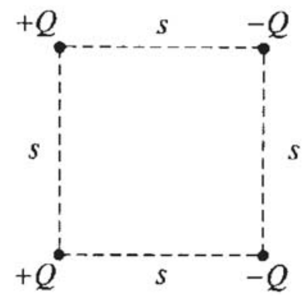
e. Still neglecting gravity, describe the path of the electrons from the time they leave the plates until they strike the screen. State a reason for your answer.



Arrangement 1

2001B3. Four charged particles are held fixed at the corners of a square of side s. All the charges have the same magnitude Q, but two are positive and two are negative. In Arrangement 1, shown above, charges of the same sign are at opposite corners. Express your answers to parts a. and b. in terms of the given quantities and fundamental constants.

a. For Arrangement 1, determine the following.
   i. The electrostatic potential at the center of the square
   ii. The magnitude of the electric field at the center of the square



Arrangement 2

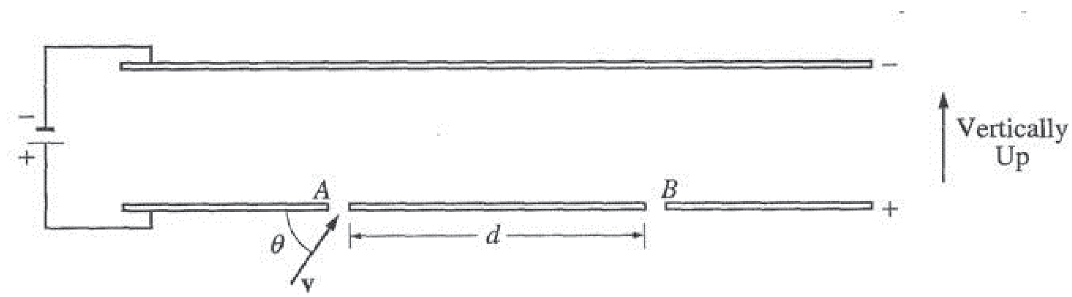
The bottom two charged particles are now switched to form Arrangement 2, shown above, in which the positively charged particles are on the left and the negatively charged particles are on the right.

b. For Arrangement 2, determine the following.
   i. The electrostatic potential at the center of the square
   ii. The magnitude of the electric field at the center of the square

c. In which of the two arrangements would more work be required to remove the particle at the upper right corner from its present position to a distance a long way away from the arrangement?

_________ Arrangement 1 ___________ Arrangement 2

Justify your answer

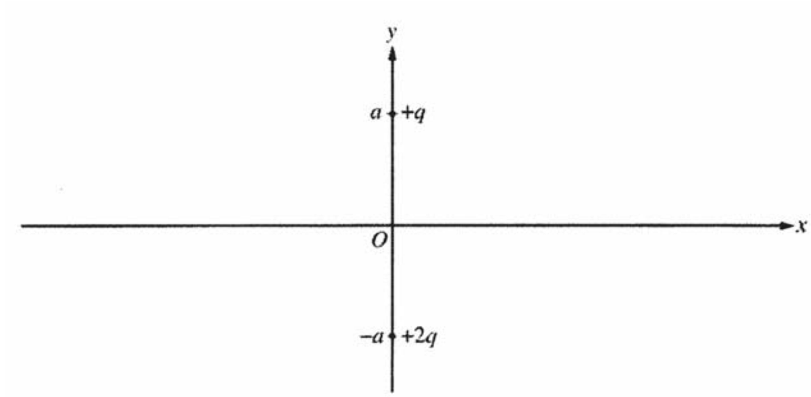


2003Bb4. An electric field E exists in the region between the two electrically charged parallel plates shown above. A beam of electrons of mass m, charge q, and velocity v enters the region through a small hole at position A. The electrons exit the region between the plates through a small hole at position B. Express your answers to the following questions in terms of the quantities m, q, E, θ, and v. Ignore the effects of gravity.

a. i. On the diagram of the parallel plates above, draw and label a vector to show the direction of the electric field E between the plates.
ii. On the following diagram, show the direction of the force(s) acting on an electron after it enters the region between the plates.

iii. On the diagram of the parallel plates above, show the trajectory of an electron that will exit through the small hole at position B.

b. Determine the magnitude of the acceleration of an electron after it has entered the region between the parallel plates.
c. Determine the total time that it takes the electrons to go from position A to position B.
d. Determine the distance d between positions A and B.
e. Now assume that the effects of gravity cannot be ignored in this problem. How would the distance where the electron exits the region between the plates change for an electron entering the region at A? Explain your reasoning.

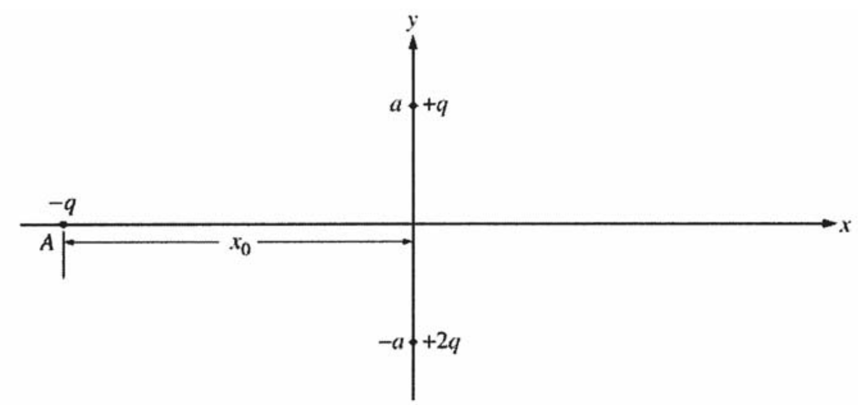


2005B3 Two point charges are fixed on the y–axis at the locations shown in the figure above. A charge of +q is located at y = +a and a charge of +2q is located at y = –a. Express your answers to parts a. and b. in terms of q, a, and fundamental constants.

a. Determine the magnitude and direction of the electric field at the origin.

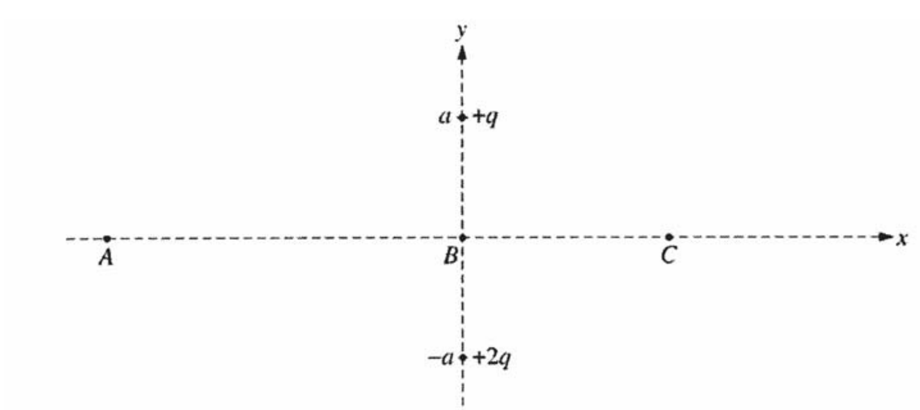
b. Determine the electric potential at the origin.

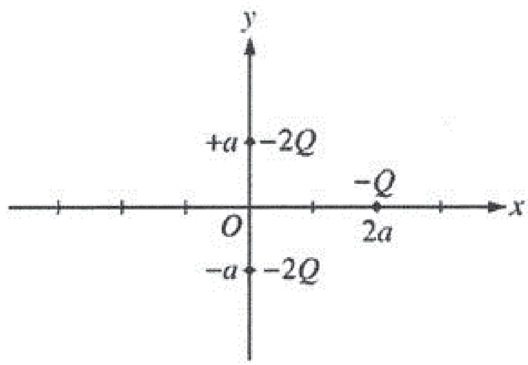
A third charge of –q is first placed at an arbitrary point A ($x = -x_o$) on the x–axis as shown in the figure below.



c. Write expressions in terms of q, a, x_0, and fundamental constants for the magnitudes of the forces on the –q charge at point A caused by each of the following.
 i. The +q charge
 ii. The +2q charge

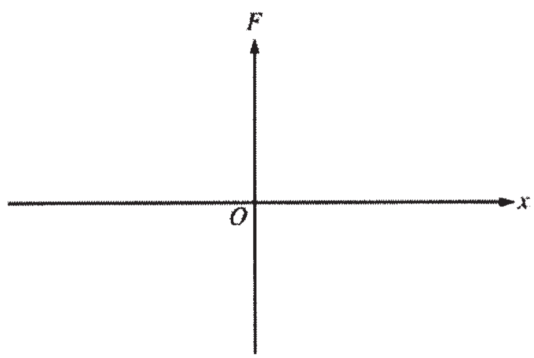
d. The –q charge can also be placed at other points on the x–axis. At each of the labeled points (A, B, and C) in the following diagram, draw a vector to represent the direction of the net force on the –q charge due to the other two charges when it is at those points.

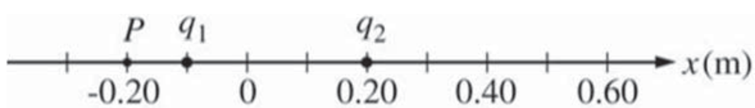




2005Bb3 The figure above shows two point charges, each of charge –2Q, fixed on the y–axis at y = +a and at y = –a. A third point charge of charge –Q is placed on the x–axis at x = 2a. Express all algebraic answers in terms of Q, a, and fundamental constants.

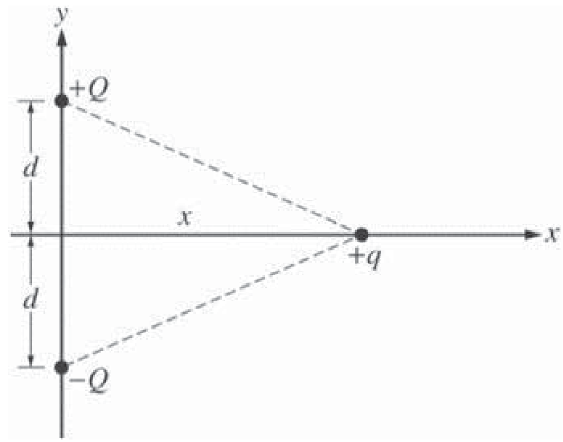
a. Derive an expression for the magnitude of the net force on the charge –Q due to the other two charges, and state its direction.
b. Derive an expression for the magnitude of the net electric field at the origin due to all three charges, and state its direction.
c. Derive an expression for the electrical potential at the origin due to all three charges.
d. On the axes below, sketch a graph of the force F on the –Q charge caused by the other two charges as it is moved along the x–axis from a large positive position to a large negative position. Let the force be positive when it acts to the right and negative when it acts to the left.





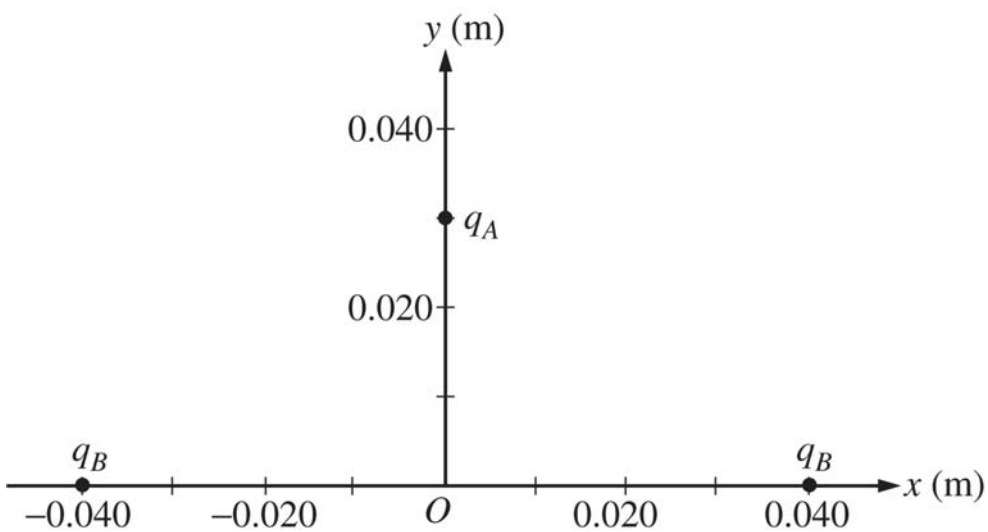
2006B3. Two point charges, q_1 and q_2, are placed 0.30 m apart on the x–axis, as shown in the figure above. Charge q_1 has a value of -3.0×10^{-9} C. The net electric field at point P is zero.

a. What is the sign of charge q_2?
____ Positive ____ Negative
Justify your answer.
b. Calculate the magnitude of charge q_2.
c. Calculate the magnitude of the electric force on q_2 and indicate its direction.
d. Determine the x–coordinate of the point on the line between the two charges at which the electric potential is zero.
e. How much work must be done by an external force to bring an electron from infinity to the point at which the electric potential is zero? Explain your reasoning.



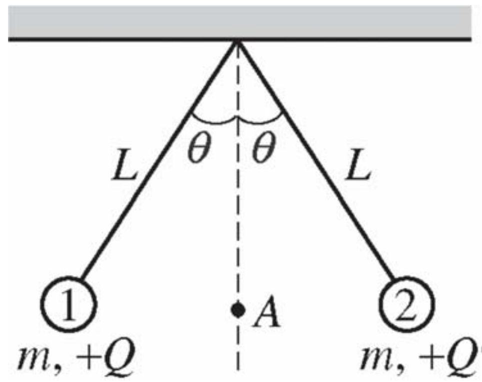
2006Bb3. Three electric charges are arranged on an x–y coordinate system, as shown above. Express all algebraic answers to the following parts in terms of Q, q, x, d, and fundamental constants.

a. On the diagram, draw vectors representing the forces F_1 and F_2 exerted on the +q charge by the +Q and –Q charges, respectively.
b. Determine the magnitude and direction of the total electric force on the +q charge.
c. Determine the electric field (magnitude and direction) at the position of the +q charge due to the other two charges.
d. Calculate the electric potential at the position of the +q charge due to the other two charges.
e. Charge +q is now moved along the positive x–axis to a very large distance from the other two charges. The magnitude of the force on the +q charge at this large distance now varies as $1/x^3$. Explain why this happens.



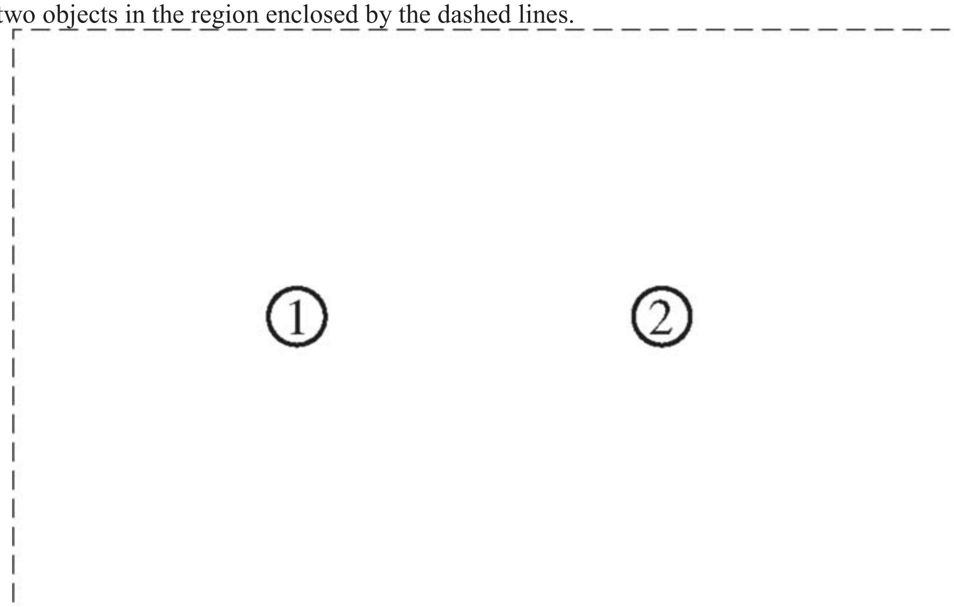
2009B2B.(modified) Three particles are arranged on coordinate axes as shown above. Particle A has charge q_A = –0.20 nC , and is initially on the y–axis at y = 0.030 m. The other two particles each have charge q_B = +0.30 nC and are held fixed on the x–axis at x = –0.040 m and x = +0.040 m, respectively.

a. Calculate the magnitude of the net electric force on particle A when it is at y = 0.030 m, and state its direction.
b. Particle A is then released from rest. Qualitatively describe its motion over a long time.



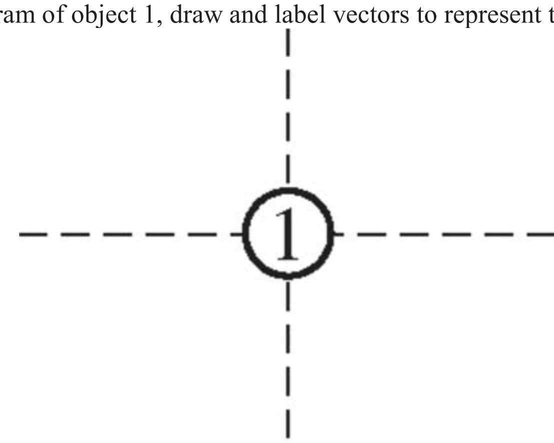
2009B2. Two small objects, labeled 1 and 2 in the diagram above, are suspended in equilibrium from strings of length L. Each object has mass m and charge +Q. Assume that the strings have negligible mass and are insulating and electrically neutral. Express all algebraic answers in terms of m, L, Q, q , and fundamental constants.

a. On the following diagram, sketch lines to illustrate a 2–dimensional view of the net electric field due to the two objects in the region enclosed by the dashed lines.

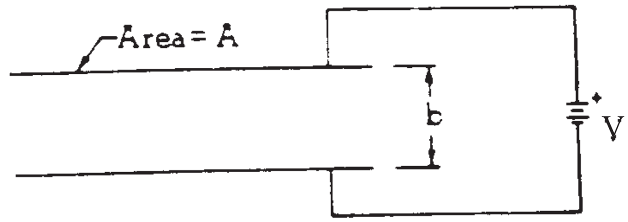


b. Derive an expression for the electric potential at point A, shown in the diagram at the top of the page, which is midway between the charged objects.

c. On the following diagram of object 1, draw and label vectors to represent the forces on the object.



d. Using the conditions of equilibrium, write—but do not solve—two equations that could, together, be solved for q and the tension T in the left–hand string.

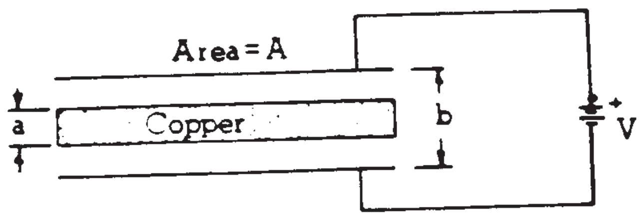


*1974E2. A parallel–plate capacitor with spacing b and area A is connected to a battery of voltage V as shown above. Initially the space between the plates is empty. Make the following determinations in terms of the given symbols.

a. Determine the electric field between the plates.

b. Determine the charge stored on each capacitor plate.

A copper slab of thickness a is now inserted midway between the plates as shown below.



c. Determine the electric field in the spaces above and below the slab.

d. Determine the ratio of capacitances $\frac{C_{with\ copper}}{C_{original}}$ when the slab is inserted

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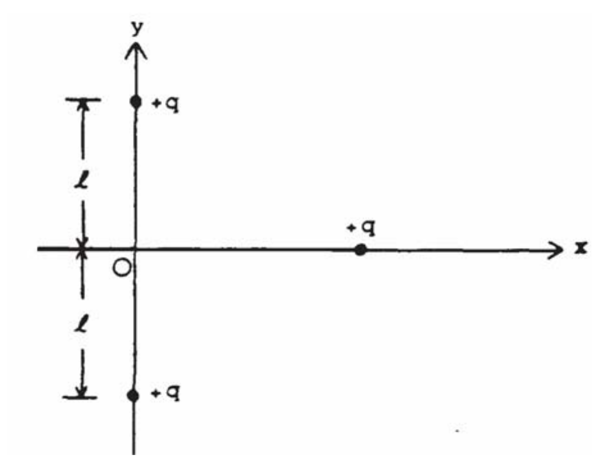
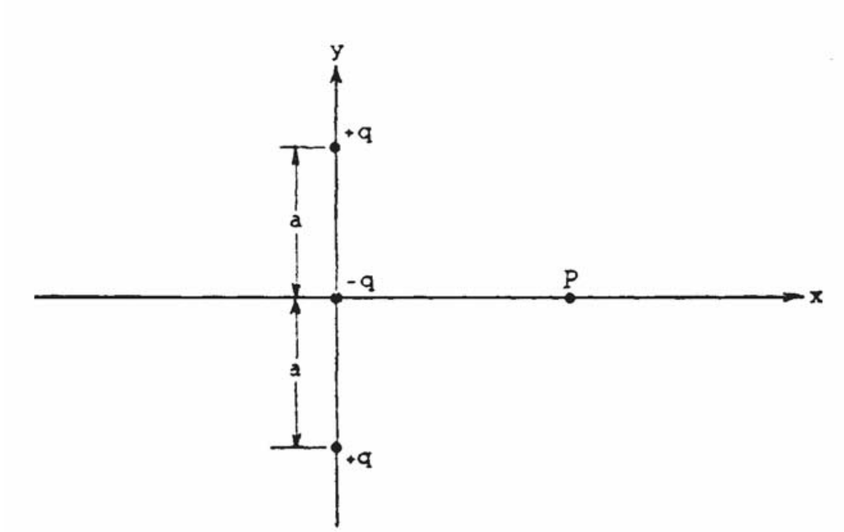


1975E1. Two stationary point charges +q are located on the y–axis as shown above. A third charge +q is brought in from infinity along the x–axis.

a. Express the potential energy of the movable charge as a function of its position on the x–axis.

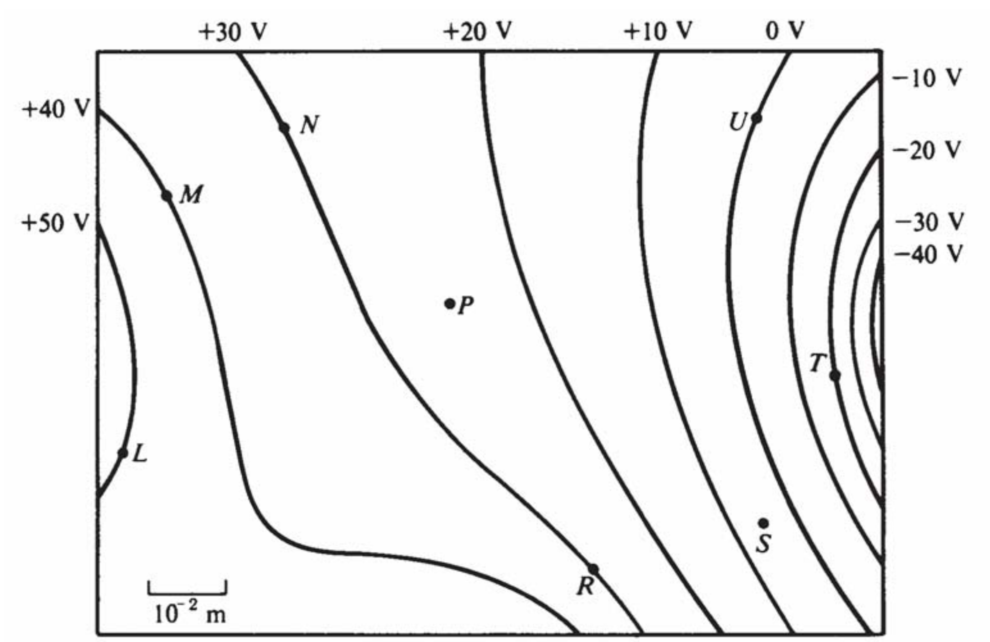
b. Determine the magnitude and direction of the force acting on the movable charge when it is located at the position x = *l*

c. Determine the work done <u>by the electric field</u> as the charge moves from infinity to the origin.



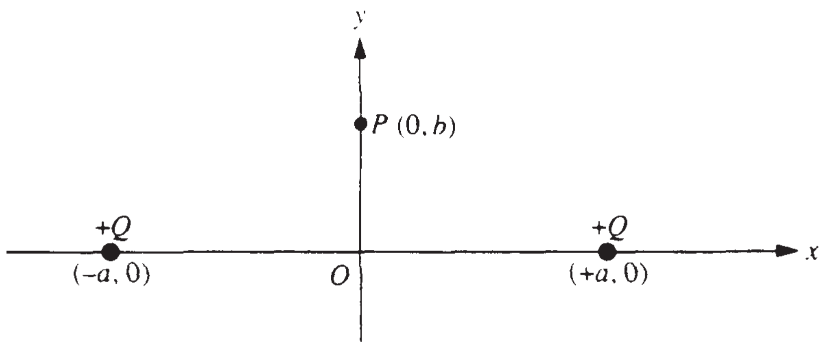
1982E1 (modified) Three point charges are arranged on the y–axis as shown above. The charges are +q at (0, a), –q at (0, 0), and +q at (0, –a). Any other charge or material is infinitely far away.

a. Determine the point(s) on the x–axis where the electric potential due to this system of charges is zero.
b. Determine the x and y components of the electric field at a point P on the x–axis at a distance x from the origin.



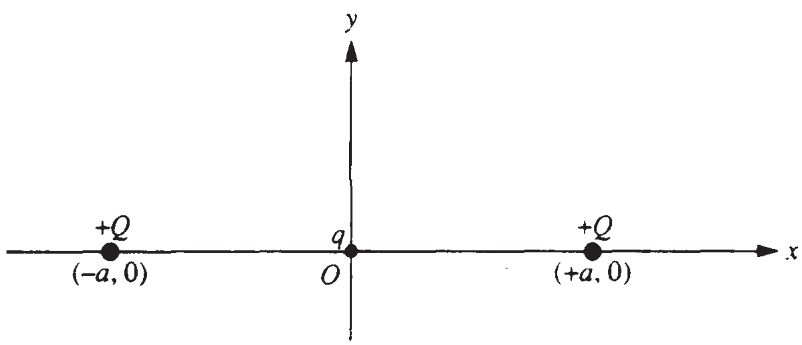
*1986E1. Three point charges produce the electric equipotential lines shown on the diagram above.

a. Draw arrows at points L, N. and U on the diagram to indicate the direction of the electric field at these points.
b. At which of the lettered points is the electric field E greatest in magnitude? Explain your reasoning.
c. Compute an approximate value for the magnitude of the electric field E at point P.
d. Compute an approximate value for the potential difference, $V_M - V_S$, between points M and S.
e. Determine the work done by the field if a charge of $+5 \times 10^{-12}$ coulomb is moved from point M to point R.
f. If the charge of $+5 \times 10^{-12}$ coulomb were moved from point M first to point S, and then to point R, would the answer to e. be different, and if so, how?



1991E1. Two equal positive charges Q are fixed on the x–axis. one at +a and the other at –a, as shown above. Point P is a point on the y–axis with coordinates (0, b). Determine each of the following in terms of the given quantities and fundamental constants.

a. The electric field E at the origin *O*.
b. The electric potential V at the origin *O*.
c. The magnitude of the electric field E at point *P*.



A small particle of charge q ($q << Q$) and mass m is placed at the origin, displaced slightly, and then released. Assume that the only subsequent forces acting are the electric forces from the two fixed charges Q. at x = +a and x = –a. and that the particle moves only in the xy –plane. In each of the following cases, describe briefly the motion of the charged particle after it is released. Write an expression for its speed when far away if the resulting force pushes it away from the origin.

d. q is positive and is displaced in the +x direction.
e. q is positive and is displaced in the +y direction.
f. q is negative and is displaced in the +y direction.

2000E2 (modified) Three particles, A, B, and C, have equal positive charges Q and are held in place at the vertices of an equilateral triangle with sides of length *l*, as shown in the figures below. The dotted lines represent the bisectors for each side. The base of the triangle lies on the x–axis, and the altitude of the triangle lies on the y–axis.

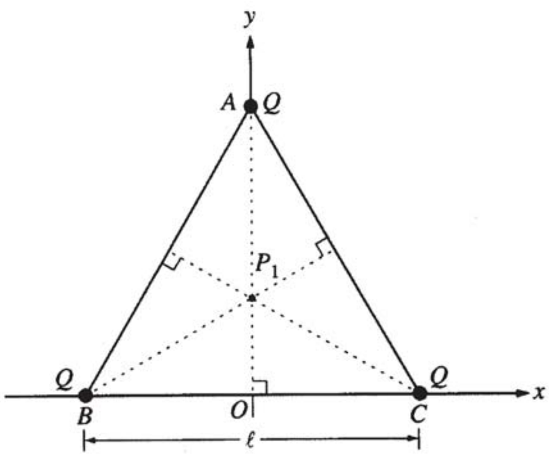


Figure 1

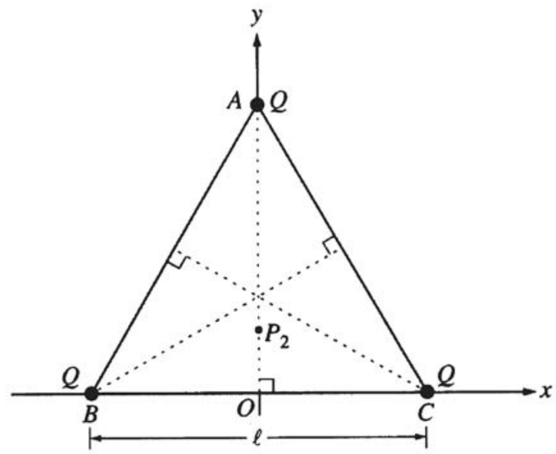


Figure 2

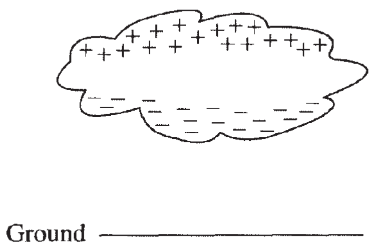
a. i. Point P_1, the intersection of the three bisectors, locates the geometric center of the triangle and is one point where the electric field is zero. On Figure 1 above, draw the electric field vectors E_A, E_B, and E_C at P, due to each of the three charges. Be sure your arrows are drawn to reflect the relative magnitude of the fields.

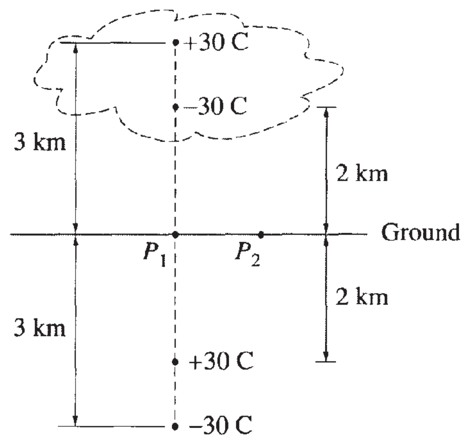
ii. Another point where the electric field is zero is point P_2 at $(0, y_2)$. On Figure 2 above, draw electric field vectors E_A, E_B, and E_C at P_2 due to each of the three point charges. Indicate below whether the magnitude of each of these vectors is greater than, less than, or the same as for point P_1.

	Greater than at P_1	Less than at P_1	The same as at P_1
E_A			
E_B			
E_C			

b. Explain why the x–component of the total electric field is zero at any point on the y–axis.

c. Write a general expression for the electric potential V at any point on the y–axis inside the triangle in terms of Q, *l*, and y.





2001E1. A thundercloud has the charge distribution illustrated above left. Treat this distribution as two point charges, a negative charge of –30 C at a height of 2 km above ground and a positive charge of +30 C at a height of 3 km. The presence of these charges induces charges on the ground. Assuming the ground is a conductor, it can be shown that the induced charges can be treated as a charge of +30 C at a depth of 2 km below ground and a charge of –30 C at a depth of 3 km, as shown above right. Consider point P_1, which is just above the ground directly below the thundercloud, and point P_2, which is 1 km horizontally away from P_1.

a. Determine the direction and magnitude of the electric field at point P_1.

b. i. On the diagram, clearly indicate the direction of the electric field at point P_2

ii. How does the magnitude of the field at this point compare with the magnitude at point P_1? Justify your answer:

____ Greater ____Equal ____ Less

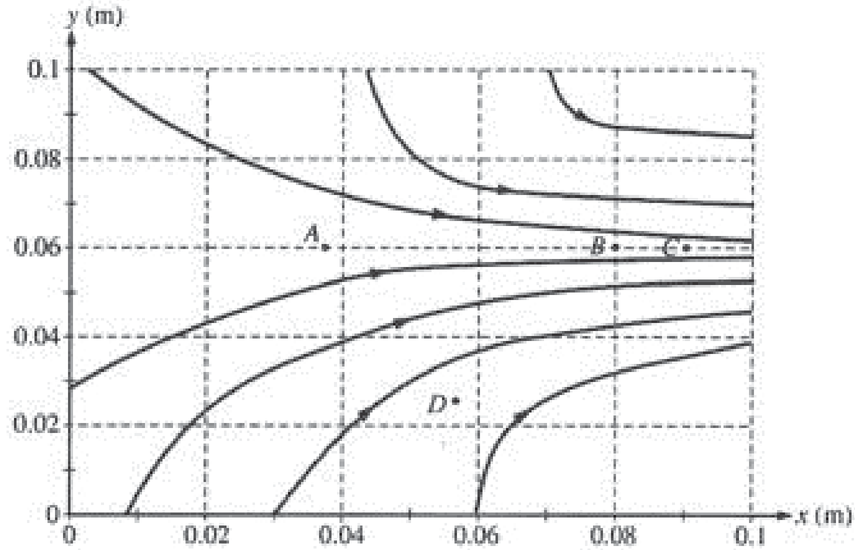
c. Letting the zero of potential be at infinity, determine the potential at these points.

i. Point P_1

ii. Point P_2

d. Determine the electric potential at an altitude of 1 km directly above point P_1.

e. Determine the total electric potential energy of this arrangement of charges.



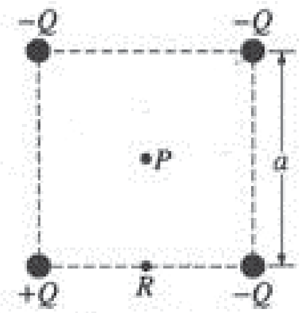
*2005E1. Consider the electric field diagram above.

a. Points A, B, and C are all located at y = 0.06 m .
 i. At which of these three points is the magnitude of the electric field the greatest? Justify your answer.
 ii. At which of these three points is the electric potential the greatest? Justify your answer.

b. An electron is released from rest at point B.
 i. Qualitatively describe the electron's motion in terms of direction, speed, and acceleration.
 ii. Calculate the electron's speed after it has moved through a potential difference of 10 V.

c. Points B and C are separated by a potential difference of 20 V. Estimate the magnitude of the electric field midway between them and state any assumptions that you make.

d. On the diagram, draw an equipotential line that passes through point D and intersects at least three electric field lines.



2006E1. The square of side a above contains a positive point charge +Q fixed at the lower left corner and negative point charges –Q fixed at the other three corners of the square. Point P is located at the center of the square.

a. On the diagram, indicate with an arrow the direction of the net electric field at point P.

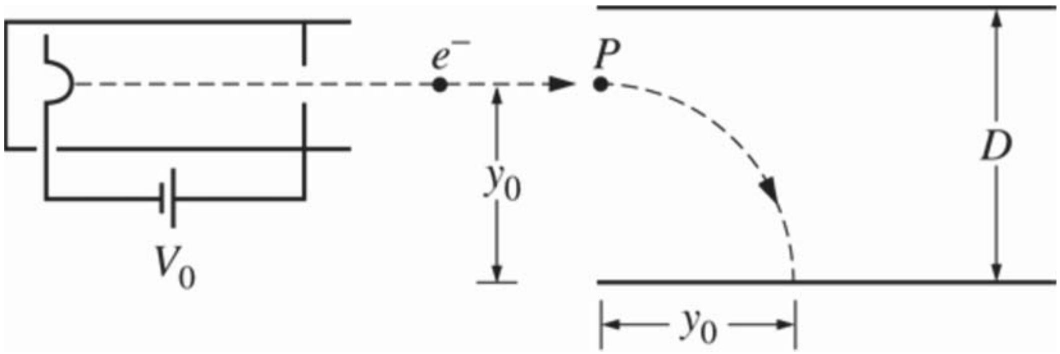
b. Derive expressions for each of the following in terms of the given quantities and fundamental constants.
   i. The magnitude of the electric field at point P
   ii. The electric potential at point P

c. A positive charge is placed at point P. It is then moved from point P to point R, which is at the midpoint of the bottom side of the square. As the charge is moved, is the work done on it by the electric field positive, negative, or zero?

   ____Positive ____Negative ____Zero

   Explain your reasoning.

d. i: Describe one way to replace a single charge in this configuration that would make the electric field at the center of the square equal to zero. Justify your answer.
   ii. Describe one way to replace a single charge in this configuration such that the electric potential at the center of the square is zero but the electric field is not zero. Justify your answer.



2009E2 (modified) Electrons created at the filament at the left end of the tube represented above are accelerated through a voltage V_0 and exit the tube. The electrons then move with constant speed to the right, as shown, before entering a region in which there is a uniform electric field between two parallel plates separated by a distance D. The electrons enter the field at point P, which is a distance y_0 from the bottom plate, and are deflected toward that plate. Express your answers to the following in terms of V_0, D, y_0, and fundamental constants.

a. Calculate the speed of the electrons as they exit the tube.

b. i. Calculate the magnitude of the electric field required to cause the electrons to land the distance y_0 from the edge of the plate.
ii. Indicate the direction of the electric field.

____ To the left	____ To the right
____ Toward the top of the page	____ Toward the bottom of the page
____ Into the page	____ Out of the page

Justify your answer.

c. Calculate the potential difference between the two plates required to produce the electric field determined in part b.

ANSWERS - AP Physics Multiple Choice Practice – Electrostatics

	Solution	Answer
1.	Since charge is free to move around on/in a conductor, excess charges will repel each other to the outer surface	D
2.	The net charge on the two spheres is +Q so when they touch and separate, the charge on each sphere (divided equally) is ½ Q. $F \propto Q_1Q_2$ so before contact $F \propto (2Q)(Q) = 2Q^2$ and after contact $F \propto (½ Q)(½ Q) = ¼ Q^2$ or 1/8 of the original force	D
3.	Newton's third law	B
4.	$F_g = Gm_1m_2/r^2$ and $F_E = kq_1q_2/r^2$. The nuclear force does not have a similar relationship.	A,B
5.	$F_E \propto q_1q_2/r^2$; if q_1 and $q_2 \times 2$; $F \times 4$ and if $r \div 2$, $F \times 4$ making the net effect $F \times 4 \times 4$	D
6.	By symmetry, the force on an electron at the center from the top half will be straight down and the force from the bottom half will also be straight down	B
7.	While spheres X and Y are in contact, electrons will repel away from the rod out of sphere X into sphere Y.	C
8.	While spheres 1 and 2 are in contact, electrons will repel away from the rod out of sphere 1 into sphere 2.	C
9.	The force vectors from the two +Q charges point down and to the left (away from the charges) so the resultant force points down and left	D
10.	The two vectors, each of magnitude F, point at right angles to each other so the resultant field is $\sqrt{2}F$	C
11.	$F_E = F_C$ and $q_1 = q_2 = e$ so we have $ke^2/R^2 = mv^2/R$ and we multiply both sides by ½ R so the right side becomes ½ mv^2 (the kinetic energy). Choices C and E could have been eliminated because they are negative, and kinetic energy cannot be negative. Choices A & D are dimensionally incorrect (D has the units of a force, not energy, and A has the units of electric potential)	A
12.	In I, charge separation occurs (negative charges repel to the leaves). The whole process describes charging by induction, where the electrons leave the electroscope to ground (the finger) and once contact with ground is broken, the electroscope is left with a positive charge (III)	D
13.	Charged objects attract object with an opposite charge, but also neutral objects by separation of charges.	B,C
14.	$F \propto 1/r^2$	A
15.	The distance between the +q charge and each charge is d. The force on the +q charge from each charge is in the same direction, making the net force $kq^2/d^2 + k(3q^2)/d^2$	B
16.	The rod will attract the same charge from each sphere to the side closer to the rod.	B
17.	$F \propto 1/r^2$; if $r \times 4$, $F \div 16$	D
18.	If the leaves are positive, further separation means they are becoming more positive, which implies electrons are leaving the leaves, attracted to the top plate of the electroscope. This will occur if the object is positively charged.	A
19.	Vector addition. Since all the charges are positive, the forces due to charges 2 and 4 point in opposite directions, making the magnitude of the net force along the x axis 2 N. Combine this with a net force along the y axis of 6 N using the Pythagorean theorem	A

20. If a positive rod attracts ball A, it is either negative or neutral. For ball B to also attract ball A means ball B can be charged positive or negative (if ball A is neutral) or neutral (if ball A is positive) D

21. $F \propto 1/r^2$; if $r \times 0.4$ then $F \div 0.4^2$ A

22. The process described is charging by induction which gives the electroscope in this case a net negative charge. Bringing a negative charge near the top of the electroscope will cause electrons to repel to the leaves. Since the leaves are already negative, this will cause them to separate further. B

23. $F \propto 1/r^2$ A

24. Newton's third law requires the forces be equal and opposite. This eliminates choices A, B and C. Since they both positive, the force is repulsive. D

25. Only electrons are transferred in static charging processes. C

26. Any charge will experience a net force of zero where the electric field is zero. This must be where the fields from each charge point in opposite directions and also closer to the smaller charge, which is to the left of the +Q charge (the answer will be to the left of –1 m). Let the distances to the +Q and the –2Q charge be x and (X + 2), respectively. This gives $E_1 = E_2$ and $kQ/x^2 = k(2Q)/(x + 2)^2$. Solve for x and add the extra 1 m to the origin. A

27. $F \propto q_1q_2/r^2$; the original force $F \propto 100Q^2/d^2$. The new charges are 15Q and 5Q making the new force $F \propto 75Q^2/(2d)^2 = 19Q^2/d^2$ A

28. Assuming C remains constant and $U_C = \frac{1}{2} CV^2$, for U_C to double V must increase by $\sqrt{2}$ D

29. Initially, when B is removed, A and C are equally and oppositely charged and B is neutral. Touching B to A gives B ½ the charge of A (split equally). The charge on B is then ½ that of C and oppositely charged. When B and C touch, the total charge between them is ½ the charge of C and the same sign as C. Each sphere then has ¼ of the charge of C after contact is made. This makes the end result that the charge on sphere B is ¼ the original charge of A and the same sign as sphere C, which is opposite that of A C

30. $\Sigma F = 0$ so we have $T + k(q)(q)/d^2 - Mg = 0$ giving $T = Mg - kq^2/d^2$ D

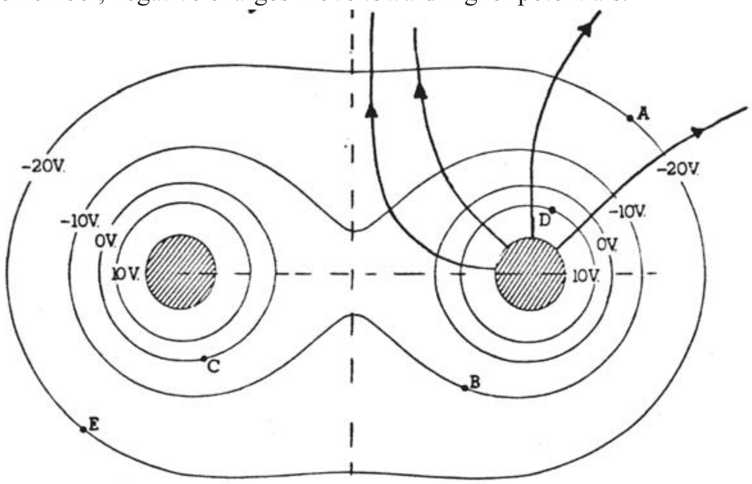
AP Physics Free Response Practice – Electrostatics – ANSWERS

WARNING: Only Electric Force is on AP Physics 1

1974B5

a. Since the potential increases as you near the cylinder on the right, it must also have a positive charge. Remember, negative charges move toward higher potentials.

b.



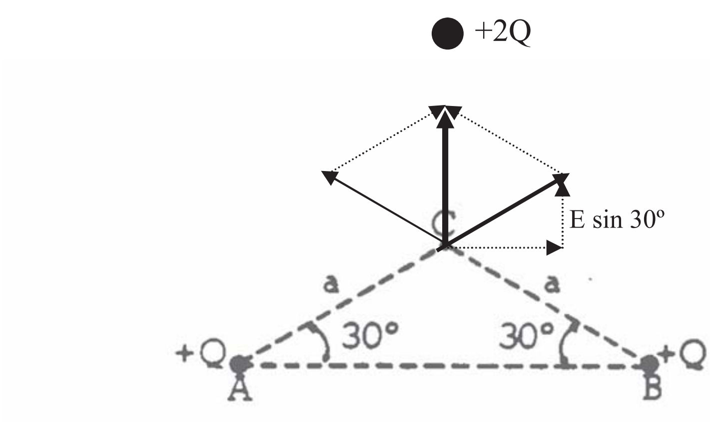
c. $V_A - V_B = (-20\text{ V}) - (-10\text{ V}) = -10\text{ V}$

d. $W_{AED} = W_{AD} = -q\Delta V = -(0.5\text{ C})(30\text{ V}) = -15\text{ J}$

1975B2

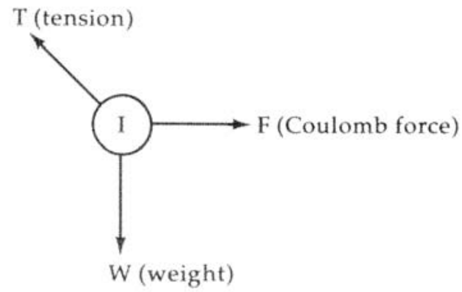
a. $V_C = kQ/a + kQ/a = 2kQ/a$; $W = -q\Delta V = -(+q)(V_\infty - V_C) = -q(0 - 2kQ/a) = 2kQq/a$

b. Looking at the diagram below, the fields due to the two point charges cancel their x components and add their y components, each of which has a value $(kQ/a^2)\sin 30º = ½\ kQ/a^2$ making the net E field (shown by the arrow pointing upward) $2 \times ½\ kQ/a^2 = kQ/a^2$. For this field to be cancelled, we need a field of the same magnitude pointing downward. This means the positive charge +2Q must be placed directly above point C at a distance calculated by $k(2Q)/d^2 = kQ/a^2$ giving $d = \sqrt{2}a$



1979B7

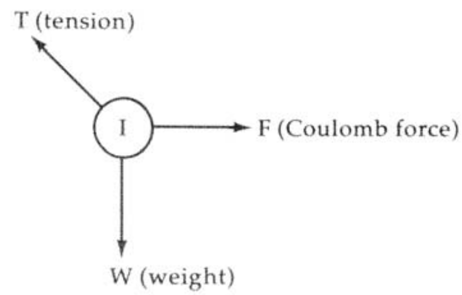
a.



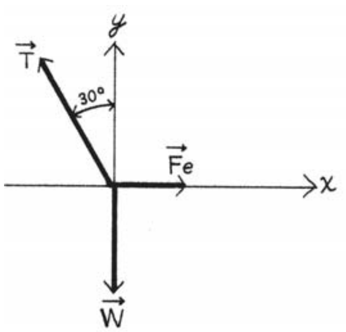
b. Resolving the tension into components we have T cos θ = W and T sin θ = F
where W = mg and $F = kq^2/r^2$ and r = 2*l* sin θ giving $F = kq^2/(4l^2 \sin^2\theta)$
Dividing the two expressions we get $\tan\theta = F/mg = kq^2/(4l^2 \sin^2\theta\ mg)$
solving yields $q^2 = 4mgl^2 (\sin^2\theta)(\tan\theta)/k$

1981 B3

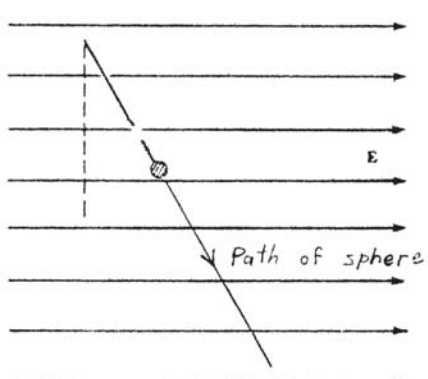
a.



b.



T cos 30º = mg so T = 0.058 N
T sin θ = F_E = Eq gives E = 5.8 × 10^3 N/C

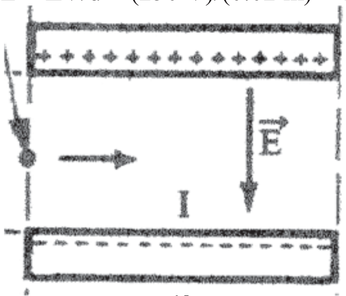


c. After the string is cut, the only forces are gravity, which acts down, and the electrical force which acts to the right. The resultant of these two forces causes a constant acceleration along the line of the string. The path is therefore down and to the right, along the direction of the string as shown above.

1985B3

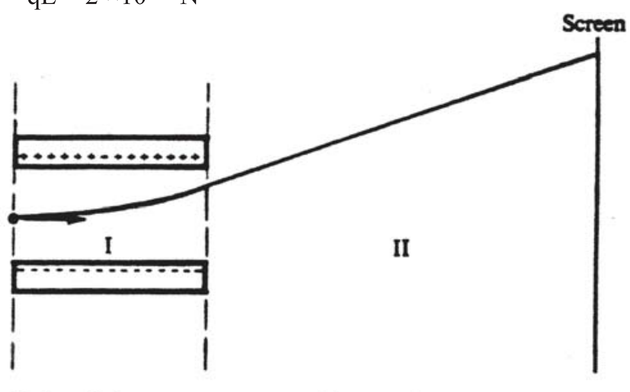
a. $K = (2 \times 10^3 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = 3.2 \times 10^{16}$ J
   $K = \frac{1}{2} mv^2$ gives $v = 2.7 \times 10^7$ m/s

b. $E = \Delta V/d = (250 \text{ V})/(0.02 \text{ m}) = 1.25 \times 10^4$ V/m



c. $F = qE = 2 \times 10^{-15}$ N

d.



Path curves parabolically toward the upper plate in region I and moves in a straight line in region II.

1987B2

a. $V = kQ/r = 9 \times 10^4$ V
b. $W = q\Delta V$ (where V at infinity is zero) = 0.09 J
c. $F = kqQ/r^2 = 0.3$ N
d. Between the two charges, the fields from each charge point in opposite directions, making the resultant field the difference between the magnitudes of the individual fields.
   $E = kQ/r^2$ gives $E_I = 1.2 \times 10^6$ N/C to the right and $E_{II} = 0.4 \times 10^6$ N/C to the left
   The resultant field is therefore $E = E_I - E_{II} = 8 \times 10^5$ N/C to the right
e. From conservation of momentum $m_I v_I = m_{II} v_{II}$ and since the masses are equal we have $v_I = v_{II}$.
   Conservation of energy gives $U = K = 2(\frac{1}{2} mv^2) = 0.09$ J giving v = 6 m/s

---

1989B2

a. $E = kQ/r^2$ and since the field is zero $E_1 + E_2 = 0$ giving $k(Q_1/r_1^2 + Q_2/r_2^2) = 0$
   This gives the magnitude of $Q_2 = Q_1(r_2^2/r_1^2) = 2\mu C$ and since the fields must point inopposite directions from each charge at point P, Q_2 must be negative.
b. $F = kQ_1Q_2/r^2 = 3.6$ N to the right (they attract)
c. $U = kQ_1Q_2/r = -0.72$ J
d. between the charges we have a distance from Q_1 of x and from Q_2 of (0.2 m – x)
   $V = kQ_1.x + kQ_2/(0.2 \text{ m} - x) = 0$, solving for x gives x = 0.16 m
e. $W = q\Delta V$ where $\Delta V = V_\infty - V_R = 0$ so W = 0

1990B2

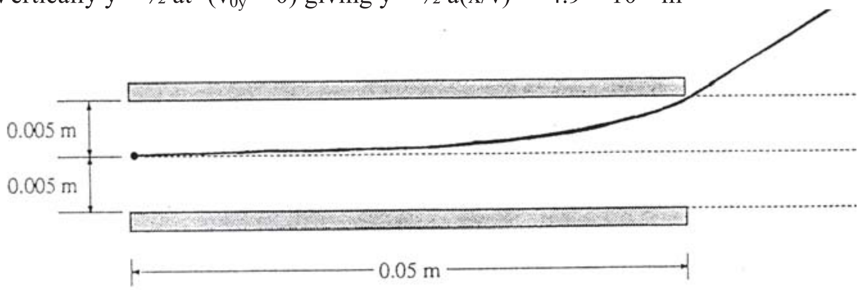
a. $E = V/d = 2 \times 10^4$ V/m

b. $F = Eq = ma$ gives $a = qE/m = 3.5 \times 10^{15}$ m/s^2

c. Horizontally: $x = vt$ giving $t = x/v$

Vertically $y = \frac{1}{2} at^2$ ($v_{0y} = 0$) giving $y = \frac{1}{2} a(x/v)^2 = 4.9 \times 10^{-3}$ m

d.

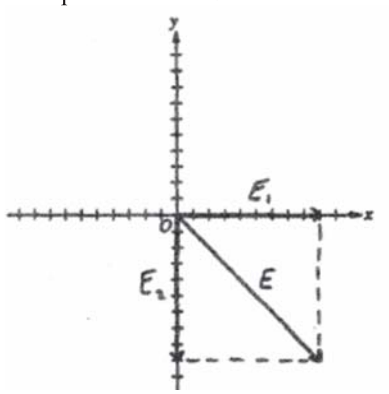


1993B2

a. i. $E = kq/r^2 = 9000$ N/C

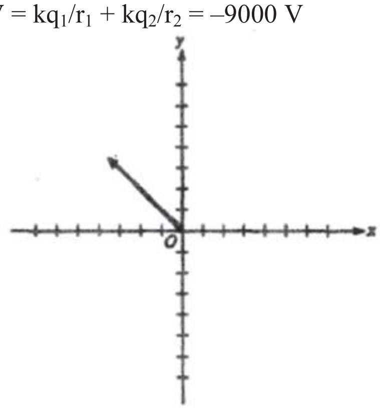
ii. $E = kq/r^2 = 9000$ N/C

iii.



b. $V = kq_1/r_1 + kq_2/r_2 = -9000$ V

c.



Since the charge is negative, the force acts opposite the direction of the net E field.

d. $W = q\Delta V = 0.036$ J

1996B6

a. $\Sigma F = 0$ gives $qE = mg$ and $q = mg/E = 3.27 \times 10^{-19}$ C

b. The drop must have a net negative charge. The electric force on a negative charge acts opposite the direction of the electric field.

c. $V = Ed = 100$ V

d. The drop moves upward. The reduced mass decreases the downward force of gravity on the drop while if the charge remains the same, the upward electric force is unchanged.

2002B5B

a. Electric field lines point away from positive charges and toward negative charges. The plate on the left is negative and the plate on the right is positive.

b. $V = Ed = 100$ V

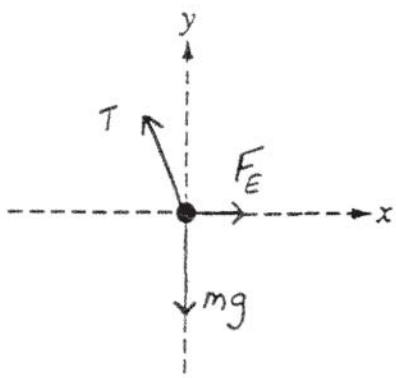
c. $C = \varepsilon_0 A/d = 1.3 \times 10^{-10}$ F

d. $F = qE = 8 \times 10^{-16}$ N to the right (opposite the direction of the electric field)

e. The potential difference between the center and one of the plates is 50 V.
$W = qV = ½ mv^2$ gives $v = 4.2 \times 10^6$ m/s

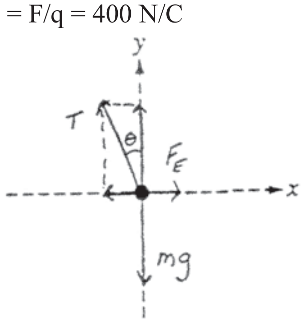
1998B2

a.



b. $E = F/q = 400$ N/C

c.



$T \sin\theta = F_E$ and $T \cos\theta = mg$. Dividing gives $\tan\theta = F/mg$ and $\theta = 18°$.
From the diagram $\sin\theta = x/(0.30\text{ m})$ giving $x = 0.09$ m

d. i. $a_x = F/m = 3.2\text{ m/s}^2$; $a_y = 9.8\text{ m/s}^2$
$a = \sqrt{a_x^2 + a_y^2} = 10.3\text{ m/s}^2$; $\tan\theta = (9.8\text{ m/s}^2)/(3.2\text{ m/s}^2) = 72°$ below the x axis
(or 18° to the right of the y axis, the same as the angle of the string)
ii. The ball moves in a straight line down and to the right

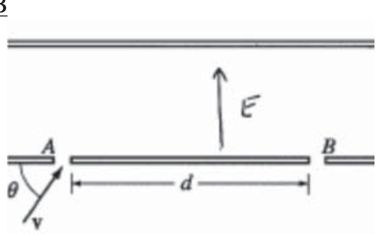
1999B2

a. $W = qV = \frac{1}{2} mv^2$ gives $V = mv^2/2q = 1.0 \times 10^4$ V
b. Electrons travel toward higher potential making the upper plate at the higher potential.
c. i. $x = v_x t$ gives $t = 6.7 \times 10^{-10}$ s
   ii. $F = ma = qE$ and $E = V/d$ gives $a = qV/md$ and $y = \frac{1}{2} at^2$ ($v_{0y} = 0$) gives $y = qVt^2/2md = 6.5 \times 10^{-4}$ m
d. F_g is on the order of 10^{-30} N (mg) and $F_E = qE = qV/d$ is around 10^{-14} N so $F_E \gg F_g$
e. Since there is no more electric force, the path is a straight line.

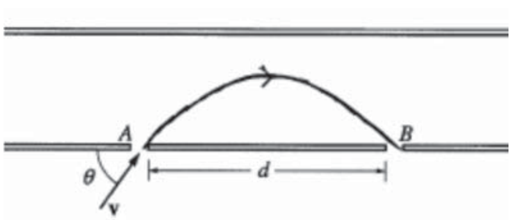
2001B3

a. i. $V = \Sigma kQ/r = k(-Q/r + -Q/r + Q/r + Q/r) = 0$
   ii. The fields from the charges on opposing corners cancels which gives E = 0
b. i. $V = \Sigma kQ/r = k(-Q/r + -Q/r + Q/r + Q/r) = 0$
   ii. The field from each individual charge points along a diagonal, with an *x*-component to the right. The vertical components cancel in pairs, and the *x*-components are equal in magnitude. Each x component being $E = kQ/r^2 \cos 45^\circ$ and the distance from a corner to the center of $r^2 = s^2/2$ gives
   $E = 4E_x = 4\frac{kQ}{s^2/2}\frac{\sqrt{2}}{2} = 4\sqrt{2}kQ/s^2$
c. Arrangement 1. The force of attraction on the upper right charge is greater in arrangement 1 because the two closest charges are both positive, whereas in arrangement 2 one is positive and one is negative.

2003B4B

a. i.


   ii.


   iii.


b. $F = ma = qE$ gives $a = qE/m$
c. The acceleration is downward and at the top of the path, $v_y = v_{0y} - at = 0$ and $v_{0y} = v \sin\theta$ which gives $t_{top} = v \sin\theta/a$ or $t_{total} = 2t_{top} = 2v \sin\theta/a$ and substituting a from part b gives $t = (2mv \sin\theta)/qE$
d. $d = x_x t$ where $v_x = v \cos\theta$ giving $d = (2mv^2 \sin\theta \cos\theta)/qE$
e. The distance would be less because gravity, acting downward, will increase the electron's downward acceleration, decreasing the time spent in the field.

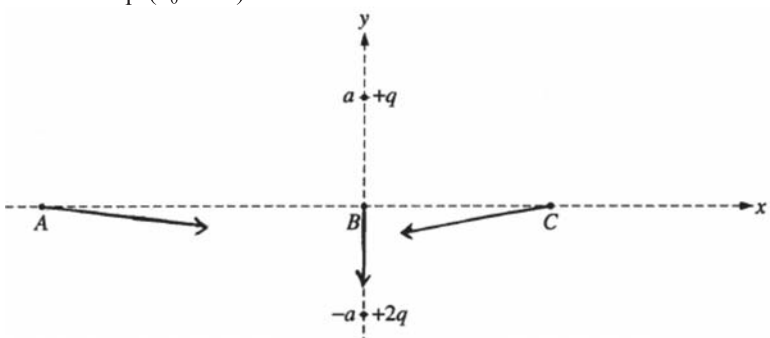
2005B3

a. $E = kq/r^2$ and the field from each charge points in opposite directions, with the larger field contribution pointing upward. $E_O = k(2q)/a^2 - kq/a^2 = kq/a^2$ upward (+y)

b. $V_O = \Sigma kq/r = k(2q)/a + kq/a = 3kq/a$

c. $F = kq_1q_2/r^2$ where in this case $r^2 = x_0^2 + a^2$
   i. $F = kq^2/(x_0^2 + a^2)$
   ii. $F = 2kq^2/(x_0^2 + a^2)$

d.



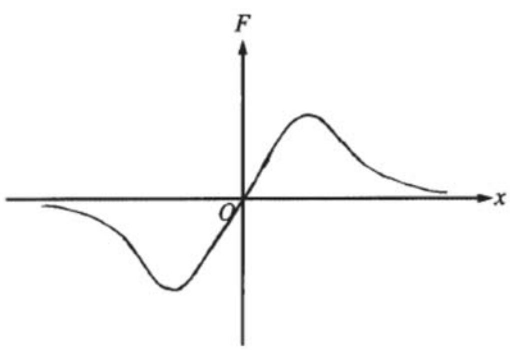
2005B3B

a. The distance between the charges is $r = \sqrt{a^2 + (2a)^2} = \sqrt{5}a$ The y components of the forces due to the two –2Q charges cancel so the magnitude of the net force equals the sum of the x components, where $F_x = F \cos \theta$ and $\cos \theta = 2a/r = 2/\sqrt{5}$
   Putting this all together gives $F_x = 2 \times (kQ(2Q)/r^2) \cos \theta = 8kQ^2/5\sqrt{5}a^2$ to the right (+x)

b. The contribution to the field from the –2Q charges cancel. This gives $E = kQ/(2a)^2 = kQ/4a^2$ to the right (+x)

c. $V = \Sigma kQ/r = k(-2Q)/a + k(-2Q)/a + k(-Q)/2a = -9kQ/2a$

d. At the origin the force is zero (they cancel). As the charge moves away from the origin, the force first increases as the x components grow, then decrease as the distance grows larger.



2006B3

a. Positive. The electric field due to q_1 points to the right since q_1 is negative. For the electric field to be zero at point P, the field form q_2 must point to the left, away from q_2 making q_2 positive.

b. $\mathbf{E}_1 + \mathbf{E}_2 = 0$ so setting the fields from each charge equal in magnitude gives
   $kq_1/d_1^2 = kq_2/d_2^2$, or $q_2 = q_1(d_2^2/d_1^2) = 4.8 \times 10^{-8}$ C

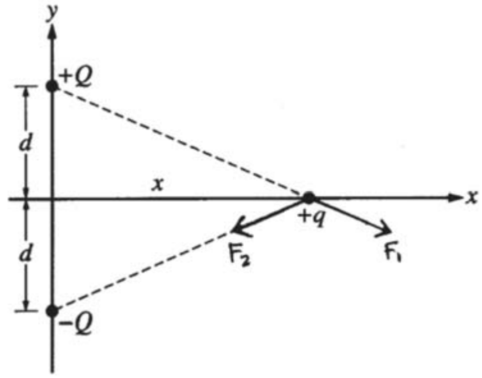
c. $F = kq_1q_2/r^2 = 1.4 \times 10^{-5}$ N to the left

d. $V_1 + V_2 = 0 = kq_1/r_1 + kq_2/r_2$ and let $r_2 = d$ and $r_1 = (0.3 \text{ m} - d)$
   solving yields d = 0.28 m to the left of q_2 which is at x = 0.20 m – 0.28 m = –0.08 m

e. $W = q\Delta V$ and since $\Delta V = 0$, W = 0

2006B3B

a.



b. The x components of the forces cancel so the net force is the sum of the y components, which are equal in magnitude and direction. $F_{net} = 2 \times F \cos\theta$ where θ is the angle between the y axis and the dashed line in the diagram above. $\cos\theta = d/r = d/\sqrt{x^2 + d^2}$
This gives $F_{net} = 2 \times kqQ/r^2 \times \cos\theta = 2kqQd/(x^2 + d^2)^{3/2}$

c. E = F/q at the point where q_1 lies. $E = 2kQd/(x^2 + d^2)^{3/2}$

d. Since the charges Q and –Q are equidistant from the point and V = ΣkQ/r, the potential V = 0

e. As x gets large, the distance to the charges r and the value of x become similar, that is $\sqrt{x^2 + d^2} \approx x$.
Substituting this into the answer to b. yields $F = 2kqQd/x^3$

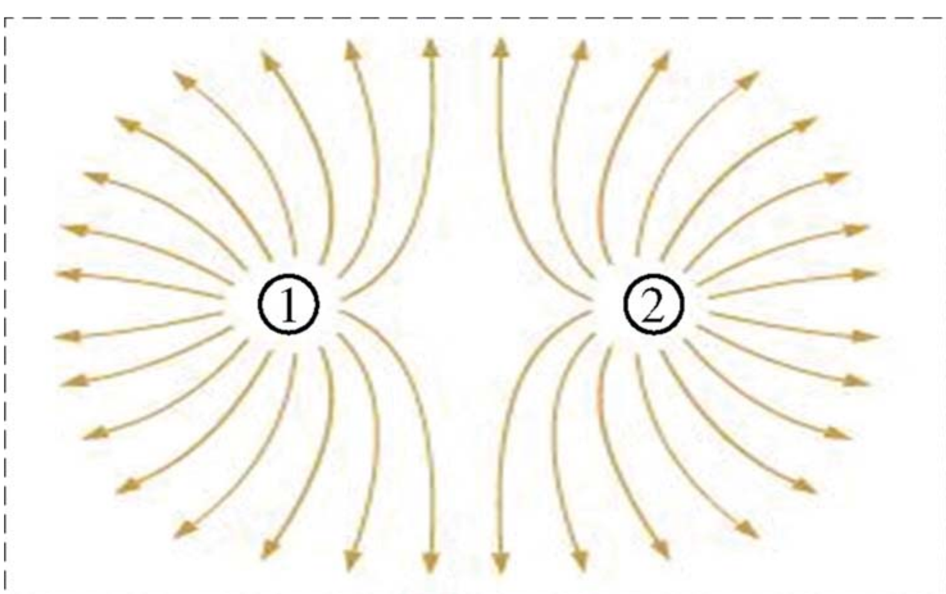
2009B2B

a. The x components of the forces due to the charges q_B cancel making the net force equal to the sum of the y components which are equal in magnitude and both point downward. The distance between q_A and either q_B is found by the Pythagorean theorem to be 0.05 m. $F_y = F \sin\theta$ where θ is the angle between the line joining q_A and q_B and the x axis, giving $\sin\theta = 3/5$.
This gives $F_{net} = 2 \times F_y = 2\,(kq_Aq_B/r^2) \times \sin\theta = 2.6 \times 10^{-7}$ N down (–y)

b. Particle A will accelerate downward, but as the particle approaches the origin, the force and the acceleration will decrease to zero at the origin. It will then pass through the origin, with a net force now pointing upward, where it will eventually slow down and reverse direction, repeating the process. The short answer is the particle will oscillate vertically about the origin.

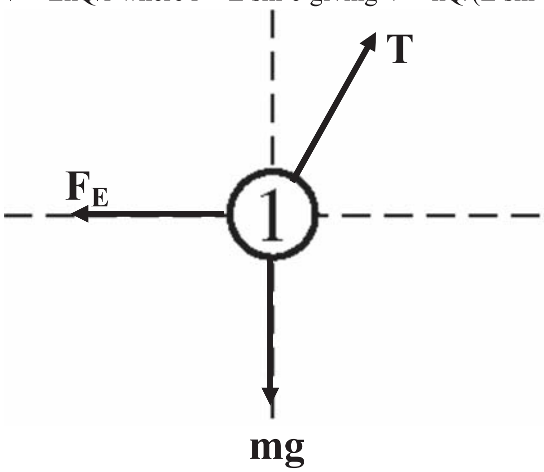
2009B2

a.



b. $V = \Sigma kQ/r$ where $r = L \sin\theta$ giving $V = kQ/(L \sin\theta) + kQ/(L \sin\theta) = 2kQ/(L \sin\theta)$

c.



d. $\Sigma F_y = 0$; $T \cos\theta = mg$
$\Sigma F_x = 0$; $T \sin\theta = F_E = kQ^2/(2L \sin\theta)^2$

1974E2

a. $E = V/d = V/b$

b. $C = \varepsilon_0 A/d = \varepsilon_0 A/b$; $Q = CV = \varepsilon_0 AV/b$

c. This arrangement acts as two capacitors in series, which each have a potential difference ½ V. Using $E = V/d$ where $d = ½(b – a)$ for each of the spaces above and below. This gives $E = V/d = (½ V)/½(b – a) = V/(b – a)$

d. With the copper inserted, we have two capacitors in series, each with a spacing ½(b – a). The capacitance of each is then $\varepsilon_0 A/(½(b – a))$ and in series, two equal capacitors have an equivalent capacitance of ½ C makinf the total capacitance with the copper inserted $½\varepsilon_0 A/(½(b – a)) = \varepsilon_0 A/(b – a)$ making the ratio $b/(b – a)$. Notice the final capacitance is effectively a new single capacitor with an air gap of (b – a). Imagine sliding the copper slab up to touch the top plate, this is the same result. This is why adding capacitors in series decreases the capacitance as if the gap between the plates was increased.

1975E1

a. To find V along the x axis we use $V = \Sigma kq/r$ where $r = \sqrt{l^2 + x^2}$ giving $V = 2kq/\sqrt{l^2 + x^2}$ and $U_E = qV$ so as a function of x we have $U_E = 2kq^2/\sqrt{l^2 + x^2}$

b. Along the x axis, the y components of the forces cancel and the net force is then the sum of the x components of the forces. Since x = l in this case, the forces make an angle of 45º to the x axis and we have $F = 2 \times F_x = 2 \times F \times \cos 45º = 2 \times kq^2/(\sqrt{l^2 + l^2})^2 \times \cos 45º = kq^2/\sqrt{2}l^2$

c. At the origin, the potential is $V = kq/l + kq/l = 2kq/l$ and with $V_\infty = 0$ we have $W = –q\Delta V = –2kq^2/l$

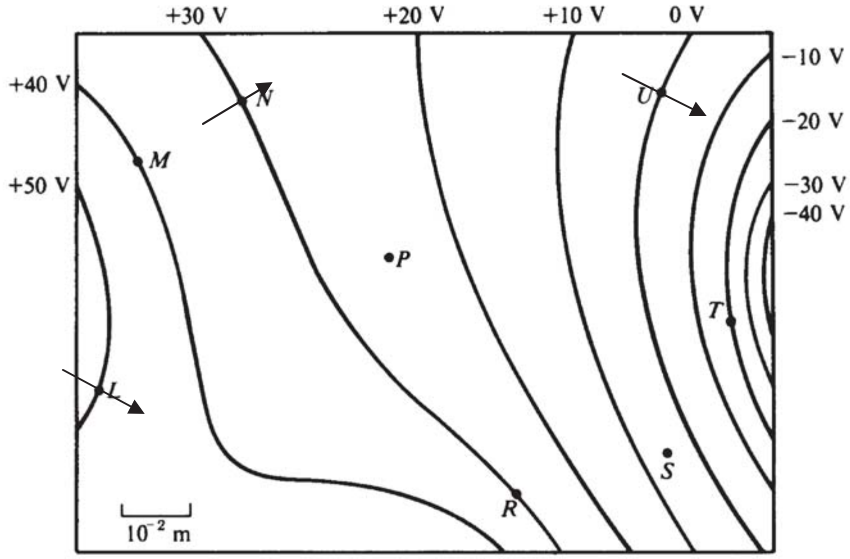
1982E1

a. $V = \Sigma kq/r = –kq/x + 2kq/\sqrt{a^2 + x^2} = 0$ which gives $1/x = 2/\sqrt{a^2 + x^2}$ cross multiplying and squaring gives $4x^2 = a^2 + x^2$ yielding $x = \pm a/\sqrt{3}$

b. $E = kq/r^2$ and by symmetry, the y components cancel. The x components of the electric field from the positive charges points to the right and has magnitude $(kq/r^2) \cos\theta$ where $\cos\theta = x/r = x/\sqrt{x^2 + a^2}$ and the x component of the electric field from the –q charge points to the left with magnitude kq/x^2 making the net field
$E = 2kqx/(x^2 + a^2)^{3/2} – kq/x^2$

1986E1

a.



The field lines point perpendicular to the equipotential lines from high to low potential.

b. The magnitude of the field is greatest at point T because the equipotential lines are closest together, meaning ΔV has the largest gradient, which is related to the strength of the electric field.

c. $E = \Delta V/d = (10\text{ V})/(0.02\text{ m}) = 500\text{ V/m}$

d. $V_M - V_S = 40\text{ V} - 5\text{ V} = 35\text{ V}$

e. $W = -q\Delta V$ and $\Delta V = -10\text{ V}$ which gives $W = 5 \times 10^{-11}\text{ J}$

f. The work done is independent of the path so the answer would be the same.

1991E1

a. $E = kQ/a^2$ for each charge, but each vector points in the opposite direction so E = 0

b. $V = kQ/a + kQ/a = 2kQ/a$

c. the distance to point P from either charge is $r = \sqrt{a^2 + b^2}$ and the magnitude of E is $kQ/r^2 = kQ/(a^2 + b^2)$
The x components cancel so we have only the y components which are E sin θ where $\sin\theta = b/\sqrt{a^2 + b^2}$ and adding the 2 y components from the two charges gives $E_{net} = 2kQb/(a^2 + b^2)^{3/2}$

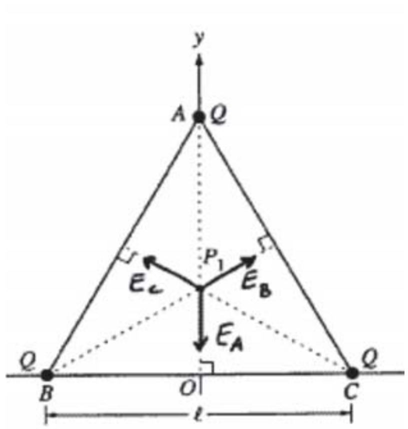
d. The particle will be pushed back toward the origin and oscillate left and right about the origin.

e. The particle will accelerate away from the origin.
The potential of at the center is 2kQ/a and far away $V_\infty = 0$. To find the speed when far away we use $W = q\Delta V = K = ½ mv^2$ which gives $v = 2\sqrt{\frac{kQq}{ma}}$

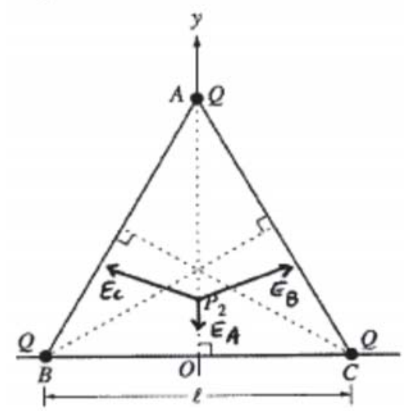
f. The particle will be pulled back toward the origin and oscillate up and down around the origin.

2000E2

a. i.



ii.



	Greater than at P_1	Less than at P_1	The same as at P_1
E_A		✓	
E_B	✓		
E_C	✓		

b. The x components cancel due to the symmetry about the y axis.

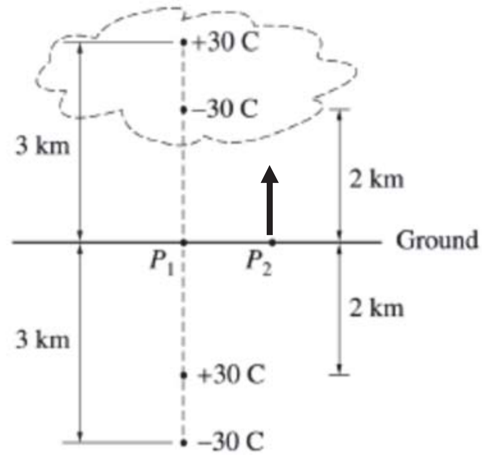
c. $V = \Sigma kQ/r = kQ_A/r_A + kQ_B/r_B + kQ_C/r_C$ where the terms for B and C are equal so we have $V = kQ_A/r_A + 2Q/r_B$

and using the proper geometry for the distances gives $V = k\left|\frac{Q}{\frac{\sqrt{3}l}{2}-y} + \frac{2Q}{\sqrt{\frac{l^2}{4}+y^2}}\right|$

2001E1

a. E is the vector sum of kQ/r^2. Let fields directed upward be positive and fields directed downward be negative. This gives $E = k[-30\ C/(3000\ m)^2 + 30\ C/(2000\ m)^2 + 30\ C/(2000\ m)^2 - 30\ C/(3000\ m)^2] = 75{,}000\ N/C$ upward

b. i.



ii. Because it is a larger distance from the charges, the magnitude is less.

c. i. By symmetry, the potentials cancel and V = 0

ii. By symmetry, the potentials cancel and V = 0

d. $V = \Sigma kQ/r = k[30\ C/(2000\ m) - 30\ C/(1000\ m) + 30\ C/(3000\ m) - 30\ C/(4000\ m)] = -1.12 \times 10^8\ V$

e. $U = kq_1q_2/r$ for *each pair* of charges
$= k[(30)(-30)/1000 + (30)(30)/5000 + (30)(-30)/6000 + -30(30)/4000 + -30(-30)/5000 + 30(-30)/1000] =$
$-1.6 \times 10^{10}\ J$

2005E1

a. i. The magnitude of the field is greatest at point C because this is where the field lines are closest together.

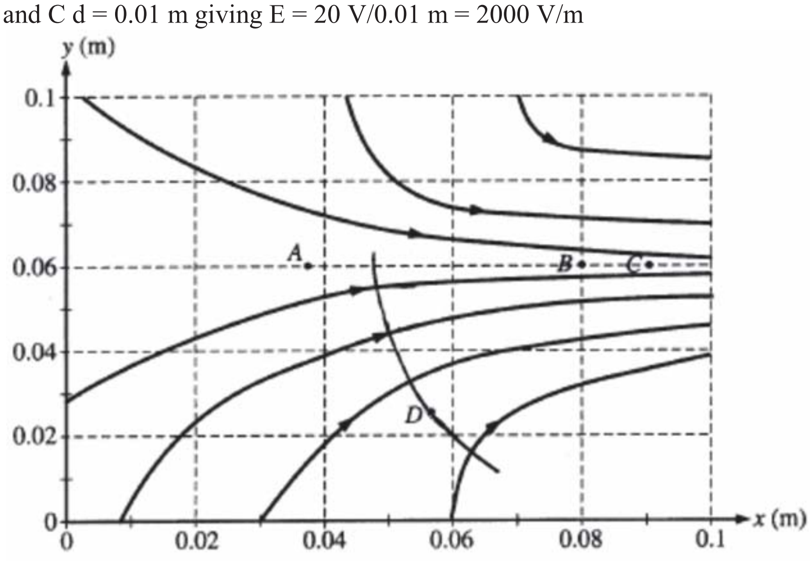
ii. The potential is greatest at point A. Electric field lines point from high to low potential.

b. i. The electron moves to the left, against the field lines. As the field gets weaker the electron's acceleration to the left decreases in magnitude, all the while gaining speed to the left.

ii. $W = q\Delta V = ½\ mv^2$ gives $v = 1.9 \times 10^6\ m/s$

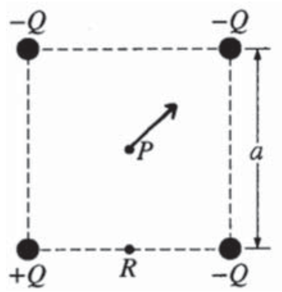
c. If we assume the field is nearly uniform between B and C we can use $E = \Delta V/d$ where the distance between B and C d = 0.01 m giving E = 20 V/0.01 m = 2000 V/m

d.



2006E1

a.



b. i. The fields at point *P* due to the upper left and lower right negative charges are equal in magnitude and opposite in direction so they sum to zero. The fields at point *P* due to the other two charges are equal in magnitude and in the same direction so they add.
Using $r^2 = a^2/2$ we have $E = 2 \times kQ/r^2 = 4kQ/a^2$

ii. $V = \Sigma kQ/r = k(-Q - Q - Q + Q)/r = -2kQ/r$ with $r = a/\sqrt{2}$ giving $V = -2\sqrt{2}kQ/a$

c. Negative. The field is directed generally from *R* to *P* and the charge moves in the opposite direction. Thus, the field does negative work on the charge.

d. i. Replace the top right negative charge with a positive charge OR replace the bottom left positive charge with a negative charge. The vector fields/forces all cancel from oppositely located same charge pairs.

ii. Replace the top left negative charge with a positive charge OR replace the bottom right negative charge with a positive charge. The scalar potentials all cancel from equidistant located opposite charge pairs. The field vectors in these cases will not cancel.

2009E2

a. $W = qV_0 = \frac{1}{2} mv^2$ giving $v = \sqrt{\frac{2eV_0}{m}}$

b. i. The time to travel horizontally a distance y_0 is found from $v = d/t$ giving $t = d/v = y_0/\sqrt{\frac{2eV_0}{m}}$

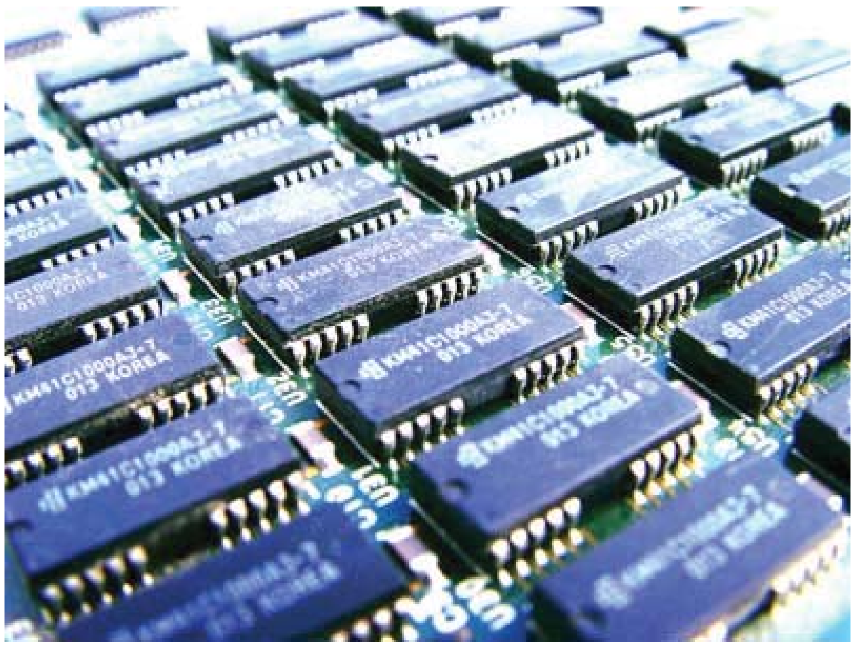
The downward acceleration of the electron is found from $F = qE = ma$ giving $a = eE/m$ and using $y = \frac{1}{2} at^2$ and substituting the values found earlier we have $y = y_0 = \frac{1}{2} (eE/m)(y_0^2)/(2eV_0/m)$ which yields $E = 4V_0/y_0$

ii. For the electron to accelerate downward requires the electric field to point upward, toward the top of the page since negative charges experience forces opposite electric field lines.

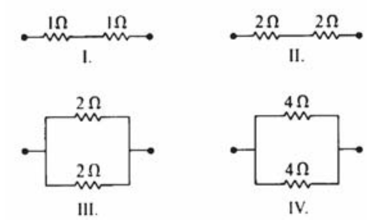
c. $\Delta V = ED = (4D/y_0)V_0$

Chapter 11

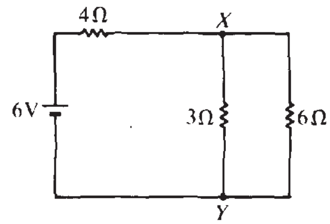
Circuits



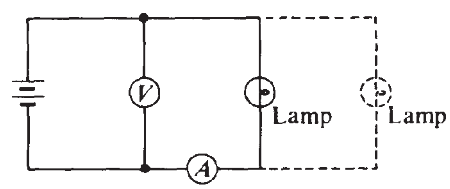
AP Physics Multiple Choice Practice – Circuits



1. **Multiple Correct.** Which two arrangements of resistors shown above have the same resistance between the terminals? Select two answers:
   (A) I
   (B) II
   (C) III
   (D) IV

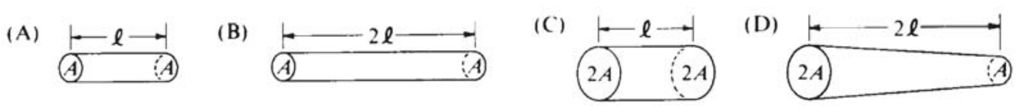


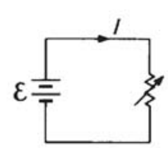
2. In the circuit shown above, what is the value of the potential difference between points X and Y if the 6–volt battery has no internal resistance?
   (A) 2 V (B) 3 V (C) 4 V (D) 6V

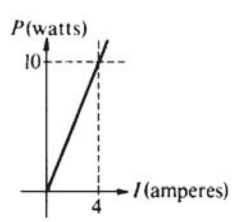


3. A lamp, a voltmeter V, an ammeter A, and a battery with zero internal resistance are connected as shown above. Connecting another lamp in parallel with the first lamp as shown by the dashed lines would
   (A) increase the ammeter reading (B) decrease the ammeter reading
   (C) increase the voltmeter reading (D) decrease the voltmeter reading

4. The five resistors shown below have the lengths and cross–sectional areas indicated and are made of material with the same resistivity. Which has the greatest resistance?

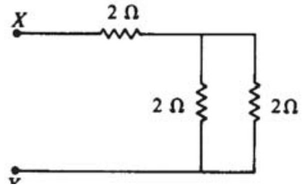






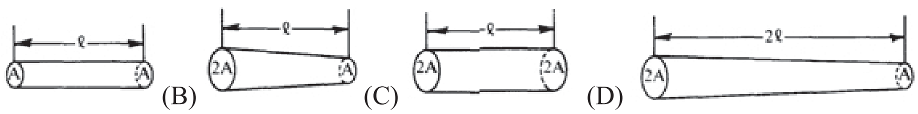
5. The circuit shown above left is made up of a variable resistor and a battery with negligible internal resistance. A graph of the power P dissipated in the resistor as a function of the current I supplied by the battery is given above right. What is the emf of the battery?
   (A) 0.025 V (B) 2.5 V (C) 6.25 V (D) 40 V

6. An immersion heater of resistance R converts electrical energy into thermal energy that is transferred to the liquid in which the heater is immersed. If the current in the heater is I, the thermal energy transferred to the liquid in time t is
(A) IRt (B) I^2Rt (C) IRt^2 (D) IR/t

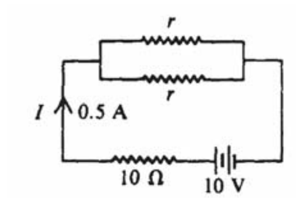


7. The total equivalent resistance between points X and Y in the circuit shown above is
(A) 3 Ω (B) 4 Ω (C) 5 Ω (D) 6 Ω

8. The five resistors shown below have the lengths and cross–sectional areas indicated and are made of material with the same resistivity. Which resistor has the least resistance?

(A) 

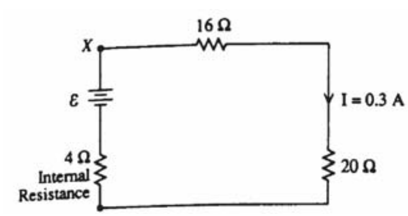
(B) (C) (D)



9. In the circuit shown above, the value of r for which the current I is 0.5 ampere is
(A) 1 Ω (B) 5 Ω (C) 10 Ω (D) 20 Ω

10. Kirchhoff's loop rule for circuit analysis is an expression of which of the following?
(A) Conservation of charge (B) Conservation of energy (C) Ampere's law
(D) Ohm's law

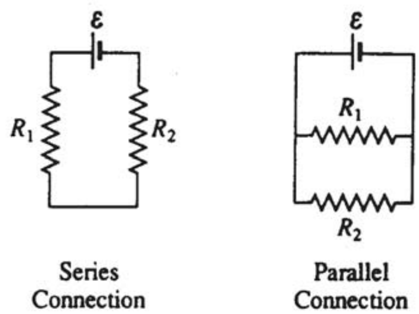
Questions 11-12



The above circuit diagram shows a battery with an internal resistance of 4.0 ohms connected to a 16–ohm and a 20–ohm resistor in series. The current in the 20–ohm resistor is 0.3 amperes

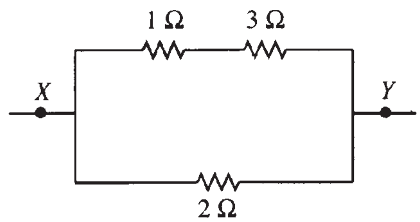
11. What is the emf of the battery?
(A) 1.2 V (B) 6.0 V (C) 10.8 V (D) 12.0 V

12. What power is dissipated by the 4–ohm internal resistance of the battery?
(A) 0.36 W (B) 1.2 W (C) 3.2 W (D) 3.6 W



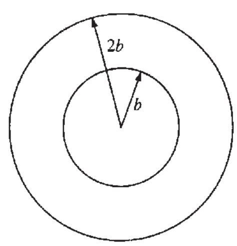
13. In the diagrams above, resistors R_1 and R_2 are shown in two different connections to the same source of emf ε that has no internal resistance. How does the power dissipated by the resistors in these two cases compare?
    (A) It is greater for the series connection.
    (B) It is greater for the parallel connection.
    (C) It is different for each connection, but one must know the values of R_1 and R_2 to know which is greater.
    (D) It is different for each connection, but one must know the value of ε to know which is greater.

<u>Questions 14-15</u> refer to the following diagram that shows part of a closed electrical circuit.



14. The electrical resistance of the part of the circuit shown between point X and point Y is
    (A) 4/3 Ω (B) 2 Ω (C) 4 Ω (D) 6 Ω

15. When there is a steady current in the circuit, the amount of charge passing a point per unit of time is
    (A) the same everywhere in the circuit
    (B) greater in the 1 Ω resistor than in the 2 Ω resistor
    (C) greater at point X than at point Y
    (D) greater in the 2 Ω resistor than in the 3 Ω resistor

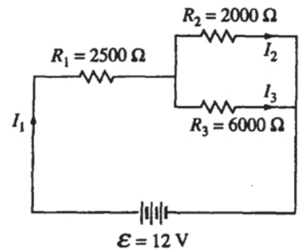


16. Two concentric circular loops of radii *b* and *2b*, made of the same type of wire, lie in the plane of the page, as shown above. The total resistance of the wire loop of radius *b* is *R*. What is the resistance of the wire loop of radius *2b*?
    (A) *R/4* (B) *R/2* (C) *2R* (D) *4R*

17. A wire of length *L* and radius *r* has a resistance *R*. What is the resistance of a second wire made from the same material that has a length *L/2* and a radius *r/2*?
    (A) *4R* (B) *2R* (C) *R* (D) *R/4*

18. The operating efficiency of a 0.5 A, 120 V electric motor that lifts a 9 kg mass against gravity at an average velocity of 0.5 m/s is most nearly
    (A) 13% (B) 25% (C) 53% (D) 75 %

Questions 19-20

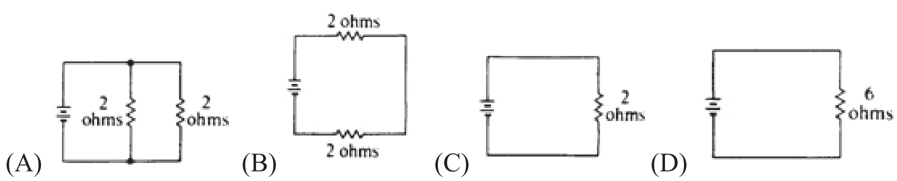


19. What is the current I_1?
(A) 1.0 mA (B) 2.0 mA (C) 3.0 mA (D) 6.0 mA

20. How do the currents I_1, I_2, and I_3 compare?
(A) $I_1 > I_2 > I_3$ (B) $I_1 > I_3 > I_2$ (C) $I_2 > I_1 > I_3$ (D) $I_3 > I_1 > I_2$

21. When lighted, a 100–watt light bulb operating on a 110–volt household circuit has a resistance closest to
(A) $10^{-2}\ \Omega$ (B) $10^{-1}\ \Omega$ (C) $10\ \Omega$ (D) $100\ \Omega$

Questions 22-24



The batteries in each of the circuits shown above are identical and the wires have negligible resistance.

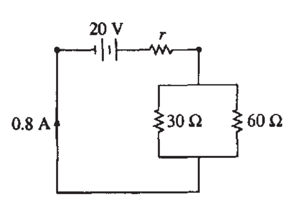
22. In which circuit is the current furnished by the battery the greatest?
(A) A (B) B (C) C (D) D

23. In which circuit is the equivalent resistance connected to the battery the greatest?
(A) A (B) B (C) C (D) D

24. Which circuit dissipates the least power?
(A) A (B) B (C) C (D) D

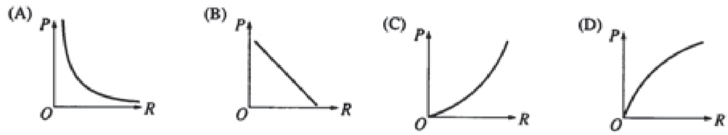
25. The power dissipated in a wire carrying a constant electric current I may be written as a function of the length l of the wire, the diameter d of the wire, and the resistivity ρ of the material in the wire. In this expression, the power dissipated is directly proportional to which of the following?
(A) l only (B) d only (C) l and ρ only (D) d and ρ only

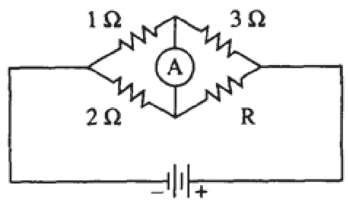
26. A wire of resistance R dissipates power P when a current I passes through it. The wire is replaced by another wire with resistance $3R$. The power dissipated by the new wire when the same current passes through it is
(A) P/9 (B) P/3 (C) 3P (D) 6P



27. A 30–ohm resistor and a 60–ohm resistor are connected as shown above to a battery of emf 20 volts and internal resistance r. The current in the circuit is 0.8 ampere. What is the value of r?
(A) 0.22 Ω (B) 4.5 Ω (C) 5 Ω (D) 16Ω

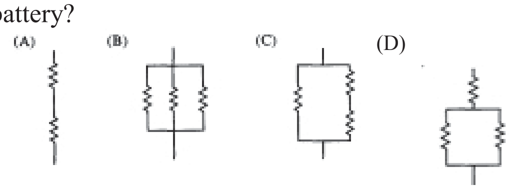
28. A variable resistor is connected across a constant voltage source. Which of the following graphs represents the power P dissipated by the resistor as a function of its resistance R?



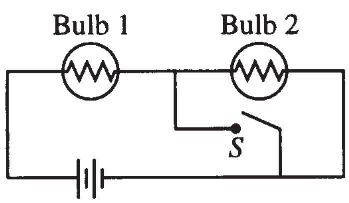


29. If the ammeter in the circuit above reads zero, what is the resistance R ?
(A) 1.5 Ω (B) 4 Ω (C) 5 Ω (D) 6Ω

30 Which of the following combinations of 4Ω resistors would dissipate 24 W when connected to a 12 Volt battery?



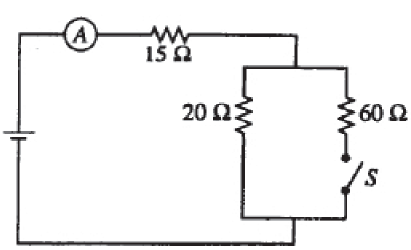
31. A narrow beam of protons produces a current of 1.6×10^{-3} A. There are 10^9 protons in each meter along the beam. Of the following, which is the best estimate of the average speed of the protons in the beam?
(A) 10^{-15} m/s (B) 10^{-12} m/s (C) 10^{-7} m/s (D) 10^7 m/s



32. The circuit in the figure above contains two identical lightbulbs in series with a battery. At first both bulbs glow with equal brightness. When switch S is closed, which of the following occurs to the bulbs?

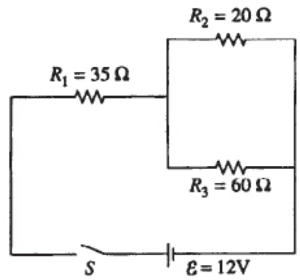
<u>Bulb I</u>	<u>Bulb 2</u>
(A) Goes out	Gets brighter
(B) Gets brighter	Goes out
(C) Gets brighter	Gets slightly dimmer
(D) Gets slightly dimmer	Gets brighter

33. A hair dryer is rated as 1200 W, 120 V. Its effective internal resistance is
(A) 0.1 Ω (B) 10 Ω (C) 12 Ω (D) 120 Ω



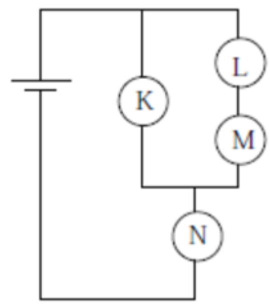
34. When the switch S is open in the circuit shown, the reading on the ammeter A is 2.0 A. When the switch is closed, the reading on the ammeter is
(A) doubled
(B) increased slightly but not doubled
(C) decreased slightly but not halved
(D) halved

35. Two conducting cylindrical wires are made out of the same material. Wire X has twice the length and twice the diameter of wire Y. What is the ratio R_x/R_y of their resistances?
(A) ½ (B) 1 (C) 2 (D) 4



36. In the circuit shown above, the equivalent resistance of the three resistors is
(A) 15Ω (B) 20 Ω (C) 50 Ω (D) 115 Ω

Questions 37-40



Four identical light bulbs K, L, M, and N are connected in the electrical circuit shown above.

37. Rank the current through the bulbs.
(A) L = M > K = N
(B) L > M > K > N
(C) N > K > L = M
(D) N > L = M > K

38. In order of decreasing brightness (starting with the brightest), the bulbs are:
(A) L = M > K = N
(B) L > M > K > N
(C) N > K > L = M
(D) N > L = M > K

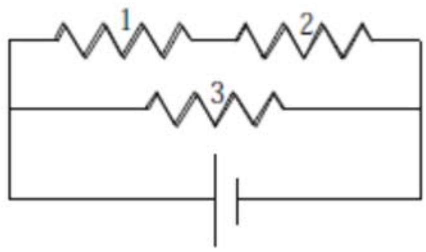
39. Bulb K burns out. Which of the following statements is true?
(A) All the light bulbs go out.
(B) Bulb N becomes brighter.
(C) The brightness of bulb N remains the same.
(D) Bulb N becomes dimmer but does not go out.

40. Bulb M burns out. Which of the following statements is true?
(A) All the light bulbs go out.
(B) Bulb N goes out but at least one other bulb remains lit.
(C) The brightness of bulb N remains the same.
(D) Bulb N becomes dimmer but does not go out.

41. When two resistors, having resistance R_1 and R_2, are connected in parallel, the equivalent resistance of the combination is 5 Ω. Which of the following statements about the resistances is correct?
(A) Both R_1 and R_2 are greater than 5 Ω.
(B) Both R_1 and R_2 are equal to 5 Ω.
(C) Both R_1 and R_2 are less than 5 Ω.
(D) One of the resistances is greater than 5 Ω, one of the resistances is less than 5 Ω.

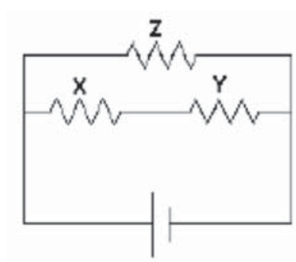
42. Three resistors – R_1, R_2, and R_3 – are connected in series to a battery. Suppose R_1 carries a current of 2.0 A, R_2 has a resistance of 3.0 Ω, and R_3 dissipates 6.0 W of power. What is the voltage across R_3?
(A) 1.0 V (B) 3.0 V (C) 6.0 V (D) 12 V

43. When a single resistor is connected to a battery, a total power P is dissipated in the circuit. How much total power is dissipated in a circuit if n identical resistors are connected in series using the same battery? Assume the internal resistance of the battery is zero.
(A) n^2P (B) nP (C) P (D) P/n



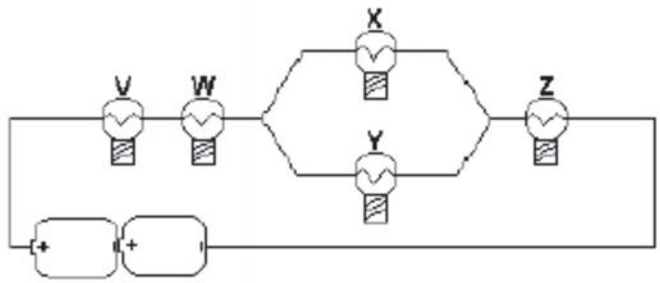
44. Consider the compound circuit shown above. The three bulbs 1, 2, and 3 – represented as resistors in the diagram – are identical. Which of the following statements are true? Select two correct answers.
(A) Bulb 3 is brighter than bulb 1 or 2.
(B) Bulb 3 has more current passing through it than bulb 1 or 2.
(C) Bulb 3 has the same voltage drop across it than bulb 1.
(D) Bulb 3 has the same voltage drop across it than bulb 2.

45. Wire I and wire II are made of the same material. Wire II has twice the diameter and twice the length of wire I. If wire I has resistance R, wire II has resistance
(A) $R/8$ (B) $R/4$ (C) $R/2$ (D) R



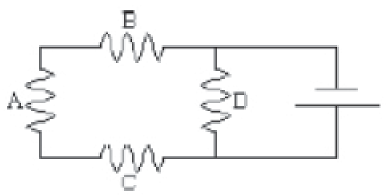
46. Given the simple electrical circuit above, if the current in all three resistors is equal, which of the following statements must be true?
(A) X, Y, and Z all have equal resistance
(B) X and Y have equal resistance
(C) X and Y added together have the same resistance as Z
(D) X and Y each have more resistance than Z

47. Wire Y is made of the same material but has twice the diameter and half the length of wire X. If wire X has a resistance of R then wire Y would have a resistance of
(A) $R/8$ (B) R (C) $2R$ (D) $8R$



48. The diagram above represents a simple electric circuit composed of 5 identical light bulbs and 2 flashlight cells. Which bulb (or bulbs) would you expect to be the brightest?
(A) V only
(B) V and W only
(C) V and Z only
(D) V, W and Z only

49. Three different resistors R_1, R_2 and R_3 are connected in parallel to a battery. Suppose R_1 has 2 V across it, R_2 = 4 Ω, and R_3 dissipates 6 W. What is the current in R_3?
(A) 0.5 A (B) 2 A (C) 3 A (D) 12 A

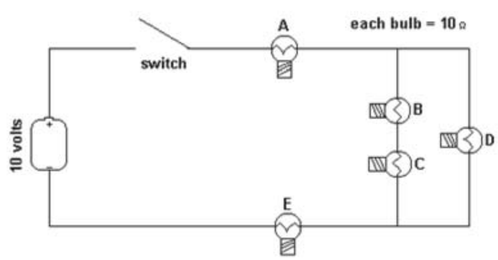


50. If all of the resistors in the simple circuit to the left have the same resistance, which would dissipate the greatest power?
(A) resistor A
(B) resistor B
(C) resistor C
(D) resistor D

51. Each member of a family of six owns a computer rated at 500 watts in a 120 V circuit. If all computers are plugged into a single circuit protected by a 20 ampere fuse, what is the maximum number of the computers can be operating at the same time?
(A) 2 (B) 3 (C) 4 (D) 5 or more

<u>Questions 52-53</u>

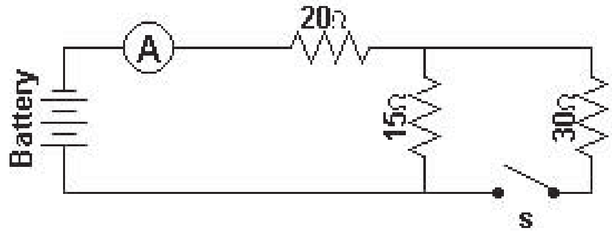
Five identical light bulbs, each with a resistance of 10 ohms, are connected in a simple electrical circuit with a switch and a 10 volt battery as shown in the diagram below.



52. The steady current in the above circuit would be closest to which of the following values?
(A) 0.2 amp (B) 0.37 amp (C) 0.5 amp (D) 2.0 amp

53. Which bulb (or bulbs) could burn out without causing other bulbs in the circuit to also go out?
(A) only bulb D (B) only bulbs C or D
(C) only bulb E (D) only bulbs A or E

Questions 54-56



An ideal battery, an ideal ammeter, a switch and three resistors are connected as shown. With the switch open as shown in the diagram the ammeter reads 2.0 amperes.

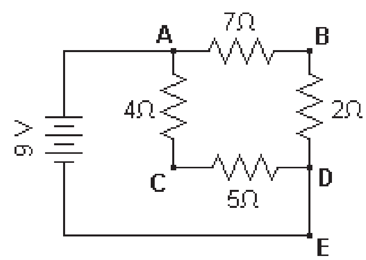
54. With the switch open, what would be the potential difference across the 15 ohm resistor?
(A) 30 V (B) 60 V (C) 70 V (D) 110V

55. With the switch open, what must be the voltage supplied by the battery?
(A) 30 V (B) 60 V (C) 70 V (D) 110 V

56. When the switch is closed, what would be the current in the circuit?
(A) 1.1 A (B) 2.0 A (C) 2.3 A (D) 3.0 A

57. How much current flows through a 4 ohm resistor that is dissipating 36 watts of power?
(A) 2.25 amps (B) 3.0 amps (C) 4.24 amps (D) 9.0 amps

Questions 58-59

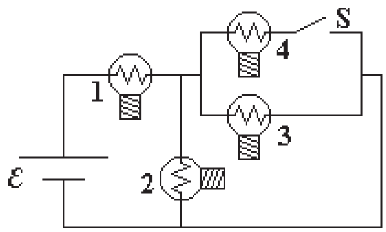


A 9–volt battery is connected to four resistors to form a simple circuit as shown above.

58. How would the current through the 2 ohm resistor compare to the current through the 4 ohm resistor?
(A) one–forth as large
(B) four times as large
(C) twice as large
(D) equally as large

59. What would be the potential at point B with respect to point C in the above circuit?
(A) +7 V (B) +3 V (C) –3 V (D) –7 V

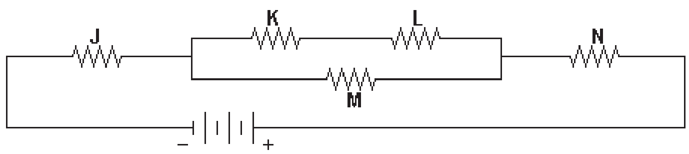
60. A cylindrical resistor has length L and radius r. This piece of material is then drawn so that it is a cylinder with new length $2L$. What happens to the resistance of this material because of this process?
(A) the resistance is quartered.
(B) the resistance is halved.
(C) the resistance is doubled.
(D) the resistance is quadrupled.



61. A circuit is connected as shown. All light bulbs are identical. When the switch in the circuit is closed illuminating bulb #4, which other bulb(s) also become brighter?
(A) Bulb #1 only (B) Bulb #2 only (C) Bulbs #2 and #3 only (D) Bulbs #1, #2, and #3

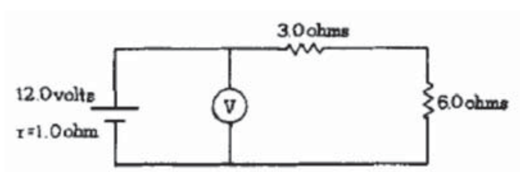
62. A cylindrical graphite resistor has length L and cross–sectional area A. It is to be placed into a circuit, but it first must be cut in half so that the new length is ½ L. What is the ratio of the new resistivity to the old resistivity of the cylindrical resistor?
(A) 4 (B) 2 (C) 1 (D) ½

<u>Questions 63-64</u>
The diagram below shows five identical resistors connected in a combination series and parallel circuit to a voltage source.

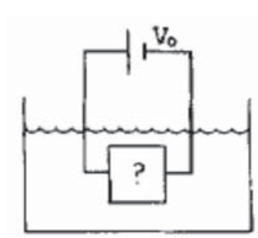


63. Through which resistor(s) would there be the greatest current?
(A) J only (B) M only (C) N only (D) J&N only

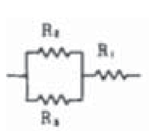
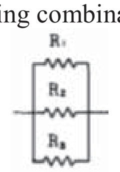
64. Which resistor(s) have the greatest rate of energy dissipation?
(A) J only (B) M only (C) N only (D) J&N only

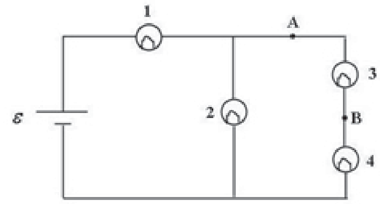


65. In the circuit above the voltmeter V draws negligible current and the internal resistance of the battery is 1.0 ohm. The reading of the voltmeter is
(A) 10.5 V (B) 10.8 V (C) 11.6 V (D) 12.0 V



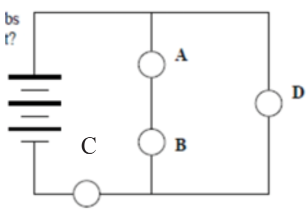
66. Suppose you are given a constant voltage source V_0 and three resistors R_1, R_2, and R_3 with $R_1 > R_2 > R_3$. If you wish to heat water in a pail which of the following combinations of resistors will give the most rapid heating?

(A)  (B)  (C)  (D) 

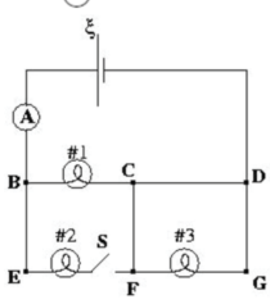


67. For the circuit shown, a shorting wire of negligible resistance is added to the circuit between points A and B. When this shorting wire is added, bulb #3 goes out. Which bulbs (all identical) in the circuit brighten?
(A) Only Bulb 2 (B) Only Bulb 4 (C) Only Bulbs 1 and 4 (D) Bulbs 1, 2 and 4

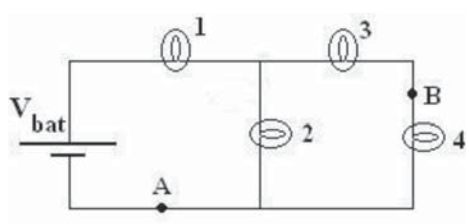
68. A student wants to make a brighter light bulb. He decides to modify the filament. How should the filament of a light bulb be modified in order to make the light bulb produce more light at a given voltage?
(A) Increase the resistivity only.
(B) Increase the diameter only.
(C) Decrease the diameter only.
(D) the length only.



69. In the circuit diagram to the left, all of the bulbs are identical. Which bulb will be the brightest?
(A) A (B) B (C) C (D) D

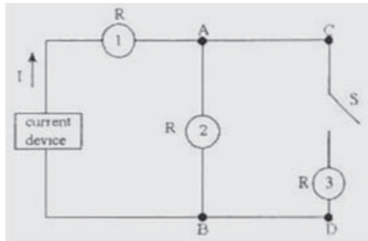


70. For the circuit shown, the ammeter reading is initially *I.* The switch in the circuit then is closed. Consequently:
(A) The ammeter reading decreases.
(B) The potential difference between *E* and *F* increases.
(C) The potential difference between *E* and *F* stays the same.
(D) Bulb #3 lights up more brightly.

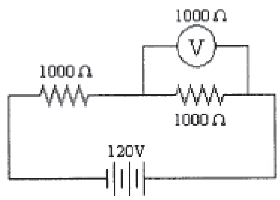


71. For the circuit shown, when a shorting wire (no resistance) connects the points labeled A and B, which of the numbered light bulbs become brighter? Assume that all four bulbs are identical and have resistance R .
(A) Bulb 2 only (B) Bulb 3 only (C) Bulbs 1 and 3 only (D) Bulbs 1, 2, and 3

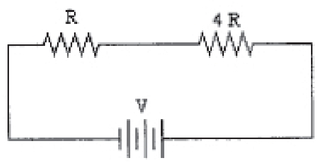
72. Consider a simple circuit containing a battery and three light bulbs. Bulb *A* is wired in parallel with bulb *B* and this combination is wired in series with bulb *C*. What would happen to the brightness of the other two bulbs if bulb *A* were to burn out?
(A) Both would get brighter.
(B) Bulb *B* would get brighter and bulb *C* would get dimmer.
(C) Bulb *B* would get dimmer and bulb *C* would get brighter.
(D) Only bulb *B* would get brighter



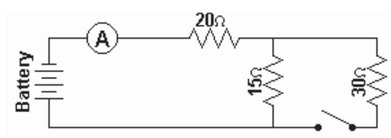
73. In the circuit shown above, a constant current device is connected to some identical light bulbs. After the switch S in the circuit is closed, which statement is correct about the circuit?
(A) Bulb #2 becomes brighter. (B) Bulb #1 becomes dimmer.
(C) All three bulbs become equally brighter. (D) The voltage between points C and D is decreased.



74. Two 1000 Ω resistors are connected in series to a 120–volt electrical source. A voltmeter with a resistance of 1000 Ω is connected across the last resistor as shown. What would be the reading on the voltmeter?
(A) 80 V (B) 60 V (C) 40 V (D) 30 V



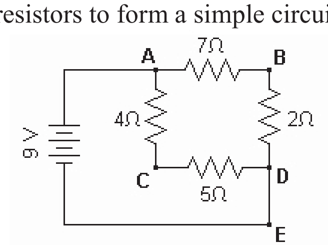
75. Two resistors, one with resistance R and the second with resistance $4R$ are placed in a circuit with a voltage V. If resistance R dissipates power P, what would be the power dissipated by the $4R$ resistance?
(A) 4 P (B) 2 P (C) 1/2 P (D) 1/4 P



76. A battery, an ammeter, three resistors, and a switch are connected to form the simple circuit shown above. When the switch is closed what would happen to the potential difference across the 15 ohm resistor?
(A) it would equal the potential difference across the 20 ohm resistor
(B) it would be twice the potential difference across the 30 ohm resistor
(C) it would equal the potential difference across the 30 ohm resistor
(D) it would be half the potential difference across the 30 ohm resistor

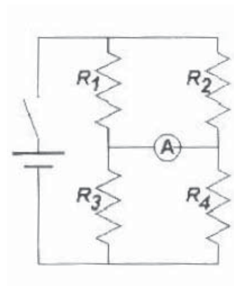
Questions 77-78

A 9–volt battery is connected to four resistors to form a simple circuit as shown below.



77. What would be the current at point E in the circuit?
(A) 2 amp (B) 4 amp (C) 5 amp (D) 7 amp

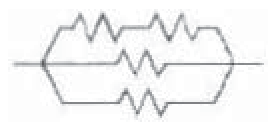
78. What would be the potential at point B with respect to point D?
(A) +2 V (B) +4 V (C) +5 V (D) +7 V



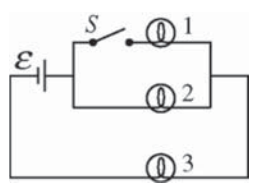
79. Four resistors, R_1, R_2, R_3, and R_4, are connected in the circuit diagram above. When the switch is closed, current flows in the circuit. If no current flows through the ammeter when it is connected as shown, what would be the value of R_3?

(A) $\frac{R_1+R_4}{(R_1+R_2)(R_3+R_4)}$ (B) $\frac{(R_1+R_2)(R_4)}{(R_2+R_4)}$ (C) $\frac{R_1+R_2}{R_4}$ (D) $R_1\frac{R_4}{R_2}$

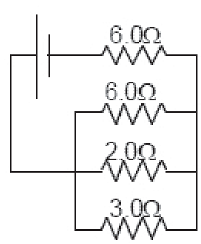
80. Given 4 identical resistors of resistance R, which of the following circuits would have an equivalent resistance of 4/3 R?

(A)  (B)  (C) 

(D) 

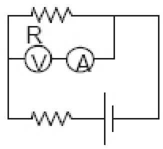


81. The three lightbulbs in the circuit above are identical, and the battery has zero internal resistance. When switch S is closed to cause bulb 1 to light, which of the other two bulbs increase(s) in brightness?
(A) Neither bulb
(B) Bulb 2 only
(C) Bulb 3 only
(D) Both bulbs

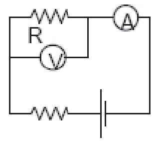


82. In the electric circuit shown above, the current through the 2.0 Ω resistor is 3.0 A. Approximately what is the emf of the battery?
(A) 51 V (B) 42 V (C) 36 V (D) 24 V

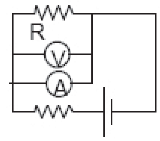
83. Which of the following wiring diagrams could be used to experimentally determine R using Ohm's Law? Assume an ideal voltmeter and an ideal ammeter.



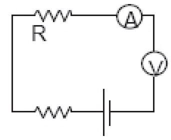
(A)



(B)

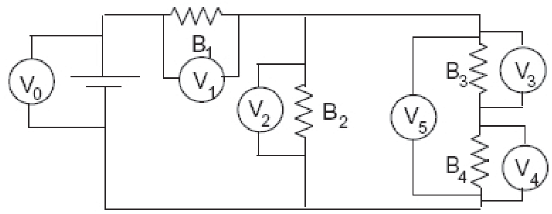


(C)



(D)

Questions 84-85

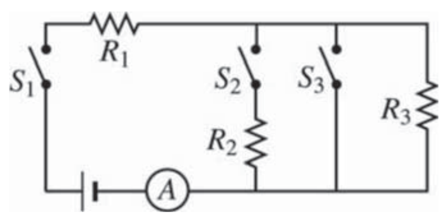


B_1, B_2, B_3, and B_4 are identical light bulbs. There are six voltmeters connected to the circuit as shown. All voltmeters are connected so that they display positive voltages. Assume that the voltmeters do not affect the circuit.

84. If B_2 were to burn out, opening the circuit, which voltmeter(s) would read zero volts?
(A) none would read zero (B) only V_2 (C) only V_3 and V_4 (D) only V_2, V_4, and V_5

85. If B_2 were to burn out, opening the circuit, what would happen to the reading of V_1? Let V be its original reading when all bulbs are functioning and let $\underline{V}$ be its reading when B_2 is burnt out.
(A) $\underline{V} > 2V$ (B) $2V > \underline{V} > V$ (C) $V > \underline{V} > V/2$ (D) $V/2 > \underline{V}$

Questions 86-87



In the circuit above, the resistors all have the same resistance. The battery, wires, and ammeter have negligible resistance. A closed switch also has negligible resistance.

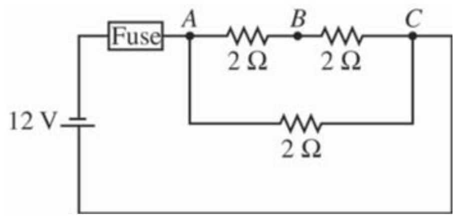
86. Closing which of the switches will produce the greatest power dissipation in R_2?
(A) S_1 only (B) S_2 only (C) S_1 and S_2 only (D) S_1 and S_3 only

87. Closing which of the switches will produce the greatest reading on the ammeter?
(A) S_2 only (B) S_3 only (C) S_1 and S_2 (D) S_1 and S_3

88. Closing which of the switches will produce the greatest voltage across R_3?
(A) S_1 only (B) S_2 only (C) S_1 and S_2 only (D) S_1 and S_3 only

89. Two cables can be used to wire a circuit. Cable A has a lower resistivity, a larger diameter, and a different length than cable B. Which cable should be used to minimize heat loss if the same current is maintained in either cable?
(A) Cable A
(B) Cable B
(C) The heat loss is the same for both.
(D) It cannot be determined without knowing the length of each cable.

Questions 90-91



An electric circuit consists of a 12 V battery, an ideal 10 A fuse, and three 2 Ω resistors connected as shown above.

90. What would be the reading on a voltmeter connected across points *A* and *C* ?
(A) 12 V (B) 6 V (C) 3 V (D) 2 V

91. What would be the reading on an ammeter inserted at point *B* ?
(A) 9 A (B) 6 A (C) 3 A (D) 2 A

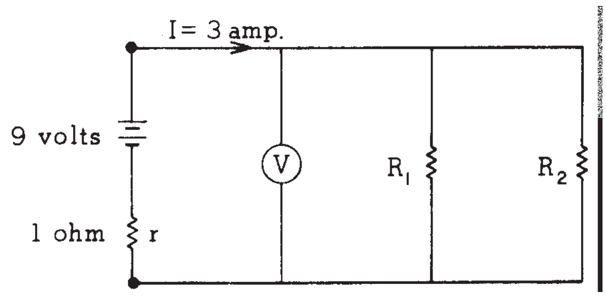
92. A length of wire of resistance *R* is connected across a battery with zero internal resistance. The wire is then cut in half and the two halves are connected in parallel. When the combination is reconnected across the battery, what happens to the resultant power dissipated and the current drawn from the battery?

	Power	**Current**
(A)	Doubles	Doubles
(B)	Quadruples	Doubles
(C)	Doubles	Quadruples
(D)	Quadruples	Quadruples

93. A fixed voltage is applied across the length of a tungsten wire. An increase in the power dissipated by the wire would result if which of the following could be increased?
(A) The resistivity of the tungsten
(B) The cross-sectional area of the wire
(C) The length of the wire
(D) The temperature of the wire

AP Physics Free Response Practice – Circuits

WARNING ONLY CIRCUITS WITH RESISTORS ARE ON AP PHYSICS 1

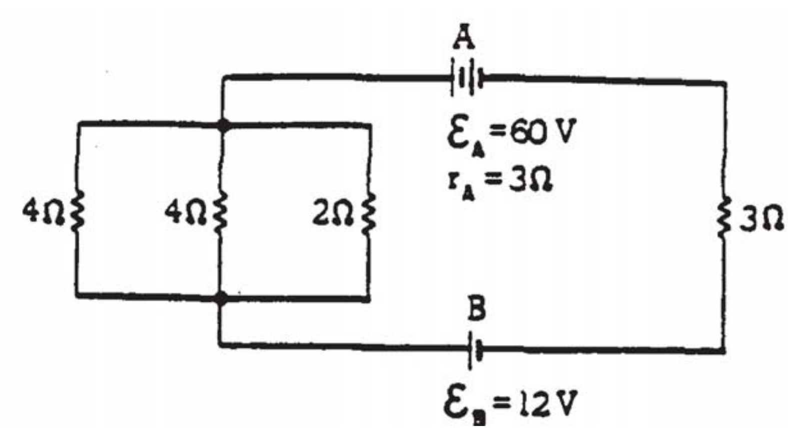


1976B3. In the circuit shown above, the current delivered by the 9-volt battery of internal resistance 1 ohm is 3 amperes. The power dissipated in R_2 is 12 watts.

a. Determine the reading of voltmeter V in the diagram.

b. Determine the resistance of R_2.

c. Determine the resistance of R_1.

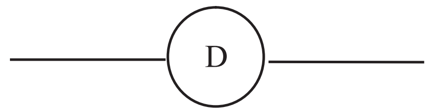


1981B4. A circuit consists of battery A of emf $\mathcal{E}_A$ = 60 volts and internal resistance r_A = 3 ohms; battery B of emf $\mathcal{E}_B$ = 12 volts and internal resistance r_B = 1 ohm; and four resistors connected as shown in the diagram above.

a. Calculate the current in the 2-ohm resistor.

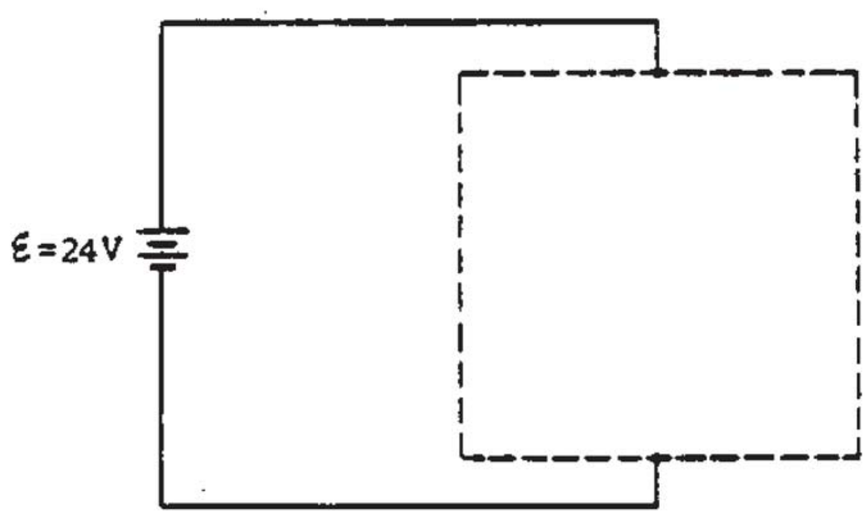
b. Calculate the power dissipated in the 3-ohm resistor.

c. Calculate the terminal voltage of battery B.

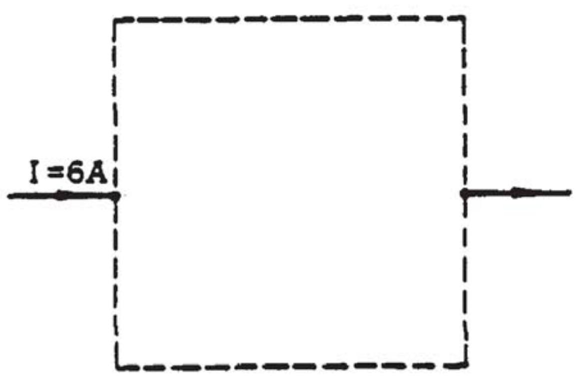


1980B2. The electrical device whose symbol is shown above requires a terminal voltage of 12 volts and a current of 2 amperes for proper operation.

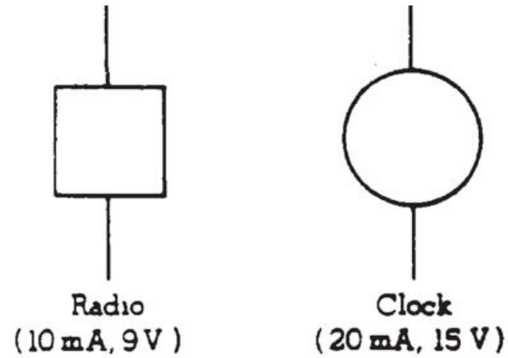
a. Using only this device and one or more 3-ohm resistors design a circuit so that the device will operate properly when the circuit is connected across a battery of emf 24 volts and negligible internal resistance. Within the dashed-line box in the diagram below, draw the circuit using the symbol for the device and the appropriate symbol for each 3-ohm resistor.



b. Using only this device and one or more 3-ohm resistors, design a circuit so that the device will operate properly when connected to a source that supplies a fixed current of 6 amperes. Within the dashed-line box in the diagram below, draw the circuit using the symbol for the device and the appropriate symbol.

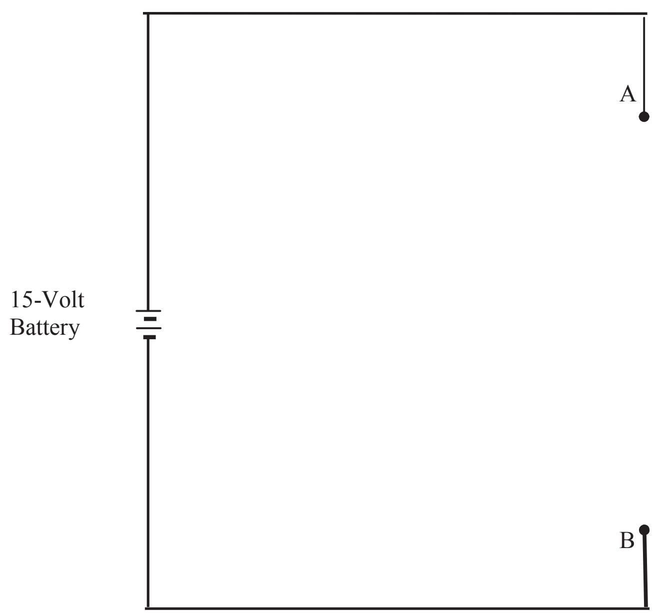


c. Calculate the power dissipation In each 3-ohm resistor used in the circuit in part b..



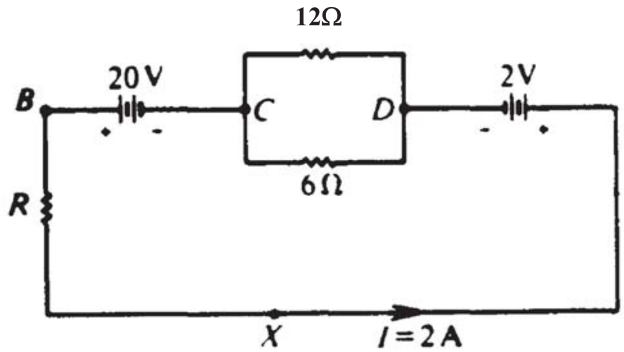
1982B4. A cabin contains only two small electrical appliances: a radio that requires 10 milliamperes of current at 9 volts, and a clock that requires 20 milliamperes at 15 volts. A 15-volt battery with negligible internal resistance supplies the electrical energy to operate the radio and the clock.

a. Complete the diagram below to show how the radio, the clock, and a single resistor R can be connected between points A and B so that the correct potential difference is applied across each appliance. Use the symbols in the diagram above to indicate the radio and the clock.



b. Calculate the resistance of R.

c. Calculate the electrical energy that must be supplied by the battery to operate the circuits for 1 minute.



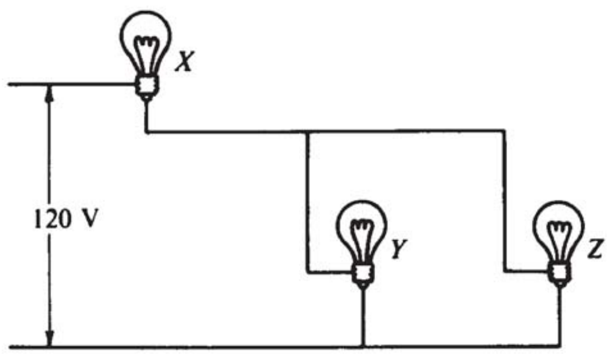
1983B3. The circuit shown above is constructed with two batteries and three resistors. The connecting wires may be considered to have negligible resistance. The current I is 2 amperes.

a. Calculate the resistance R.

b. Calculate the current in the
   i. 6-ohm resistor
   ii. 12-ohm resistor

c. The potential at point X is 0 volts. Calculate the electric potential at points B. C, and D in the circuit.

d. Calculate the power supplied by the 20-volt battery.



1986B3. In the circuit shown above, X, Y. and Z represent three light bulbs, each rated at 60 watts, 120 volts. Assume that the resistances of the bulbs are constant and do not depend on the current.

a. What is the resistance of each bulb?

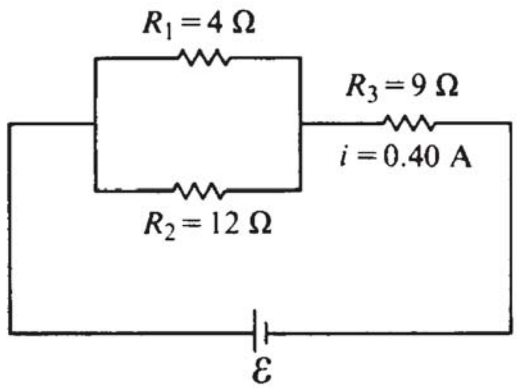
b. What is the equivalent resistance of the three light bulbs when arranged as shown?

c. What is the total power dissipation of this combination when connected to a 120-volt source as shown?

d. What is the current in bulb X?

e. What is the potential difference across bulb X?

f. What is the potential difference across bulb Zd ?

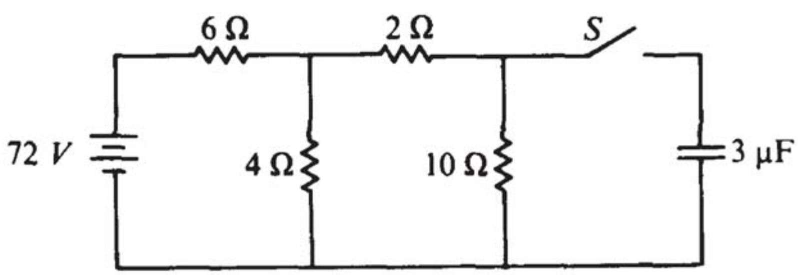


1987B4. Three resistors are arranged in a circuit as shown above. The battery has an unknown but constant emf $\mathcal{E}$ and a negligible internal resistance.

a. Determine the equivalent resistance of the three resistors.

The current I in resistor R_3 is 0.40 ampere.

b. Determine the emf $\mathcal{E}$ (Voltage) of the battery.
c. Determine the potential difference across resistor R_1
d. Determine the power dissipated in resistor R_1
e. Determine the amount of charge that passes through resistor R_3 in one minute.



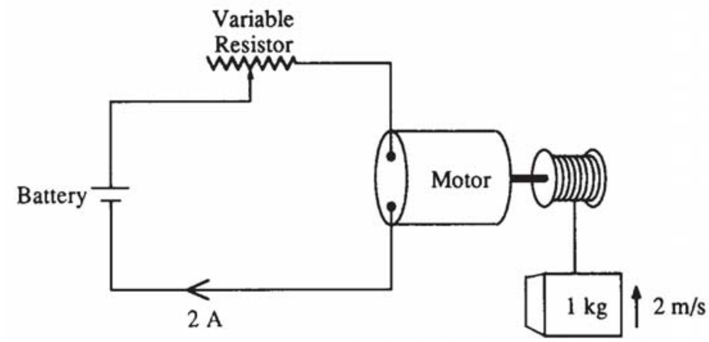
1988B3. The circuit shown above includes a switch S, which can be closed to connect the 3-microfarad capacitor in parallel with the 10-ohm resistor or opened to disconnect the capacitor from the circuit.

Case 1: Switch S is open. The capacitor is not connected. Under these conditions determine:

a. the current in the battery
b. the current in the 10-ohm resistor
c. the potential difference across the 10-ohm resistor

Case II: Switch S is closed. The capacitor is connected. After some time, the currents reach constant values. Under these conditions determine:

d. the charge on the capacitor
e. the energy stored in the capacitor



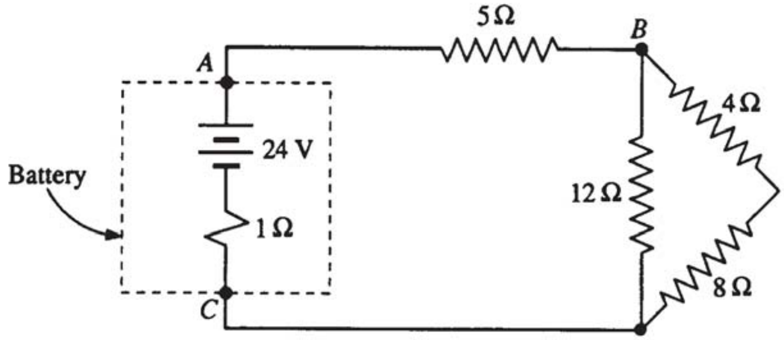
1989B3. A series circuit consists of a battery of negligible internal resistance, a variable resistor, and an electric motor of negligible resistance. The current in the circuit is 2 amperes when the resistance in the circuit is adjusted to 10 ohms. Under these conditions the motor lifts a l-kilogram mass vertically at a constant speed of 2 meters per second.

a. Determine the electrical power that is
   i. dissipated in the resistor
   ii. used by the motor in lifting the mass
   iii. supplied by the battery

b. Determine the potential difference across
   i. the resistor
   ii. the motor
   iii. the battery

The resistor is now adjusted until the mass rises vertically at a constant speed of 3 meters per second. The voltage drop across the motor is proportional to the speed of the motor, and the current remains constant.

c. Determine the voltage drop across the motor.
d. Determine the new resistance in the circuit.



1990B3. A battery with an emf of 24 volts and an internal resistance of 1 ohm is connected to an external circuit as shown above. Determine each of the following:

a. the equivalent resistance of the combination of the 4-ohm, 8-ohm, and 12-ohm resistors
b. the current in the 5-ohm resistor
c. the terminal voltage, V_{AC} of the battery
d. the rate at which energy is dissipated in the 12-ohm resistor
e. the magnitude of the potential difference V_{BC}
f. the power delivered by the battery to the external circuit

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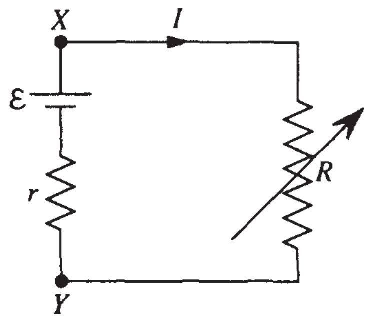
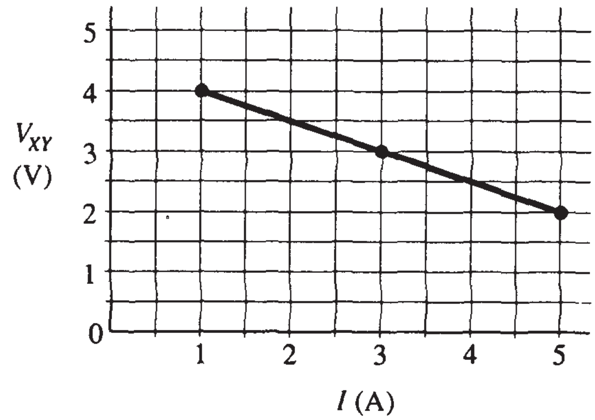




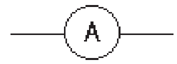
1991B4. A battery with emf $\mathcal{E}$ and internal resistance r is connected to a variable resistance R at points X and Y. as shown above on the left. Varying R changes both the current I and the terminal voltage V_{XY}. The quantities I and V_{XY} are measured for several values of R and the data are plotted in a graph, as shown above on the right.

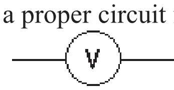
a. Determine the emf $\mathcal{E}$ of the battery.

b. Determine the internal resistance r of the battery.

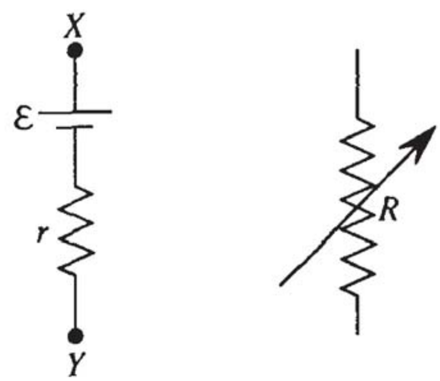
c. Determine the value of the resistance R that will produce a current I of 3 amperes.

d. Determine the maximum current that the battery can produce.

e. The current and voltage measurements were made with an ammeter and a voltmeter. On the diagram below, show a proper circuit for performing these measurements. Use 

to represent the ammeter and 

to represent the voltmeter.



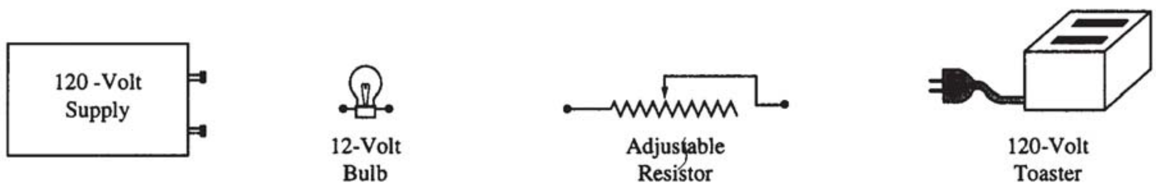
1995B2. A certain light bulb is designed to dissipate 6 watts when it is connected to a 12-volt source.

a. Calculate the resistance of the light bulb.

b. If the light bulb functions as designed and is lit continuously for 30 days, how much energy is used? Be sure to indicate the units in your answer.

The 6-watt, 12-volt bulb is connected in a circuit with a 1,500-watt, 120-volt toaster; an adjustable resistor; and a 120-volt power supply. The circuit is designed such that the bulb and the toaster operate at the given values and, if the light bulb fails, the toaster will still function at these values.

c. On the diagram below, draw in wires connecting the components shown to make a complete circuit that will function as described above.



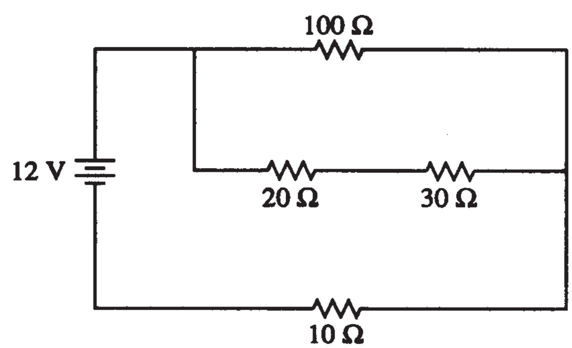
d. Determine the value of the adjustable resistor that must be used in order for the circuit to work as designed.

e. If the resistance of the adjustable resistor is increased, what will happen to the following?
   i. The brightness of the bulb. Briefly explain your reasoning.
   ii. The power dissipated by the toaster. Briefly explain your reasoning.

1996B4. A student is provided with a 12.0-V battery of negligible internal resistance and four resistors with the following resistances: 100 Ω, 30 Ω, 20 Ω, and 10 Ω. The student also has plenty of wire of negligible resistance available to make connections as desired.

a. Using all of these components, draw a circuit diagram in which each resistor has nonzero current flowing through it, but in which the current from the battery is as small as possible.

b. Using all of these components, draw a circuit diagram in which each resistor has nonzero current flowing through it, but in which the current from the battery is as large as possible (without short circuiting the battery).



The battery and resistors are now connected in the circuit shown above.

c. Determine the following for this circuit.
   i. The current in the 10-Ω resistor
   ii. The total power consumption of the circuit

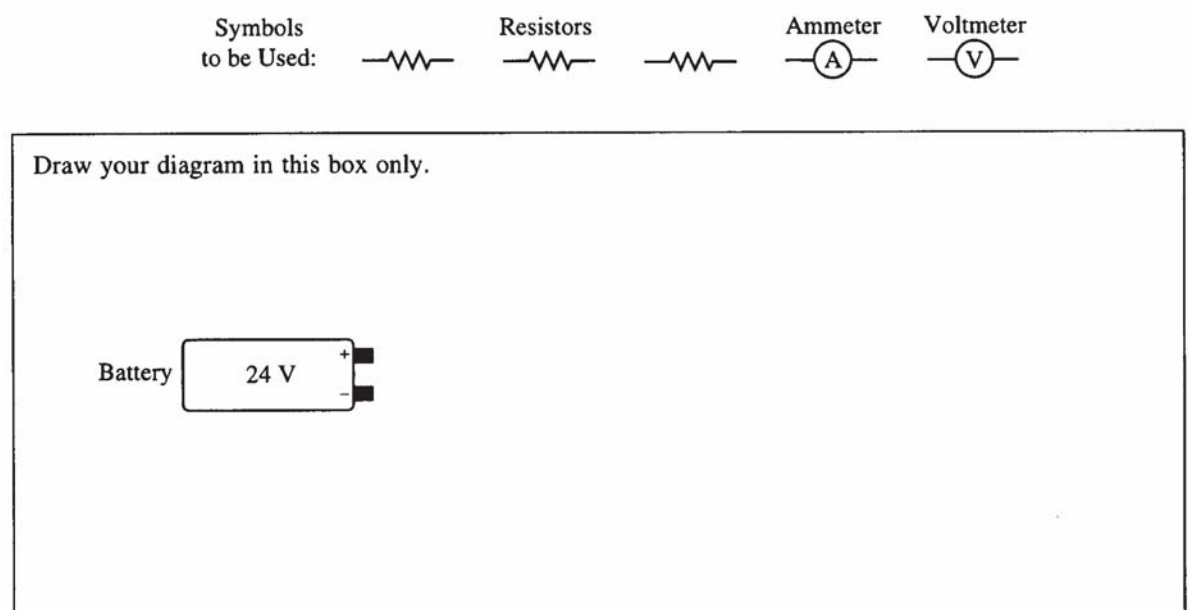
d. Assuming that the current remains constant, how long will it take to provide a total of 10 kJ of electrical energy to the circuit?

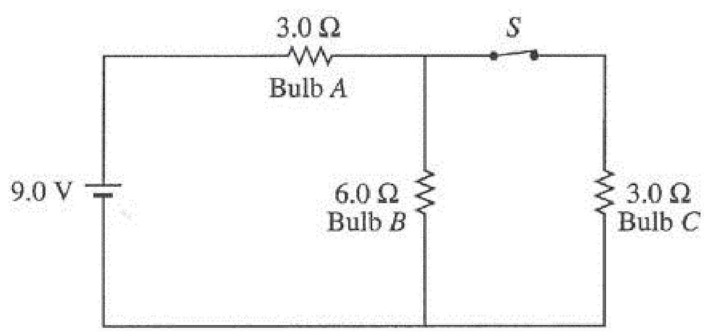
1997B4 (modified) Three identical resistors, each of resistance 30 Ω are connected in a circuit to heat water in a glass beaker. 24 V battery with negligible internal resistance provides the power. The three resistors may be connected in series or in parallel.

a. i. If they are connected in series, what power is developed in the circuit?

ii. If they are connected in parallel, what power is developed in the circuit?

b. Using the battery and one or more of the resistors, design a circuit that will heat the water at the fastest rat when the resistor(s) are placed in the water. Include an ammeter to measure the current in the circuit and a voltmeter to measure the total potential difference of the circuit. Assume the wires are insulated and have no resistance. Draw a diagram of the circuit in the box below, using the following symbols to represent the components in your diagram.





2002B3B. Lightbulbs of fixed resistance 3.0 Ω and 6.0 Ω, a 9.0 V battery, and a switch S are connected as shown in the schematic diagram above. The switch S is closed.

a. Calculate the current in bulb A.

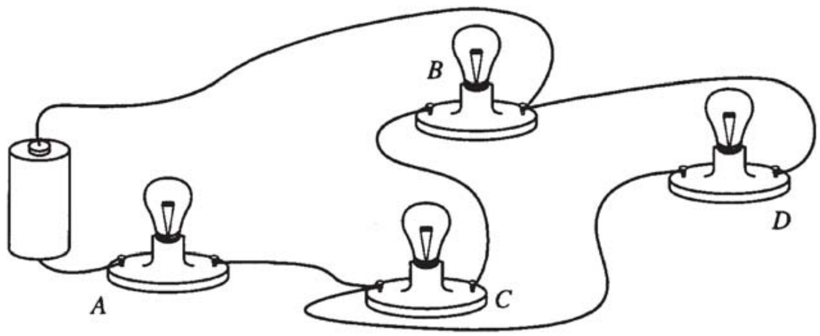
b. Which lightbulb is brightest? Justify your answer.

c. Switch S is then opened. By checking the appropriate spaces below, indicate whether the brightness of each lightbulb increases, decreases, or remains the same. Explain your reasoning for each lightbulb.

i. Bulb A: The brightness ____ increases ____decreases ____ remains the same
Explanation:

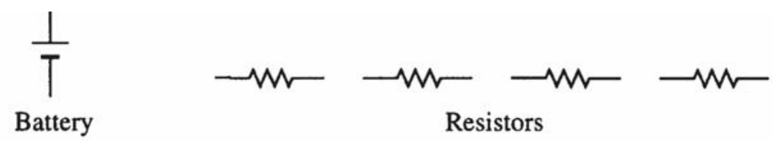
ii. Bulb B: The brightness ____increases ____decreases ____remains the same
Explanation:

iii. Bulb C: The brightness ____increases ____decreases ____ remains the same
Explanation:



1998B4 In the circuit shown above, A, B, C, and D are identical lightbulbs. Assume that the battery maintains a constant potential difference between its terminals (i.e., the internal resistance of the battery is assumed to be negligible) and the resistance of each lightbulb remains constant.

a. Draw a diagram of the circuit in the box below, using the following symbols to represent the components in your diagram. Label the resistors A, B. C, and D to refer to the corresponding lightbulbs.



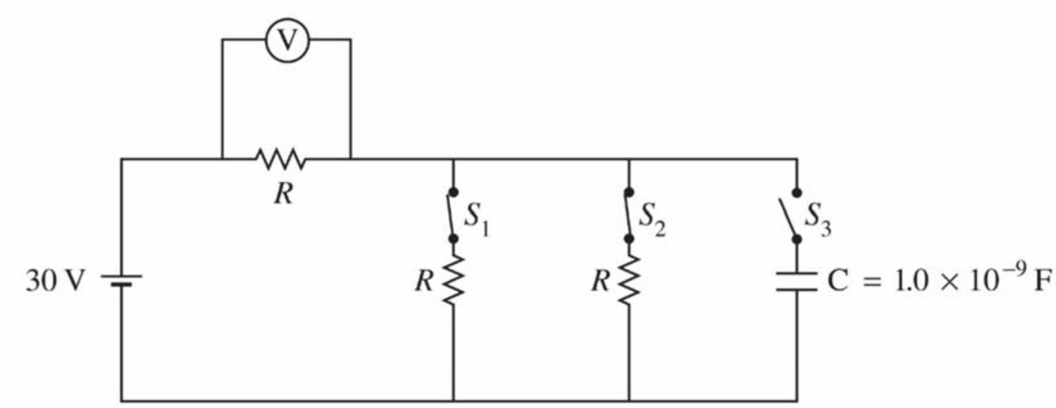
Draw your diagram in this box only.

b. List the bulbs in order of their brightnesses, from brightest to least bright. If any two or more bulbs have the same brightness, state which ones. Justify your answer.

c. Bulb D is then removed from its socket.

i. Describe the change in the brightness, if any, of bulb A when bulb D is removed from its socket. Justify your answer.

ii. Describe the change in the brightness, if any, of bulb B when bulb D is removed from its socket. Justify your answer.



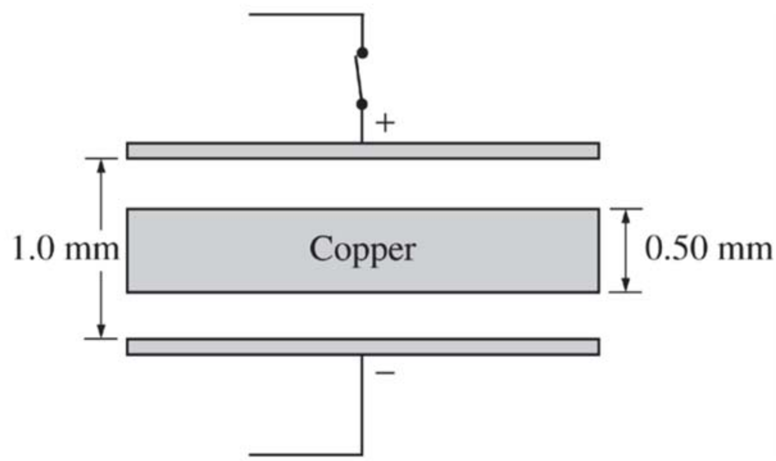
2000B3. Three identical resistors, each with resistance R, and a capacitor of 1.0×10^{-9} F are connected to a 30 V battery with negligible internal resistance, as shown in the circuit diagram above. Switches S_1 and S_2 are initially closed, and switch S_3 is initially open. A voltmeter is connected as shown.

a. Determine the reading on the voltmeter.

Switches S_1 and S_2 are now opened, and then switch S_3 is closed.

b. Determine the charge Q on the capacitor after S_3 has been closed for a very long time.

After the capacitor is fully charged, switches S_1 and S_2 remain open, switch S_3 remains closed, the plates are held fixed, and a conducting copper block is inserted midway between the plates, as shown below. The plates of the capacitor are separated by a distance of 1.0 mm, and the copper block has a thickness of 0.5 mm.



c. i. What is the potential difference between the plates?

ii. What is the electric field inside the copper block?

iii. On the diagram above, draw arrows to clearly indicate the direction of the electric field between the plates.

iv. Determine the magnitude of the electric field in each of the spaces between the plates and the copper block.

2002B3 Two lightbulbs, one rated 30 W at 120 V and another rated 40 W at 120 V, are arranged in two different circuits.

a. The two bulbs are first connected in parallel to a 120 V source.
   i. Determine the resistance of the bulb rated 30 W and the current in it when it is connected in this circuit.
   ii. Determine the resistance of the bulb rated 40 W and the current in it when it is connected in this circuit.

b. The bulbs are now connected in series with each other and a 120 V source.
   i. Determine the resistance of the bulb rated 30 W and the current in it when it is connected in this circuit.
   ii. Determine the resistance of the bulb rated 40 W and the current in it when it is connected in this circuit.

c. In the spaces below, number the bulbs in each situation described, in order of their brightness. (1= brightest, 4 = dimmest)

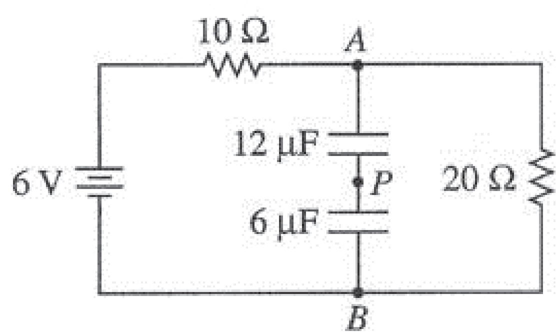
____30 W bulb in the parallel circuit

____40 W bulb in the parallel circuit

____30 W bulb in the series circuit

____40 W bulb in the series circuit

d. Calculate the total power dissipated by the two bulbs in each of the following cases.
   i. The parallel circuit
   ii. The series circuit



2003B2 A circuit contains two resistors (10 Ω and 20 Ω) and two capacitors (12 μF and 6 μF) connected to a 6 V battery, as shown in the diagram above. The circuit has been connected for a long time.

a. Calculate the total capacitance of the circuit.
b. Calculate the current in the 10 Ω resistor.
c. Calculate the potential difference between points A and B.
d. Calculate the charge stored on one plate of the 6 μF capacitor.
e. The wire is cut at point P. Will the potential difference between points A and B increase, decrease, or remain the same?

_____increase _____decrease _____remain the same

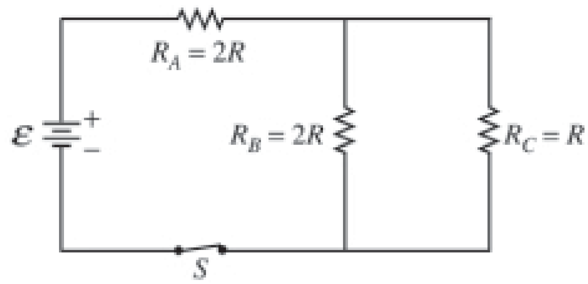
Justify your answer.

2003Bb2. A student is asked to design a circuit to supply an electric motor with 1.0 mA of current at 3.0 V potential difference.

a. Determine the power to be supplied to the motor.
b. Determine the electrical energy to be supplied to the motor in 60 s.
c. Operating as designed above, the motor can lift a 0.012 kg mass a distance of 1.0 m in 60 s at constant velocity. Determine the efficiency of the motor.

To operate the motor, the student has available only a 9.0 V battery to use as the power source and the following five resistors.

d. In the space below, complete a schematic diagram of a circuit that shows how one or more of these resistors can be connected to the battery and motor so that 1.0 mA of current and 3.0 V of potential difference are supplied to the motor. Be sure to label each resistor in the circuit with the correct value of its resistance.



2007B3. The circuit above contains a battery with negligible internal resistance, a closed switch S, and three resistors, each with a resistance of R or 2R.

a. i. Rank the currents in the three resistors from greatest to least, with number 1 being greatest. If two resistors have the same current, give them the same ranking.

_______I_A _______I_B _______I_C

ii. Justify your answers.

b. i. Rank the voltages across the three resistors from greatest to least, with number 1 being greatest. If two resistors have the same voltage across them, give them the same ranking.

_______V_A _______V_B _______V_C

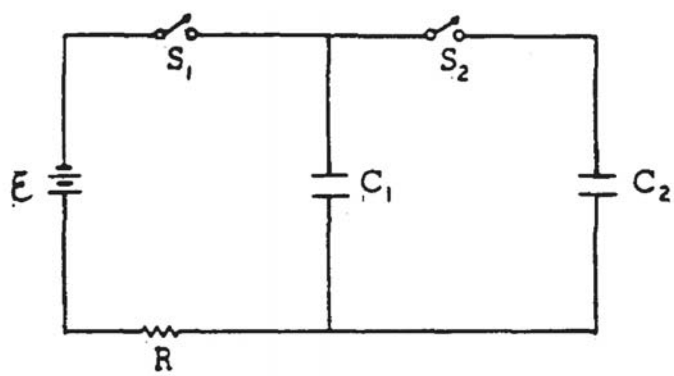
ii. Justify your answers.

For parts c. through e., use $\mathcal{E} = 12$ V and R = 200 Ω.

c. Calculate the equivalent resistance of the circuit.

d. Calculate the current in resistor R_C.

e. The switch S is opened, resistor R_B is removed and replaced by a capacitor of capacitance 2.0×10^{-6} F, and the switch S is again closed. Calculate the charge on the capacitor after all the currents have reached their final steady-state values.



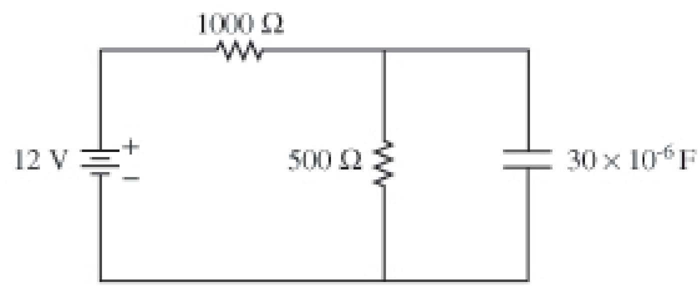
1975E2. In the diagram above, V = 100 volts; C_1 = 12 microfarads; C_2 = 24 microfarads; R = 10 ohms. Initially, C_1 and C_2 are uncharged, and all switches are open.

a. First, switch S_1 is closed. Determine the charge on C_1 when equilibrium is reached.

b. Next S_1 is opened and afterward S_2 is closed. Determine the charge on C_1 when equilibrium is again reached.

c. For the equilibrium condition of part b., determine the voltage across C_1.

d. S_2 remains closed, and now S_1 is also closed. How much additional charge flows from the battery?



B2007b3. In the circuit above, a 12.0 V battery is connected to two resistors, one of resistance 1000 Ω and the other of resistance 500 Ω . A capacitor with a capacitance of 30×10^{-6} F is connected in parallel with the 500 Ω resistor. The circuit has been connected for a long time, and all currents have reached their steady states.

a. Calculate the current in the 500 Ω resistor.

b. i. Draw an ammeter in the circuit above in a location such that it could measure the current in the 500 Ω resistor. Use the symbol (A) to indicate the ammeter.

ii. Draw a voltmeter in the circuit above in a location such that it could measure the voltage across the 1000 Ω, resistor. Use the symbol (V) to indicate the voltmeter.

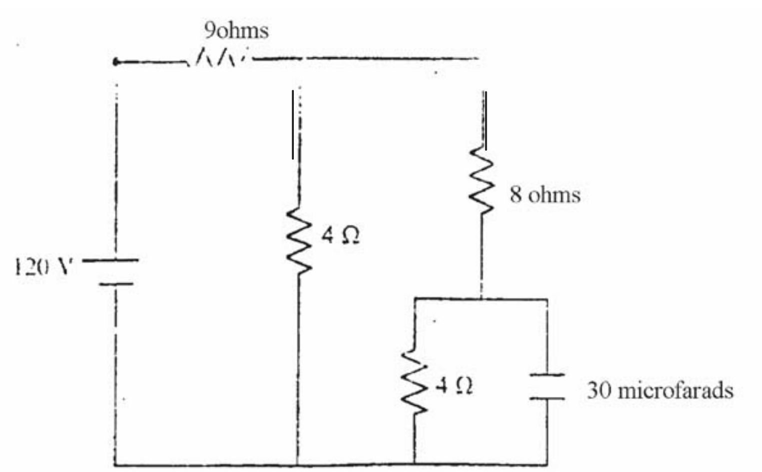
c. Calculate the charge stored on the capacitor.

d. Calculate the power dissipated in the 1000 Ω resistor.

e. The capacitor is now discharged, and the 500 Ω resistor is removed and replaced by a resistor of greater resistance. The circuit is reconnected, and currents are again allowed to come to their steady-state values. Is the charge now stored on the capacitor larger, smaller, or the same as it was in part c.?

______Larger ______Smaller ______The same as

Justify your answer.



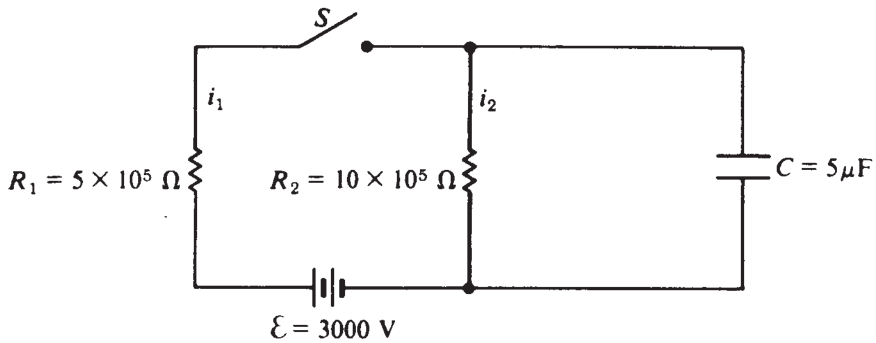
1988E2. In the circuit shown above. the battery has been connected for a long time so that the currents have steady values. Given these conditions, calculate each of the following

a. The current in the 9-ohm resistor.

b. The current in the 8-ohm resistor.

c. The potential difference across the 30-microfarad capacitor.

d. The energy stored in the 30-microfarad capacitor.



1985E2 (modified) In the circuit shown above, i_1 and i_2 are the currents through resistors R_1 and R_2, respectively. V_1, V_2, and V_c are the potential differences across resistor R_1 resistor R_2, and capacitor C, respectively. Initially the capacitor is uncharged.

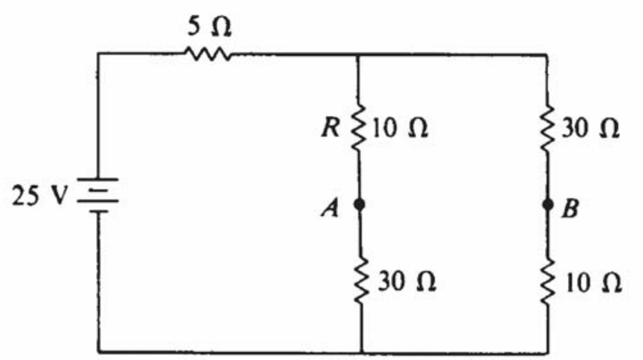
a. Calculate the current i_1 immediately after switch S is closed.

Assume switch S has been closed for a long time.

b. Calculate the current i_2.

c. Calculate the charge Q on the capacitor.

d. Calculate the energy U stored in the capacitor.



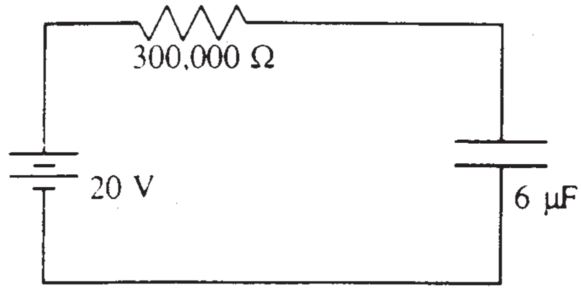
1986E2 (modified) Five resistors are connected as shown above to a 25-volt source of emf with zero internal resistance.

a. Determine the current in the resistor labeled R.

A 10-microfarad capacitor is connected between points A and B. The currents in the circuit and the charge on the capacitor soon reach constant values. Determine the constant value for each of the following.

b. The current in the resistor R

c. The charge on the capacitor



1989E3. A battery with an emf of 20 volts is connected in series with a resistor of 300,000 ohms and an air-filled parallel-plate capacitor of capacitance 6 microfarads.

a. Determine the energy stored in the capacitor when it is fully charged.

The spacing between the capacitor plates is suddenly increased (in a time short enough so the charge does not have time to readjust) to four times its original value.

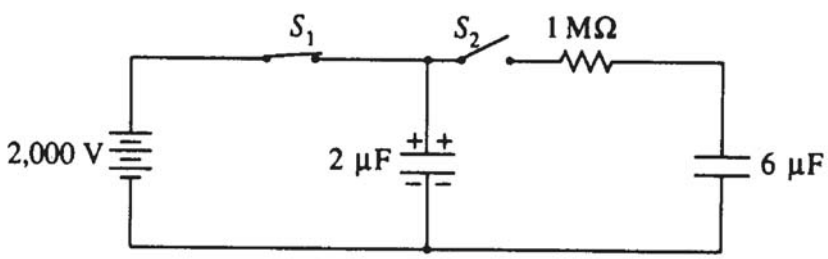
b. Determine the work that must be done in increasing the spacing in this fashion.

c. Determine the current in the resistor immediately after the spacing is increased.

After a long time. the circuit reaches a new static state.

d. Determine the total charge that has passed through the battery.

e. Determine the energy that has been added to the battery.



1992E2. The 2-microfarad (2×10^{-6} farad) capacitor shown in the circuit above is fully charged by closing switch S_1 and keeping switch S_2 open, thus connecting the capacitor to the 2,000-volt power supply.

a. Determine each of the following for this fully charged capacitor.
   i. The magnitude of the charge on each plate of the capacitor.
   ii. The electrical energy stored in the capacitor.

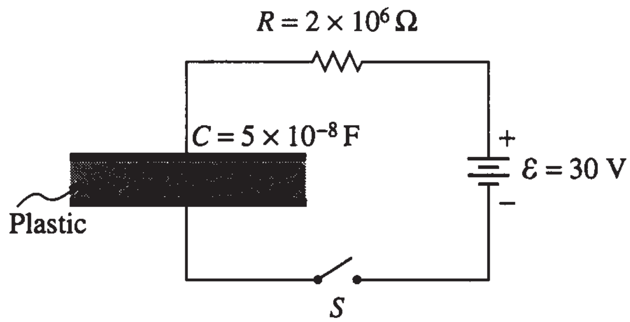
At a later time, switch S_1 is opened. Switch S_2 is then closed, connecting the charged 2-microfarad capacitor to a l-megohm (1×10^6 Ω) resistor and a 6-microfarad capacitor, which is initially uncharged.

b. Determine the initial current in the resistor the instant after switch S_2 is closed.

Equilibrium is reached after a long period of time.

c. Determine the charge on the positive plate of <u>each</u> of the capacitors at equilibrium.

d. Determine the total electrical energy stored in the two capacitors at equilibrium. If the energy is greater than the energy determined in part a. ii., where did the increase come from? If the energy is less than the energy determined in part a. ii., where did the electrical energy go?



1995E2 (modified) A parallel-plate capacitor is made from two sheets of metal, each with an area of 1.0 square meter, separated by a sheet of plastic 1.0 millimeter (10^{-3} m) thick, as shown above. The capacitance is measured to be 0.05 microfarad (5×10^{-8} F).

a. What is the dielectric constant of the plastic?

The uncharged capacitor is connected in series with a resistor R = 2×10^6 ohms, a 30-volt battery, and an open switch S, as shown above. The switch is then closed.

b. What is the initial charging current when the switch S is closed?

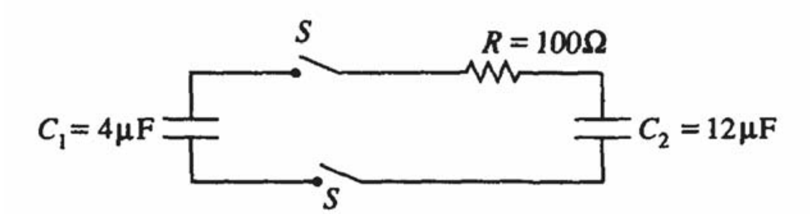
c. Determine the magnitude and sign of the final charge on the bottom plate of the fully charged capacitor.

d. How much electrical energy is stored in the fully charged capacitor?

After the capacitor is fully charged, it is carefully disconnected, leaving the charged capacitor isolated in space. The plastic sheet is then removed from between the metal plates. The metal plates retain their original separation of 1.0 millimeter.

e. What is the new voltage across the plates?

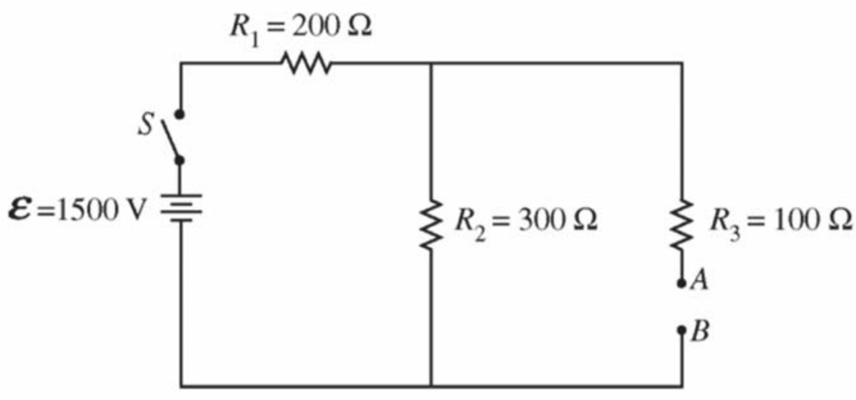
f. If there is now more energy stored in the capacitor, where did it come from? If there is now less energy, what happened to it?



1996E2 (modified) Capacitors 1 and 2, of capacitance C_1 = 4μF and C_2 = 12μF, respectively, are connected in a circuit as shown above with a resistor of resistance R = 100 Ω and two switches. Capacitor 1 is initially charged to a voltage V_0 = 50 V and capacitor 2 is initially uncharged. Both of the switches S are then closed at time t = 0.

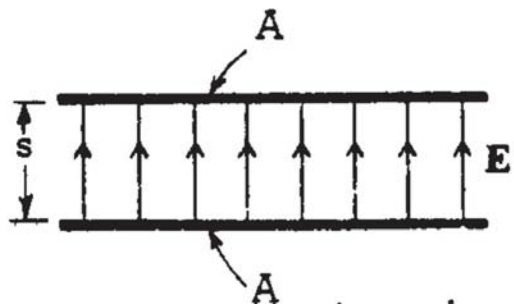
a. What are the final charges on the positive plate of each of the capacitors 1 and 2 after equilibrium has been reached?

b. Determine the difference between the initial and the final stored energy of the system after equilibrium has been reached.



2008E2 (modified) In the circuit shown above, A and B are terminals to which different circuit components can be connected.

a. Calculate the potential difference across R_2 immediately after the switch S is closed in each of the following cases.
   i. A 50 Ω resistor connects A and B.
   ii. An initially uncharged 0.80 μF capacitor connects A and B.

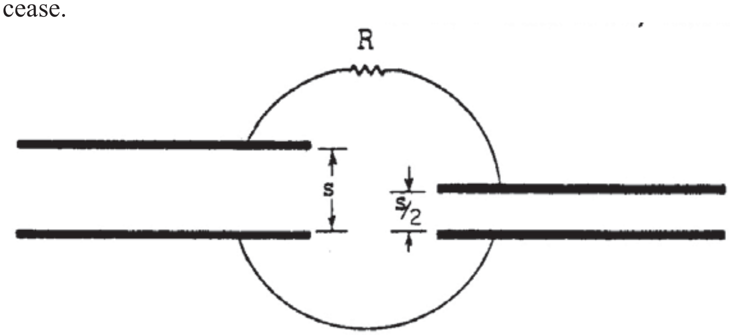


1978B3. A uniform electric field E is established between two capacitor plates, each of area A, which are separated by a distance s as shown above.

a. What is the electric potential difference V between the plates?
b. Specify the sign of the charge on each plate.

The capacitor above is then connected electrically through a resistor to a second parallel-plate capacitor, initially uncharged, whose plates have the same area A but a separation of only s/2.

c. Indicate on the diagram below the direction of the current in each wire, and explain why the current will eventually cease.



d. After the current has ceased, which capacitor has the greater charge? Explain your reasoning.
e. The total energy stored in the two capacitors after the current has ceased is less than the initial stored energy. Explain qualitatively what has become of this "lost" energy.

ANSWERS - AP Physics Multiple Choice Practice – Circuits

	Solution	Answer
1.	The resistances are as follows: I: 2 Ω, II: 4 Ω, III: 1 Ω, IV: 2 Ω	A,D
2.	The total resistance of the 3 Ω and 6 Ω in parallel is 2 Ω making the total circuit resistance 6 Ω and the total current $\mathcal{E}$/R = 1 A. This 1 A will divide in the ratio of 2:1 through the 3 Ω and 6 Ω respectively so the 3 Ω resistor receives 2/3 A making the potential difference IR = (2/3 A)(3 Ω) = 2 V.	A
3.	Adding resistors in parallel decreases the total circuit resistance, this increasing the total current in the circuit.	A
4.	R = ρL/A. Greatest resistance is the longest, narrowest resistor.	B
5.	P = I$\mathcal{E}$	B
6.	$W = Pt = I^2Rt$	B
7.	The resistance of the two 2 Ω resistors in parallel is 1 Ω. Added to the 2 Ω resistor in series with the pair gives 3 Ω	A
8.	R = ρL/A. Least resistance is the widest, shortest resistor	C
9.	The resistance of the two resistors in parallel is r/2. The total circuit resistance is then 10 Ω + ½ r, which is equivalent to $\mathcal{E}$/I = (10 V)/(0.5 A) = 20 Ω = 10 Ω + r/2	D
10.	The loop rule involves the potential and energy supplied by the battery and it's use around a circuit loop.	B
11.	Total circuit resistance (including internal resistance) = 40 Ω; total current = 0.3 A. $\mathcal{E}$ = IR	D
12.	$P = I^2r$	A
13.	With more current drawn from the battery for the parallel connection, more power is dissipated in this connection. While the resistors in series share the voltage of the battery, the resistors in parallel have the full potential difference of the battery across them.	B
14.	Resistance of the 1 Ω and 3 Ω in series = 4 Ω. This, in parallel with the 2 Ω resistor gives (2 × 4) /(2 + 4) = 8/6 Ω. Also notice the equivalent resistance must be less than 2 Ω (the 2 Ω resistor is in parallel and the total resistance in parallel is smaller than the smallest resistor) and there is only one choice smaller than 2 Ω.	A
15.	The upper branch, with twice the resistance of the lower branch, will have ½ the current of the lower branch.	D
16.	The larger loop, with twice the radius, has twice the circumference (length) and R = ρL/A	C
17.	R = ρL/A. If L ÷ 2, R ÷ 2 and is r ÷ 2 then A ÷ 4 and R × 4 making the net effect R ÷ 2 × 4	B
18.	The motor uses P = IV = 60 W of power but only delivers P = Fv = mgv = 45 W of power. The efficiency is "what you get" ÷ "what you are paying for" = 45/60	D
19.	Resistance of the 2000 Ω and 6000 Ω in parallel = 1500 Ω, adding the 2500 Ω in series gives a total circuit resistance of 4000 Ω. $I_{total} = I_1 = \mathcal{E}/R_{total}$	C
20.	I_1 is the main branch current and is the largest. It will split into I_2 and $I_3$and since I_2 moves through the smaller resistor, it will be larger than I_3.	A
21.	$P = V^2/R$	D

22. Current is greatest where resistance is least. The resistances are, in order, 1 Ω, 2 Ω, 4 Ω, 2 Ω and 6 Ω. A

23. See above D

24. Least power is for the greatest resistance ($P = \mathcal{E}^2/R$) D

25. $P = I^2R$ and $R = \rho L/A$ giving $P \propto \rho L/d^2$ C

26. $P = I^2R$ D

27. Total resistance = $\mathcal{E}/I$ = 25 Ω. Resistance of the 30 Ω and 60 Ω resistors in parallel = 20 Ω adding the internal resistance in series with the external circuit gives R_{total} = 20 Ω + r = 25 Ω C

28. $P = V^2/R$ and if V is constant $P \propto 1/R$ A

29. For the ammeter to read zero means the junctions at the ends of the ammeter have the same potential. For this to be true, the potential drops across the 1 Ω and the 2 Ω resistor must be equal, which means the current through the 1 Ω resistor must be twice that of the 2 Ω resistor. This means the resistance of the upper branch (1 Ω and 3 Ω) must be ½ that of the lower branch (2 Ω and R) giving 1 Ω + 3 Ω = ½ (2 Ω + R) D

30. To dissipate 24 W means $R = V^2/P$ = 6 Ω. The resistances, in order, are: 8 Ω, 4/3 Ω, 8/3 Ω, 12 Ω and 6 Ω D

31. Dimensional analysis: 1.6×10^{-3} A = 1.6×10^{-3} C/s ÷ 1.6×10^{-19} C/proton = 10^{16} protons/sec ÷ 10^9 protons/meter = 10^7 m/s D

32. Closing the switch short circuits Bulb 2 causing no current to flow to it. Since the bulbs were originally in series, this decreases the total resistance and increases the total current, making bulb 1 brighter. B

33. $P = V^2/R$ C

34. Closing the switch reduces the resistance in the right side from 20 Ω to 15 Ω, making the total circuit resistance decrease from 35 Ω to 30 Ω, a slight decrease, causing a slight increase in current. For the current to double, the total resistance must be cut in half. B

35. $R = \rho L/A \propto L/d^2$ where d is the diameter. $R_x/R_y = L_x/d_x^2 \div L_y/d_y^2 = (2L_y)d_y^2/[L_y(2d_y)^2] = ½$ A

36. The equivalent resistance of the 20 Ω and the 60 Ω in parallel is 15 Ω, added to the 35 Ω resistor in series gives 15 Ω + 35 Ω = 50 Ω C

37. N is in the main branch, with the most current. The current then divides into the two branches, with K receiving twice the current as L and M. The L/M branch has twice the resistance of the K branch. L and M in series have the same current. C

38. See above. Current is related to brightness ($P = I^2R$) C

39. If K burns out, the circuit becomes a series circuit with the three resistors, N, M and L all in series, reducing the current through bulb N. D

40. If M burns out, the circuit becomes a series circuit with the two resistors, N and K in series, with bulb L going out as well since it is in series with bulb M. D

41. The equivalent resistance in parallel is smaller than the smallest resistance. A

42. In series, they all have the same current, 2 A. $P_3 = I_3V_3$ B

43. $P = \mathcal{E}^2/R$. Total resistance of n resistors in series is nR making the power $P = \mathcal{E}^2/nR = P/n$ D

44.	The current through bulb 3 is twice the current through 1 and 2 since the branch with bulb 3 is half the resistance of the upper branch. The potential difference is the same across each branch, but bulbs 1 and 2 must divide the potential difference between them.	A,B
45.	$R = \rho L/A \propto L/d^2$ where d is the diameter. $R_{II}/R_I = L_{II}/d_{II}^2 \div L_I/d_I^2 = (2L_I)d_I^2/[L_I(2d_I)^2] = ½$	C
46.	For the currents in the branches to be equal, each branch must have the same resistance.	C
47.	$R \propto L/A = L/d^2$. If $d \times 2$, $R \div 4$ and if $L \div 2$, $R \div 2$ making the net effect $R \div 8$	A
48.	Bulbs in the main branch have the most current through them and are the brightest.	D
49.	In parallel, all the resistors have the same voltage (2 V). $P_3 = I_3V_3$	C
50.	Resistor D is in a branch by itself while resistors A, B and C are in series, drawing less current than resistor D.	D
51.	Each computer draws I = P/V = 4.17 A. 4 computers will draw 16.7 A, while 5 will draw over 20 A.	C
52.	Resistance of bulbs B & C = 20 Ω combined with D in parallel gives 6.7 Ω for the right side. Combined with A & E in series gives a total resistance of 26.7 Ω. $\mathcal{E} = IR$	B
53.	A and E failing in the main branch would cause the entire circuit to fail. B and C would affect each other.	A
54.	$V = IR$	A
55.	$\mathcal{E} = IR_{total}$ where $R_{total} = 35\ \Omega$	C
56.	With the switch closed, the resistance of the 15 Ω and the 30 Ω in parallel is 10 Ω, making the total circuit resistance 30 Ω and $\mathcal{E} = IR$	C
57.	$P = I^2R$	B
58.	The equivalent resistance through path ACD is equal to the equivalent resistance through path ABD, making the current through the two branches equal	D
59.	The resistance in each of the two paths is 9 Ω, making the current in each branch 1 A. From point A, the potential drop across the 7 Ω resistor is then 7 V and across the 4 Ω resistor is 4 V, making point B 3 V lower than point C	C
60.	Since the volume of material drawn into a new shape in unchanged, when the length is doubled, the area is halved. $R = \rho L/A$	D
61.	Closing the switch reduces the total resistance of the circuit, increasing the current in the main branch containing bulb 1	A
62.	*Resistivity* is dependent on the material. Not to be confused with *resistance*	C
63.	Resistors J and N are in the main branch and therefore receive the largest current.	D
64.	$P = I^2R$	D
65.	With a total resistance of 10 Ω, the total current is 1.2 A. The terminal voltage $V_T = \mathcal{E} - Ir$	B
66.	Most rapid heating requires the largest power dissipation. This occurs with the resistors in parallel.	D
67.	Shorting bulb 3 decreases the resistance in the right branch, increasing the current through bulb 4 and decreasing the total circuit resistance. This increases the total current in the main branch containing bulb 1.	C

68. For more light at a given voltage, more current is required, which requires less resistance. $R = \rho L/A$ B

69. Bulb C in the main branch receiving the total current will be the brightest C

70. Wire CD shorts out bulb #3 so it will never light. Closing the switch merely adds bulb #2 in parallel to bulb #1, which does not change the potential difference across bulb #1. C

71. Shorting bulb 4 decreases the resistance in the right branch, increasing the current through bulb 3 and in the main branch containing bulb 1. C

72. If A were to burn out, the total resistance of the parallel part of the circuit increases, causing less current from the battery and less current through bulb A. However, A and B split the voltage from the battery in a loop and with less current through bulb A, A will have a smaller share of voltage, increasing the potential difference (and the current) through bulb B. B

73. Since there is constant current, bulb 1 remains unchanged and bulbs 2 and three must now split the current. With half the current through bulb 2, the potential difference between A and B is also halved. D

74. The voltmeter is essentially another resistor. The voltmeter in parallel with the 100 Ω resistor acts as a 500 Ω resistor, which will half ½ the voltage of the 100 Ω resistor on the left. Thus the 120 V will split into 80 V for the 1000 Ω resistor and 40 V for the voltmeter combination. C

75. $P = I^2R$ and the current is the same through each resistor. A

76. The 15 Ω resistor would be in parallel with the 30 Ω resistor when the switch is closed. C

77. ACD = 9 Ω, ABD = 9 Ω so the total resistance is 4.5 Ω making the total current $\mathcal{E}/R = 2$ A. A

78. The 2 A will divide equally between the two branches with 1 A going through each branch. From B to D we have – (1 A)(2 Ω) = –2 V, with B at the higher potential A

79. For no current to flow, the potential drop across R_1 must equal the potential drop across R_2. For this to occur $I_1R_1 = I_2R_2$. Since the two branches also have the same potential difference as a whole (they are in parallel) we also have $I_1(R_1 + R_3) = I_2(R_2 + R_4)$. Solve for R_3 D

80. The resistances are, respectively, 4/3 R, 2/5 R, R, and 5/3 R A

81. Closing the switch adds another parallel branch, increasing the total current delivered by the battery. Bulb 3 will get brighter. Bulb 2, in its own loop with bulb 3 and the battery will then lose some of its share of the potential difference from the battery and will get dimmer. C

82. Using ratios, the currents in the 6 Ω and 3 Ω resistors are 1 A and 2 A. They have three times and 3/2 times the resistance of the 2 Ω resistor so they will have 1/3 and 2/3 the current. The total current is then 6 A giving a potential drop of 36 V across the 6 Ω resistor in the main branch and adding any one of the branches below with the loop rule gives 36 V + 6 V = 42 V for the battery B

83. Voltmeters must be placed in parallel and ammeters must be placed in series. B

84. Even though B_2 burns out, the circuit is still operating elsewhere as there are still closed paths. B

85. With B_2 burning out, the total resistance of the circuit increases as it is now a series circuit. This decreases the current in the main branch, decreasing V_1. For V_1 to be halved, the current must be halved which means the total resistance must be doubled, which by inspection did not happen in this case (total before = 5/3 R, total after = 3 R) C

86 S_1 must be closed to have any current. Closing S_2 will allow current in R_2 but closing R_3 would short circuit R_2. C

87. S_1 must be closed to have any current. Closing S_3 will short circuit R_3, leaving only resistor R_1, which is the lowest possible resistance. D

88. S_1 must be closed to have any current. The greatest voltage will occur with the greatest current through R_3 but closing S_2 or S_3 will draw current away from R_3. A

89. $R = \rho L/A$ D

90. Starting at A and summing potential differences *counterclockwise* to point C gives 12 V A

91. The branch with two 2 Ω resistors has a total resistance of 4 Ω and a potential difference of 12 V. $V = IR$ C

92. Before cutting the resistance is R. After cutting we have two wires of resistance ½ R which in parallel is an equivalent resistance of ¼ R. $P = V^2/R$ and $I = V/R$ D

93. $P = V^2/R$ and $R = \rho L/A$ giving $P = V^2A/\rho L$ B

AP Physics Free Response Practice – Circuits – ANSWERS

WARNING ONLY CIRCUITS WITH RESISTORS ARE ON AP PHYSICS 1

1976B3

a. $V_T = E - Ir = 6$ V

b. In parallel, each resistor gets 6 V and $P = V^2/R$ gives R = 3 Ω

c. For the 3 Ω resistor we have I = V/R = 2 A leaving 1 A for the branch with R_1. R = V/I = 6 Ω

1981B4

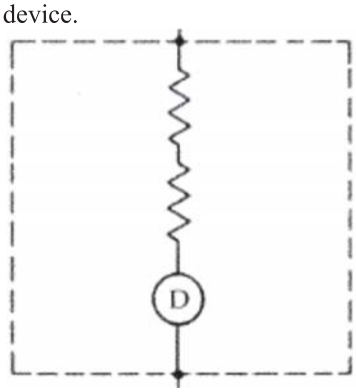
a. The two batteries are connected with opposing emfs so the total emf in the circuit is $\mathcal{E}$ = 60 V – 12 V = 48 V
The resistance of the parallel combination of resistors is $(\frac{1}{4} + \frac{1}{4} + \frac{1}{2})^{-1}$ = 1 Ω combining with the rest of the resistors in series gives a total circuit resistance of 8 Ω.
The total current is then $\mathcal{E}/R$ = 6 A. The voltage across the parallel combination of resistors is $V_P = IR_P$ = 6 V so the current through the 2 Ω resistor is I = V/R = 3 A

b. $P = I^2R$ = 108 W

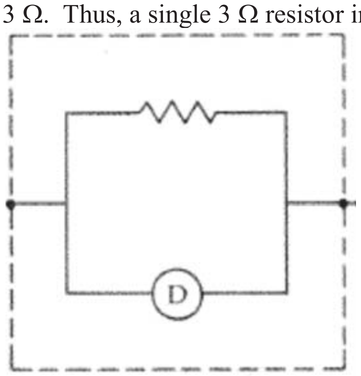
c. The current is forced through battery Bfrom the positive to the negative terminal, charging the battery. This makes the equation for the terminal voltage $V_T = \mathcal{E} + Ir$ = 18 V

1980B2

a. The resistance of the device is found from R = V/I = 6 Ω. With a 24 volt source, to provide a current of 2 A requires a total resistance of 12 Ω. For the additional 6 Ω resistance, place two 3 Ω resistors in series with the device.



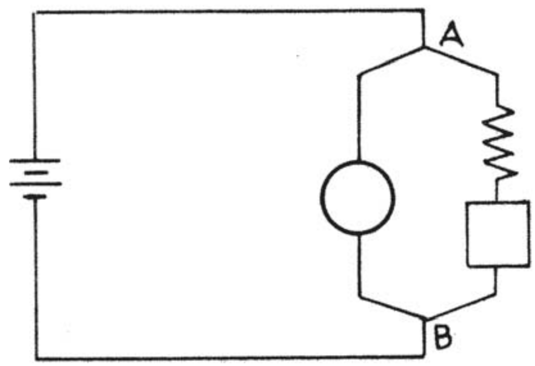
b. Since the device requires 2 A, a resistor in parallel with the device must carry a current of 6 A – 2 A = 4 A. In parallel with the device, the resistor will have a potential difference of 12 V so must have a resistance of V/I = 3 Ω. Thus, a single 3 Ω resistor in parallel will suffice.



c. $P = I^2R$ = 48 W

1982B4

a. Since the clock requires 15 V it must be directly connected between A and B. Since the radio requires less than 15 V, there must be a resistor in series with it.



b. The current through the radio (and R) is 10 mA. The voltage across the radio is 9 V, which leaves 6 V across the resistor giving R = V/I = 600 Ω

c. P = IV where V = 15 V and I = 10 mA + 20 mA = 30 mA so P = 0.45 W and energy = Pt = 27 J

1983B3

a. The two batteries are connected with opposing emfs so the total emf in the circuit is $\mathcal{E}$ = 20 V – 2 V = 18 V
The equivalent resistance of the two parallel resistors is (6 × 12)/(6 + 12) = 4 Ω and since R is in series with the pair, the total circuit resistance is (4 + R) Ω = $\mathcal{E}$/I = 9 Ω giving R = 5 Ω

b. Because the voltages of the two resistors in parallel are equal we have $6I_1 = 12I_2$ and $I_1 + I_2$ = 2 A giving
   i. 4/3 A
   ii. 2/3 A

c. Summing the potential differences form point X gives V_X + IR = 0 + (2 A)(5 Ω) = V_B = 10 V. Continuing along gives V_B – 20 V = V_C = –10 V. And V_C + (2/3 A)(12 Ω) = V_D = –2 V

d. P = $\mathcal{E}$I = 40 W

1986B3

a. P = V^2R gives R = 240 Ω

b. Bulbs Y and Z in parallel have an equivalent resistance of 120 Ω. Adding bulb X in series with the pair gives R = 360 Ω

c. $P_T = \mathcal{E}^2/R_T$ = 40 W

d. I = $\mathcal{E}$/R = 1/3 A

e. $V_X = IR_X$ = 80 V

f. The current splits equally through Y and Z. $V_Z = I_ZR_Z$ = (1/6 A)(240 Ω) = 40 V

1987B4

a. The equivalent resistance of R_1 and R_2 is (12 × 4)/(12 + 4) = 3 Ω. Adding R_3 in series with the pair gives R = 12 Ω

b. $\mathcal{E} = IR_T$ = 4.8 V

c. The voltage across resistor 1 (equal to the voltage across R_2) is the emf of the battery minus the drop across R_3 which is 4.8 V – (0.4 A)(9 Ω) = 1.2 V

d. P = V^2/R = 0.36 W

e. Q = It = (0.4 C/s)(60 s) = 24 C

1988B3

a. On the right we have two resistors in series: 10 Ω + 2 Ω = 12 Ω. This is in parallel with the 4 Ω resistor which is an equivalent resistance of 3 Ω and adding the remaining main branch resistor in series gives a total circuit resistance of 9 Ω. The current is then $I = \mathcal{E}/R_T = 8$ A

b. The voltage remaining for the parallel branches on the right is the emf of the battery minus the potential dropped across the 6 Ω resistor which is 72 V – (8 A)(6 Ω) = 24 V. Thus the current in the 10 Ω resistor is the current through the whole 12 Ω branch which is I = V/R = (24 V)/(12 Ω) = 2 A

c. $V_{10} = I_{10}R_{10} = 20$ V

d. When charged, the capacitor is in parallel with the 10 Ω resistor so $V_C = V_{10} = 20$ V and Q = CV = 60 μC

e. $U_C = \frac{1}{2} CV^2 = 6 \times 10^{-4}$ J

1989B3

a. i. $P = I^2R = (2\text{ A})^2(10\ \Omega) = 40$ W
   ii. P = Fv = mgv = 20 W (using $g = 10\text{ m/s}^2$)
   iii. $P_B = P_R + P_M = 40\text{ W} + 20\text{ W} = 60$ W

b. i. V = IR = 20 V
   ii. V = P/I = (20 W)/(2 A) = 10 V
   iii. $\mathcal{E} = V_R + V_M = 30$ V

c. Since the speed is increased by 3/2, the voltage drop increases by the same value and is now (3/2)(10 V) = 15 V

d. The new voltage across the resistor is found from $V_R = \mathcal{E} - V_M = 15$ V and $I = V_R/I = (15\text{ V})/(2\text{ A}) = 7.5\ \Omega$

1990B3

a. The 4 Ω and 8 Ω are in series so their equivalent resistance is 12 Ω. Another 12 Ω resistor in parallel makes the equivalent resistance (12 × 12)/(12 + 12) = 6 Ω

b. Adding the remaining resistors in series throughout the circuit gives a total circuit resistance of 12 Ω and the total current (which is also the current in the 5 Ω resistor) = $\mathcal{E}/R = 2$ A

c. $V_{AC} = \mathcal{E} - Ir = 22$ V

d. The current divides equally between the two branches on the right so $P_{12} = I^2R = (1\text{ A})^2(12\ \Omega) = 12$ W

e. From B to C you only have to pass through the 12 Ω resistor which gives V = (1 A)(12 Ω) = 12 V

f. $P_B = V_{AC}^2/R_{external} = (22\text{ V})^2/11\ \Omega = 44$ W

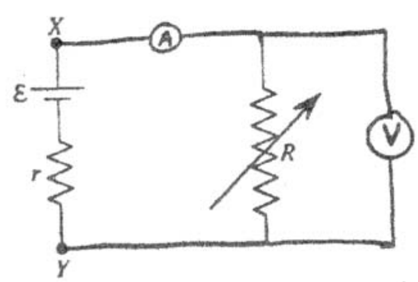
1991B4

a/b. $V_{XY} = \mathcal{E} - Ir$ and using data from the graph we can find two equations to solve simultaneously
$4\text{ V} = \mathcal{E} - (1\text{ A})r$ and $3\text{ V} = \mathcal{E} - (3\text{ A})r$ will yield the solutions $\mathcal{E} = 4.5$ V and r = 0.5 Ω

c. $V_{XY} = IR$ which gives 3 V = (3 A)R and R = 1 Ω

d. I_{max} occurs for R = 0 and $V_{XY} = 0$ which gives $\mathcal{E} = I_{max}r$ and $I_{max} = 9$ A (this is the x intercept of the graph)

e.

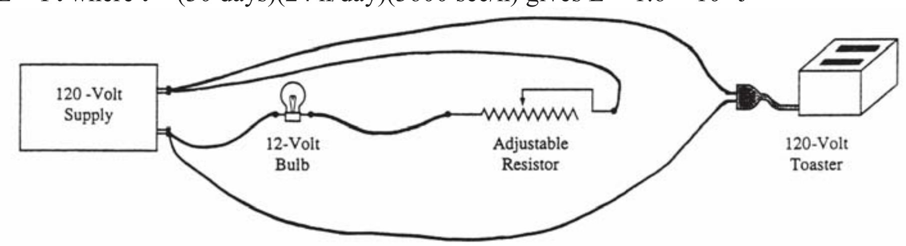


1995B2

a. $P = V^2/R$ gives $R = 24\ \Omega$

b. $E = Pt$ where t = (30 days)(24 h/day)(3600 sec/h) gives $E = 1.6 \times 10^7$ J

c.



The bulb, needing only 12 V must have a resistor in series with it and the toaster, requiring 120 V must be connected directly to the power supply.

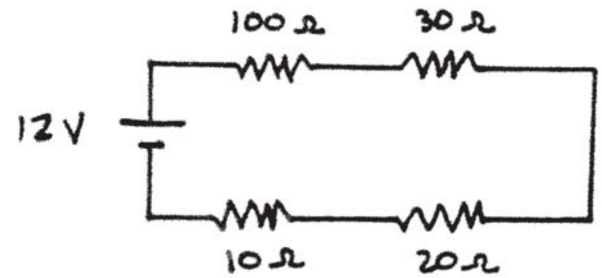
d. The current through the bulb is $I = P/V = 0.5$ A, which is also the current in the resistor, which must have 108 V across it to provide the light bulb only 12 V. $R = V/I = (108\text{ V})/(0.5\text{ A}) = 216\ \Omega$

e. i. If the resistance of the resistor is increased, the current through the branch will decrease, decreasing the brightness of the bulb.

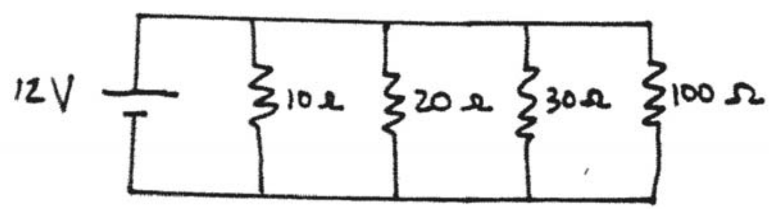
ii. Since the toaster operates in its own parallel branch, nothing will change for the toaster.

1996B4

a. For the smallest current, place the resistors in series



b. For the largest current, place the resistors in parallel



c. i. The 20 Ω and 30 Ω resistors combine in series as a 50 Ω resistor, which is in parallel with the 100 Ω resistor making their effective resistance 33.3 Ω. Adding the 10 Ω resistor in the main branch in series gives a total circuit resistance of 43 Ω. The current in the 10 Ω resistor is the total current delivered by the battery $\mathcal{E}/R = 0.28$ A

ii. $P = \mathcal{E}^2/R = 3.35$ W

d. $E = Pt$, or $t = E/P = (10 \times 10^3\text{ J})/(3.35\text{ W}) = 3 \times 10^3$ seconds

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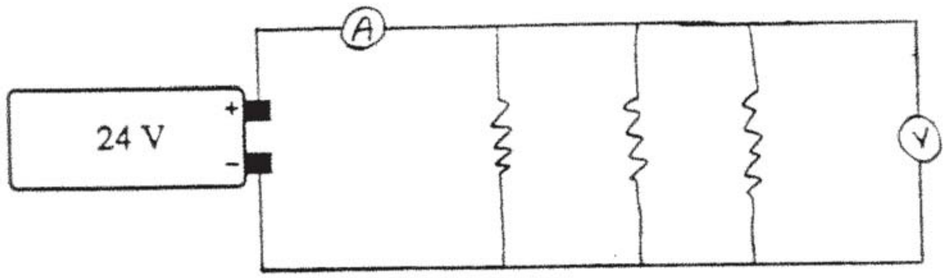
1997B4

a. i. In series $R_T = 90\ \Omega$ and $P = V^2/R = 6.4$ W

ii. In parallel $R_T = 10\ \Omega$ and $P = 57.6$ W

b. The fastest heating occurs with a parallel connection



2002B3B

a. The resistance of the 6 Ω and 3 Ω resistors in parallel is (6 × 3)/(6 + 3) = 2 Ω. Adding the 3 Ω resistor in the main branch gives a total circuit resistance of 5 Ω. The current in bulb A in the main branch is the total current delivered by the battery $I = \mathcal{E}/R = (9\text{ V})/(5\ \Omega) = 1.8$ A

b. Bulb A is the brightest. In the main branch, it receives the most current. You can also calculate the power of each resistor where $P_A = 9.7$ W, $P_B = 2.2$ W and $P_C = 4.3$ W

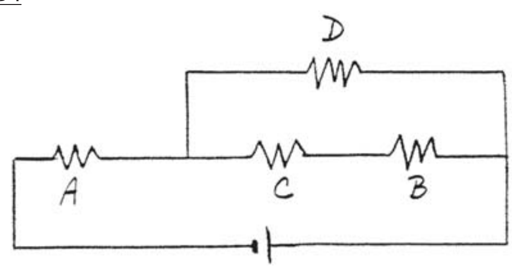
c. i. Removing Bulb C from the circuit changes the circuit to a series circuit, increasing the total resistance and decreasing the total current. With the total current decreased, bulb A is dimmer.

ii. Since bulb A receives less current, the potential drop is less than the original value and being in a loop with bulb B causes the voltage of bulb B to increase, making bulb B brighter. The current through bulb B is greater since it is no longer sharing current with bulb C.

iii. The current through bulb C is zero, bulb C goes out.

1998B4

a.



b. A > D > B = C

Bulb *A* has the largest current through it, making it brightest. The voltage across bulb *D* is the same as that across bulbs Band C combined, so it is next brightest, leaving *B* and C as least bright Bulbs *B* and C are in series, and thus have the same current through them, so they must be equally bright.

c. i. The brightness of bulb A decreases. The total resistance of the circuit increases so the current in bulb A decreases.

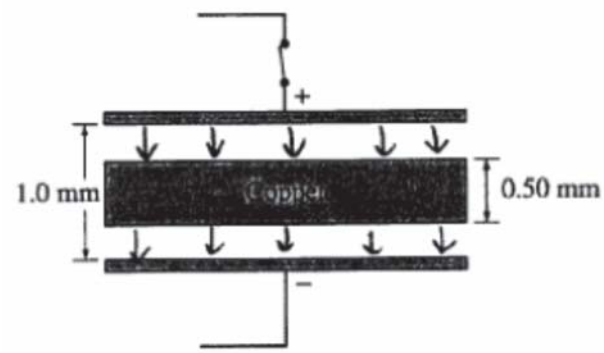
ii. The brightness of bulb B increases. The current (and the voltage) across B increases. Even though the total current decreases, it is no longer splitting to do through the branch with bulb D. Another way to look at it is since A has less current, the potential difference across A is decreased, this allows a larger share of the battery voltage to be across B and C.

2000B3

a. The equivalent resistance of the two resistors in parallel is R/2, which if ½ the resistance of the resistor in the main branch, so the parallel combination will receive half the potential difference of the main branch resistor. The 30 V of the battery will then divide into 20 V for the main branch resistor (and across the voltmeter) and 10 V each for the resistors in parallel.

b. After the switch has been closed for a long time, the voltage across the capacitor will be 30 V.
$Q = CV = 3 \times 10^8$ C

c. i. The 30 V battery is still connected across the capacitor so the potential difference remains 30 V.
ii. E = 0 inside a conductor in electrostatic equilibrium
iii.



iv. E = V/d and you can use the entire gap or just one of the two gaps; E = 30 V/(0.5 mm) or 15 V/(0.25 mm)
E = 60 V/mm or 60,000 V/m

2002B3

a. i. $P = V^2/R$ gives R = 480 Ω and V = IR gives I = 0.25 A
ii. $P = V^2/R$ gives R = 360 Ω and V = IR gives I = 0.33 A

b. i./ii. The resistances are unchanged = 480 Ω and 360 Ω. The total resistance in series is 480 Ω + 360 Ω = 840 Ω making the total current I = V/R = 0.14 A which is the same value for both resistors in series

c. The bulbs are brightest in parallel, where they provide their labeled values of 40 W and 30 W. In series, it is the larger resistor (the 30 W bulb) that glows brighter with a larger potential difference across it in series.
This gives the order from top to bottom as **2 1 3 4**

d. i. In parallel, they each operate at their rated voltage so they each provide their rated power and P_T = 30 W + 40 W = 70 W
ii. In series $P_T = V_T^2/R_T$ = 17 W

2003B2

a. For two capacitors in series the equivalent capacitance is (6 × 12)/(6 + 12) = 4 μF

b. The capacitors are fully charged so current flows through the resistors but not the capacitors. R_T = 30 Ω and I = V/R = 0.2 A

c. The potential difference between A and B is the voltage across the 20 Ω resistor. V = IR = 4 V

d. The capacitors in series store the same charge as a single 4 μF capacitor. Q = CV = (4 μF)(4 V) = 16 μC

e. Remains the same. No current is flowing from A to P to B therefore braking the circuit at point P does not affect the current in the outer loop, and therefore will not affect the potential difference between A and B.

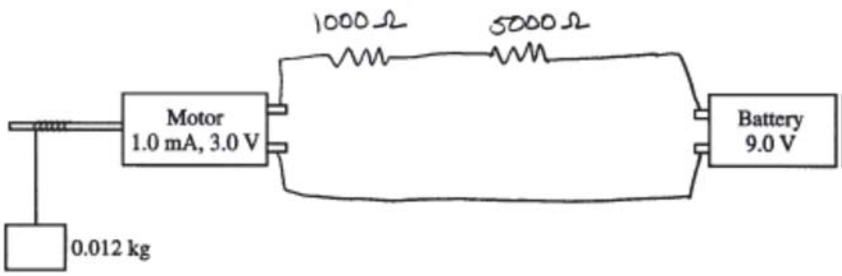
2003B2B

a. $P = IV = 3$ mW $= 3 \times 10^{-3}$ W

b. $E = Pt = 0.180$ J

c. e = "what you get"/"what you are paying for" = (power lifting the mass) ÷ (power provided by the motor)
$P_{lifting} = Fv = mgv = mgd/t = 1.96$ mW so the efficiency is 1.96/3 = 0.653 or 65.3 %

d. To reduce the battery voltage of 9 V to the motor's required voltage of 3 V, we need 6 V across the resistors. The required resistance is then V/I = (6 V)/(1 mA) = 6000 Ω. This is done with a 1000 Ω and a 5000 Ω resistor in series.



2007B3

a. i. __1__ I_A __3__ I_B __2__ I_C

ii. The total current flows through R_A and gets divided between the other two resistors with the smaller resistor R_C getting a larger current

b. i. __1__ V_A __2__ V_B __2__ V_C

ii. No resistor is greater than R_A and R_A has the full current through it. R_B and R_C are in parallel and therefore have the same potential difference.

c. For the two resistors in parallel, the equivalent resistance is $(2R \times R)/(2R + R) = 2/3\ R = 133\ \Omega$. Adding R_A in series with the pair gives $R_T = 400\ \Omega + 133\ \Omega = 533\ \Omega$

d. $I_T = I_A = \mathcal{E}/R_T = 0.0225$ A. The potential drop across A is V = IR = 9 V which leaves 3 V for the two branches in parallel. $I_C = V_C/R_C = 0.015$ A

e. In the new circuit, $I_B = 0$ at equilibrium and the circuit behaves as a simple series circuit with a total resistance of 600 Ω and a total current of $\mathcal{E}/R = 0.02$ A. The voltage across the capacitor is the same as the voltage across resistor C and $V_C = IR_C = 4$ V and $Q = CV = 8 \times 10^{-6}$ C

1975E2

a. $Q = C\mathcal{E} = 12\ \mu F \times 100\ V = 1200\ \mu C$

b. Connecting the two capacitors puts them in parallel with the same voltage so $V_1 = V_2$ and V = Q/C which gives $Q_1/C_1 = Q_2/C_2$ or $Q_1/12 = Q_2/24$ and $Q_2 = 2Q_1$. We also know the total charge is conserved so $Q_1 + Q_2 = 1200$ μC so we have $Q_1 + 2Q_1 = 1200$ μC so $Q_1 = 400$ μC

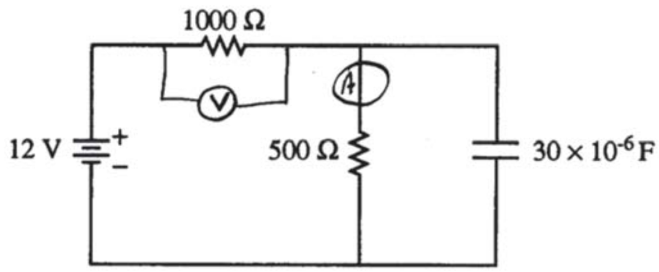
c. V = Q/C = 33.3 V

d. When the battery is reconnected, both capacitors charge to a potential difference of 100 V each. The total charge is then $Q = Q_1 + Q_2 = (C_1 + C_2)V = 3600$ μC making the *additional* charge from the battery 2400 μC.

2007B3B

a. In their steady states, no current flows through the capacitor so the total resistance is 1500 Ω and the total current is $\mathcal{E}/R_T = 8.0 \times 10^{-3}$ A

b.



c. The voltage across the capacitor is the same as the voltage across the 500 Ω resistor = IR = 4 V so we have Q = CV = 1.2×10^{-4} C

d. $P = I^2R = 6.4 \times 10^{-2}$ W

e. Larger. Replacing the 50 Ω resistor with a larger resistor lowers the steady state current, causing the voltage across the 1000 Ω resistor to decrease and the voltage across the replacement resistor to increase.

1988E2

a. In their steady states, no current flows through the capacitor so the effective resistance of the branch on the right is 8 Ω + 4 Ω = 12 Ω. This is in parallel with the 4 Ω resistor making their effective resistance (12 × 4)/(12 + 4) = 3 Ω. Adding the 9 Ω resistor in the main branch gives a total circuit resistance of 12 Ω and a total current of $\mathcal{E}/R$ = 10 A. This is the current in the 9 Ω resistor as it is in the main branch.

b. With 10 A across the 9 Ω resistor, the potential drop across it is 90 V, leaving 30 V across the two parallel branches on the right. With 30 V across the 12 Ω effective resistance in the right branch, we have a current through that branch (including the 8 Ω resistor) of V/R = 2.5 A

c. $V_C = V_4 = IR$ = (2.5 A)(4 Ω) = 10 V

d. $U_C = \frac{1}{2} CV^2$ = 1500 μJ

1985E2

a. Immediately after the switch is closed, the capacitor begins charging with current flowing to the capacitor as if it was just a wire. This short circuits R_2 making the total effective resistance of the circuit 5×10^6 Ω and the total current $\mathcal{E}/R_{eff}$ = 0.006 A

b. When the capacitor is fully charged, no current flows through that branch and the circuit behaves as a simple series circuit with a total resistance of 15×10^6 Ω and a total current of $\mathcal{E}/R$ = 0.002 A

c. The voltage across the capacitor is equal to the voltage across the 10 MΩ resistor as they are in parallel. $V_C = V_{10M} = IR$ = 2000 V and Q = CV = 0.01 C

d. $U_C = \frac{1}{2} CV^2$ = 10 J

1986E2

a. The resistance of the two parallel branches are equal at 40 Ω each making the equivalent resistance of the two branches 20 Ω. Adding the 5 Ω resistance in the main branch gives a total circuit resistance of 25 Ω and a total current of $\mathcal{E}/R$ = 1 A which will split evenly between the two equal branches giving I_R = 0.5 A

b. After the capacitor is charged, no current flows from A to B, making the circuit operate as it did initially when the capacitor was not present. Therefore the current through R is the same as calculated above at 0.5 A

c. Consider the voltage at the junction above resistor R. The potential drop from this point to point A is V = IR = (0.5 A)(10 Ω) = 5 V and to point B is (0.5 A)(30 Ω) = 15 V making the potential difference across the plates of the capacitor 15 V – 5 V = 10 V. Q = CV = (10 μF)(10 V) = 100 μC

1989E3

a. When charged, the potential difference across the capacitor is 20 V. $U_C = ½ CV^2 = 1200$ μJ

b. Given that the charge is initially unchanged, the work done is the change in the energy stored in the capacitor. Increasing the distance between plates to 4 times the initial value causes the capacitance to decrease to ¼ its initial value ($C \propto 1/d$). Since $Q_i = Q_f$ we have $C_iV_i = C_fV_f$ so $V_f = 4V_i$
$W = \Delta U_C = ½ C_fV_f^2 – ½ C_iV_i^2 = ½ (¼ C(4V)^2) – ½ CV^2 = 3600$ μJ

c. After the spacing is increased, the capacitor acts as a battery with a voltage of 4V = 80 V with its emf opposite that of the 20 V battery making the effective voltage supplied to the circuit 80 V – 20 V = 60 V.
$I = \mathcal{E}_{eff}/R = 2 \times 10^{-4}$ A

d. The charge on the capacitor initially was Q = CV = 120 μC and after the plates have been separated and a new equilibrium is reach Q = (¼C)V = 30 μC so the charge that flowed back through the battery is 120 μC – 30 μC = 90 μC

e. For the battery $U = Q_{added}V = 1800$ μJ

1992E2

a. i. $Q = CV = 4 \times 10^{-3}$ C
   ii. $U_C = ½ CV^2 = 4$ J

b. When the switch is closed, there is no charge on the 6 μF capacitor so the potential difference across the resistor equals that across the 2 μF capacitor, or 2000 V and $I = V/R = 2 \times 10^{-3}$ A

c. In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference. $V_2 = V_6$ gives $Q_2/C_2 = Q_6/C_6$ giving $Q_6 = 3Q_2$ and since total charge is conserved we have $Q_2 + Q_6 = Q_2 + 3Q_2 = 4Q_2 = 4 \times 10^{-3}$ C so $Q_2 = 1 \times 10^{-3}$ C and $Q_6 = 3 \times 10^{-3}$ C

d. $U_C = U_2 + U_6 = Q_2^2/2C_2 + Q_6^2/2C_6 = 1$ J. This is less than in part a. ii. Part of the energy was converted to heat in the resistor.

1995E2

a. $C = \kappa\varepsilon_0A/d$ so $\kappa = Cd/\varepsilon_0A = 5.65$

b. i. When the switch is closed, the voltage across the capacitor is zero thus all the voltage appears across the resistor and $I = \mathcal{E}/R = 1.5 \times 10^{-5}$ A

c. When fully charged, the current has stopped flowing and all the voltage now appears across the capacitor and $Q = CV = 1.5 \times 10^{-6}$ C and since the bottom plate is connected to the negative terminal of the battery the charge on that plate is also negative.

d. $U_C = ½ CV^2 = 2.25 \times 10^{-5}$ J

e. Since the capacitor is isolated, the charge on it remains the same. Removing the plastic reduces the capacitance to $C' = \varepsilon_0A/d = C_{original}/\kappa$ and $V = Q/C' = 170$ V

f. $U' = Q^2/2C' = Q^2/2(C/\kappa) = \kappa(Q^2/2C) = \kappa U > U_{original}$. The increase came from the work that had to be done to remove the plastic from the capacitor.

1996E2

a. The initial charge on C_1 is $Q = CV_0 = 200$μC. In equilibrium, charge is no longer moving so there is no potential difference across the resistor therefore the capacitors have the same potential difference. $V_1 = V_2$ gives $Q_1/C_1 = Q_2/C_2$ giving $Q_2 = 3Q_1$ and since total charge is conserved we have $Q_1 + Q_2 = Q_1 + 3Q_1 = 4Q_1 =$ 200 μC so $Q_1 = 50$ μC and $Q_2 = 150$ μC

b. $\Delta U = U_f – U_i = (Q_1^2/2C_1 + Q_2^2/2C_2) – ½ C_1V_0^2 = –3750$ μJ

2008E2

a. With a 50 Ω resistor, the right branch has a total resistance of 150 Ω, making the parallel combination with the 300 Ω resistor equal to (150 × 300)/(150 + 300) = 100 Ω. Adding R_1 from the main branch in series with the branches gives a total circuit resistance of 300 Ω and a total current of $\mathcal{E}/R$ = 5 A. The potential difference across R_1 is then V = IR = 1000V, leaving 500 V across the two parallel branches and across R_2.

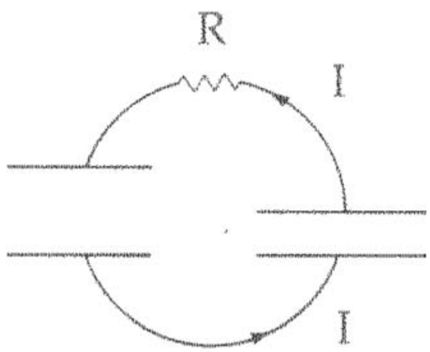
b. When the switch is closed with a capacitor between points A and B, the voltage across the capacitor is zero and the current flows through the branch as if the capacitor was a wire. This gives the effective resistance of the parallel resistors as (100 × 300)/(100 + 300) = 75 Ω and the total resistance = 275 Ω, the total current = $\mathcal{E}/R$ = 5.45 A, the voltage across R_1 = IR = 1090 V and V_2 = 1500 V – 1090 V = 410 V

1978B3

a. V = Ed = Es

b. Since the field points from the power plate to the upper plate, the lower plate is positive and the upper plate is negative.

c.



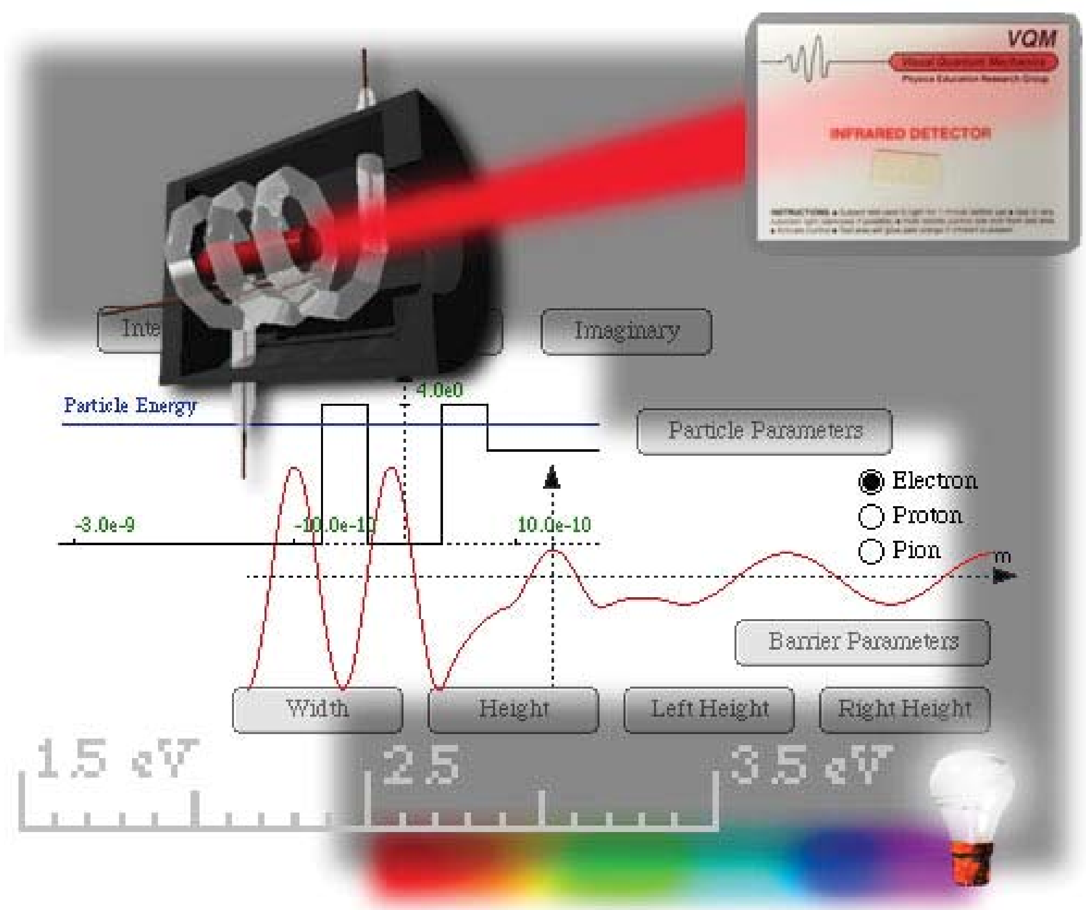
When the potential difference is the same on the two capacitors, charge will stop flowing as charge will flow only when there is a difference in potential.

d. The capacitor on the left has the smaller capacitance and since the two capacitors are in parallel, they have the same voltage. Q = CV so the larger capacitor (on the right) contains more charge.

e. The energy lost has been converted to heat through the resistor.

APPENDIX 1

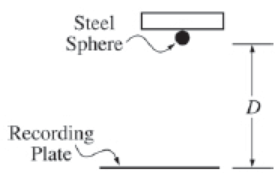
Experimental Questions



AP LAB-BASED QUESTIONS

MECHANICS

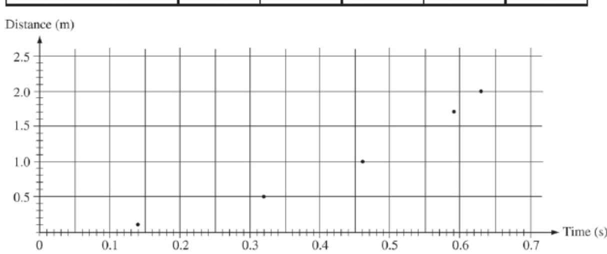
KINEMATICS (2006B2) A world-class runner can complete a 100 m dash in about 10 s. Past studies have shown that runners in such a race accelerate uniformly for a time t_u and then run at constant speed for the remainder of the race. A world-class runner is visiting your physics class. You are to develop a procedure that will allow you to determine the uniform acceleration a_u and an approximate value of t_u for the runner in a 100 m dash. By necessity your experiment will be done on a straight track and include your whole class of eleven students.
a. By checking the line next to each appropriate item in the list below, select the equipment, other than the runner and the track that your class will need to do the experiment.
_____ Stopwatches _____Tape measures _____Rulers _____Masking tape
_____Metersticks _____Starter's pistol _____String _____Chalk
b. Outline the procedure that you would use to determine a_u and t_u , including a labeled diagram of the experimental setup. Use symbols to identify carefully what measurements you would make and include in your procedure how you would use each piece of the equipment you checked in part (a).
c. Outline the process of data analysis, including how you will identify the portion of the race that has uniform acceleration, and how you would calculate the uniform acceleration.



KINEMATICS (FREE FALL) (2006bB1) A student wishing to determine experimentally the acceleration g due to gravity has an apparatus that holds a small steel sphere above a recording plate, as shown above. When the sphere is released, a timer automatically begins recording the time of fall. The timer automatically stops when the sphere strikes the recording plate.

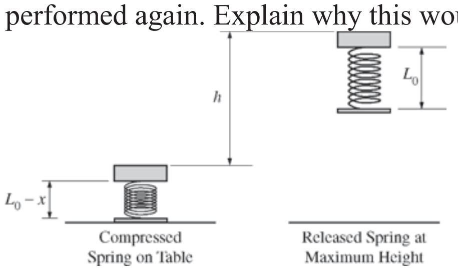
The student measures the time of fall for different values of the distance *D* shown above and records the data in the table below. These data points are also plotted on the graph.

Distance of Fall (m)	0.10	0.50	1.00	1.70	2.00
Time of Fall (s)	0.14	0.32	0.46	0.59	0.63



a. On the grid, sketch the smooth curve that best represents the student's data. The student can use these data for distance *D* and time *t* to produce a second graph from which the acceleration *g* due to gravity can be determined.

b. If only the variables *D* and *t* are used, what quantities should the student graph in order to produce a linear relationship between the two quantities?
c. Plot the data points for the quantities you have identified in part (b), and sketch the best straight-line fit to the points. Label your axes and show the scale that you have chosen for the graph.
d. Using the slope of your graph in part (c), calculate the acceleration *g* due to gravity in this experiment.
e. State one way in which the student could improve the accuracy of the results if the experiment were to be performed again. Explain why this would improve the accuracy.



ENERGY (2009 B1) In an experiment, students are to calculate the spring constant *k* of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance *x* from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height *h* above the point of maximum compression.

The students repeat the experiment several times, measuring *h* with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is *m*.

The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped.

a. Derive an expression for the height h in terms of m, x, k, and fundamental constants.

With the spring compressed a distance x = 0.020 m in each trial, the students obtained the following data for different values of m.

	m (kg)	h (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

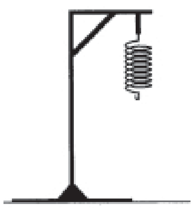
b. i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant k

ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.

c. Plot your data and draw a best-fit straight line. Label the axes and indicate the scale.

d. Using your best-fit line, calculate the numerical value of the spring constant. *(524 N/m)*

e. Describe a procedure for measuring the height h in the experiment, given that the toy is only momentarily at that maximum height.



SPRINGS (1996B2)

A spring that can be assumed to be ideal hangs from a stand, as shown above.

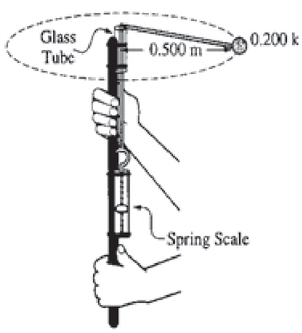
a. You wish to determine experimentally the spring constant **k** of the spring by two different methods.

i. What additional, commonly available equipment would you need for each method?

ii. What measurements would you make in each method?

iii. How would ***k*** be determined from these measurements?

b. What quantities were graphed to make a straight-line graph for each method?



Not Necessarily To Scale

CIRCULAR MOTION 1 (1997B2)

To study circular motion, two students use the hand-held device shown, which consists of a rod on which a spring scale is attached.

A polished glass tube attached at the top serves as a guide for a light cord attached the spring scale.

A rubber stopper is attached to the other end of the cord. One student swings the teal around at constant speed in a horizontal circle with a constant radius. Assume friction and air resistance al negligible.

a. Explain how the students, by using a stopwatch and the information given above, can determine the speed of the stopper as it is revolving.

b. The students find that, despite their best efforts, they cannot swing the stopper so that the cord remains exactly horizontal.

i. Draw and label vectors to represent the forces acting on the ball

ii. Explain what measurements you need to make to determine the angle that the cord makes with the horizontal.

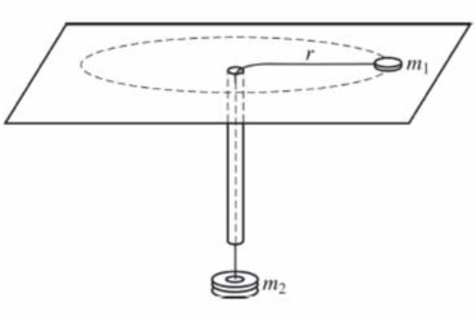
c. Perform the experiment by substituting the spring scale for five different masses.

d. Calculate the centripetal force, tangential velocity and centripetal acceleration for each case.

e. Neatly plot graphs of:

i. $\mathbf{F_c}$ vs $\mathbf{a_c}$ **ii.** $\mathbf{v^2}$ vs $\mathbf{a_c}$

f. Calculate the slopes for each graph. Clearly mark the points selected on the graph. Show all your calculations below. What do the slopes of each graph represent?



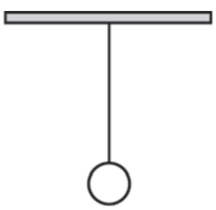
CIRCULAR MOTION 2 (2009 B1b) An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius *r*, while another student measures the period *P*.

a. Derive the equation $P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$ that relates *P* and m_2 .

The procedure is repeated, and the period *P* is determined for four different values of m_2 , where $m_1 = 0.012$ kg and $r = 0.80$ m. The data, which are presented below, can be used to compute an experimental value for *g*.

m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75

b. What quantities should be graphed to yield a straight line with a slope that could be used to determine *g* ?
c. Plot the quantities determined in part **(b),** label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.
d. Use your graph to calculate the experimental value of *g*.

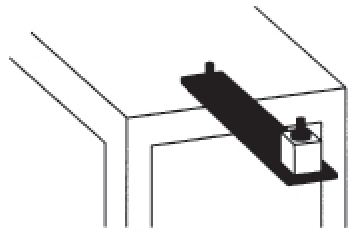


PENDULUM (2010bB2) The simple pendulum above consists of a bob hanging from a light string. You wish to experimentally determine the frequency of the swinging pendulum.
a. By checking the line next to each appropriate item on the list below, select the equipment that you would need to do the experiment.

____ Meterstick ____ Protractor ____ Additional string
____ Stopwatch ____ Photogate ____ Additional masses

b. Describe the experimental procedure that you would use. In your description, state the measurements you would make, how you would use the equipment to make them, and how you would determine the frequency from those measurements.
c. You next wish to discover which parameters of a pendulum affect its frequency. State one parameter that could be varied, describe how you would conduct the experiment, and indicate how you would analyze the data to show whether there is a dependence.
d. After swinging for a long time, the pendulum eventually comes to rest. Assume that the room is perfectly thermally insulated. How will the temperature of the room change while the pendulum comes to rest? Justify your answer.
____ It would slightly increase ____ It would slightly decrease ____ No effect. It would remain the same

e. Another pendulum using a thin, light, metal rod instead of a string is used in a clock to keep time. If the temperature of the room was to increase significantly, what effect, if any, would this have on the period of the pendulum? Justify your answer.
____ It would increase ____ It would decrease ____ No effect. It would remain the same

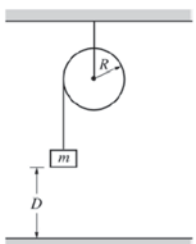


FORCE CONSTANT (1996M1) A thin, flexible metal plate attached at one end to a platform, as shown above, can be used to measure mass. When the free end of the plate is pulled down and released, it vibrates in simple harmonic motion with a period that depends on the mass attached to the plate.
To calibrate the force constant, objects of known mass are attached to the plate and the plate is vibrated, obtaining the data shown.

a. The data obtained in the experiment is shown in the table. Calculate the period.

Mass (kg)	t_{avg} for 10 vibrations (s)	T (s)	
0.10	8.86		
0.20	10.6		
0.30	13.5		
0.40	14.7		
0.50	17.7		

b. Complete the last column in the table by calculating the quantity that needs to be graphed to provide a linear relationship from the data collected.
c. Sketch a graph of the best straight-line fit to the data points.
d. From your graph clearly calculate the force constant of the metal plate.



ROTATIONAL INERTIA (2004M2) A solid disk of unknown mass and known radius ***R*** is used as a pulley in a lab experiment, as shown. A small block of mass ***m*** is attached to a string, the other end of which is attached to the pulley and wrapped around it several times.
The block of mass ***m*** is released from rest and takes a time ***t*** to fall the distance ***D*** to the floor.

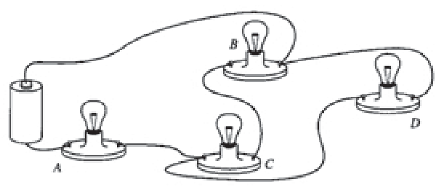
a. Calculate the **linear acceleration** *a* of the falling block in terms of the given quantities.
b. The time *t* is measured for various heights *D* and the data are recorded in the following table.

D (m)	*t* (s)
0.5	0.68
1	1.02
1.5	1.19
2	1.38

i. What quantities should be graphed in order to best determine the **acceleration** of the block? Explain your reasoning.
ii. Plot the quantities determined in **(b) i**., label the axes, and draw the best-fit line to the data.
iii. Use your graph to calculate the magnitude of the **acceleration**.

c. Calculate the **rotational inertia** of the pulley in terms of ***m*, *R*, *a***, and fundamental constants.
d. The value of acceleration found in **(b)iii,** along with numerical values for the given quantities and your answer to **(c),** can be used to determine the rotational inertia of the pulley. The pulley is removed from its support and its rotational inertia is found to be **greater** than this value. Give **one explanation** for this discrepancy.

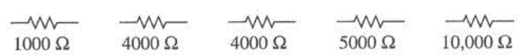
ELECTRICITY



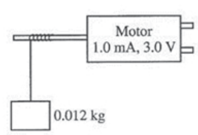
DC CIRCUITS (1998B4) In the circuit shown, A, B. C, and D are identical light bulbs. Assume that the battery maintains a constant potential difference between its terminals (i.e., the internal resistance of the battery is assumed to be negligible) and the resistance of each light bulb remains constant.

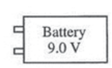
a. Draw a diagram of the circuit in the box below. **Use and label** the resistors symbols as A, B. C, and D to refer to the corresponding light bulbs.
b. List the bulbs in order of brightness, from brightest to least bright. If any two or more bulbs have the same brightness, state which ones. **Justify your answer.**
c. Bulb D is then removed from its socket.
i. Describe the change in the brightness, if any, of bulb A when bulb D is removed from its socket. **Justify your answer.**
ii. Describe the change in the brightness, if any, of bulb B when bulb D is removed from its socket. **Justify your answer.**

CIRCUITS (2003Bb2) A student is asked to design a circuit to supply an electric motor with 1.0 mA of current at 3.0 V potential difference.
a. Determine the power to be supplied to the motor.
b. Determine the electrical energy to be supplied to the motor in 60 s.
c. Operating as designed above, the motor can lift a 0.012 kg mass a distance of 1.0 m in 60 s at constant velocity. Determine the efficiency of the motor.
To operate the motor, the student has available only a 9.0 V battery to use as the power source and the following five resistors.

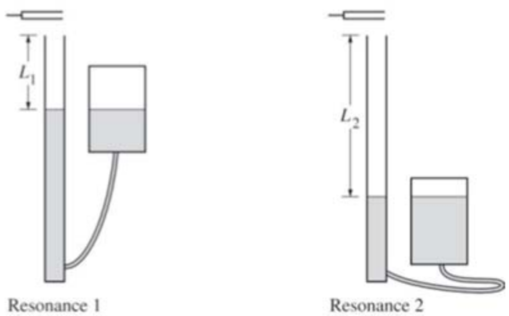


d. In the space below, complete a schematic diagram of a circuit that shows how one or more of these resistors can be connected to the battery and motor so that 1.0 mA of current and 3.0 V of potential difference are supplied to the motor. Be sure to label each resistor in the circuit with the correct value of its resistance.



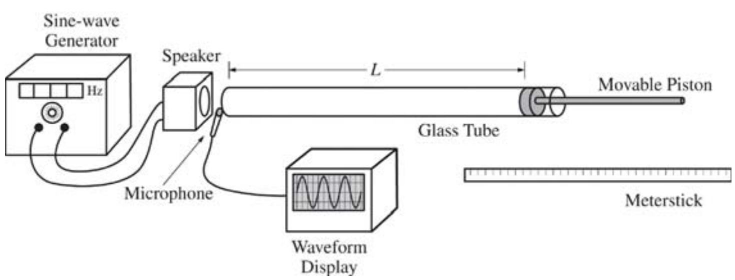


WAVES AND SOUND



SPEED OF SOUND (2004Bb3)
A vibrating tuning fork is held above a column of air, as shown in the diagrams above. The reservoir is raised and lowered to change the water level, and thus the length of the column of air. The shortest length of air column that produces a resonance is L_1 = 0.25 m, and the next resonance is heard when the air column is L_2 = 0.80 m long. The speed of sound in air at 20°C is 343 m/s and the speed of sound in water is 1490 m/s.

a. Calculate the wavelength of the standing sound wave produced by this tuning fork.
b. Calculate the frequency of the tuning fork that produces the standing wave, assuming the air is at 20°C.
c.Calculate the wavelength of the sound waves produced by this tuning fork in the water.
d.The water level is lowered again until a third resonance is heard. Calculate the length L_3 of the air column that produces this third resonance.
e. The student performing this experiment determines that the temperature of the room is actually slightly higher than 20°C. Is the calculation of the frequency in part (b) too high, too low, or still correct? _____Too high _____Too low _____Still correct **Justify your answer.**



SPEED OF SOUND (2012B6) You are given the apparatus represented in the figure. A glass tube is fitted with a movable piston that allows the indicated length L to be adjusted.

A sine-wave generator with an adjustable frequency is connected to a speaker near the open end of the tube. The output of a microphone at the open end is connected to a waveform display. You are to use this apparatus to measure the speed of sound in air.
a. Describe a procedure using the apparatus that would allow you to determine the speed of sound in air. Clearly indicate what quantities you would measure and with what instrument each measurement would be made. Represent each measured quantity with a different symbol.
b. Using the symbols defined in part **(a),** indicate how your measurements can be used to determine an experimental value of the speed of sound.
c. A more accurate experimental value can be obtained by varying one of the measured quantities to obtain multiple sets of data. Indicate one quantity that can be varied, and describe how a graph of the resulting data could be used to determine the speed of sound. Clearly identify independent and dependent variables, and indicate how the slope of the graph relates to the speed of sound.